HW #4 & Self Reflection #3 Due Tue May 7
work on Problem 1 only after Thu Recitation

Graded HW #3 handed back

Midterm 1: Avg 63.5/80  Median 66/80
Hi 80/80  Lo 29/80

Midterm grading petitions due by Thu 5/2

Transfer Function:

Output response = response due to initial
  conditions + response due to the
  external source (input) + zero
  input response + zero state
  response + zero initial
  condition

Focus on the zero state response (zero initial
  condition)

Can relate the output response \( Y(s) \) to the input \( X(s) \)

\[ Y(s) = H(s) \cdot X(s) \]

↑ Transfer function

Ex: \( H(s) = \frac{V_2(s)}{V_1(s)} \)

\[ V_2(s) = \frac{Z_2}{Z_1 + Z_2} \cdot V_1(s) \]

\[ H(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_1 + Z_2} \]

\[ Z_1 = R_1 || \frac{1}{sC_1} \]

\[ Z_2 = R_2 + \frac{1}{sC_2} \]
Analysis Problem: Given a circuit, input
Find the transfer function $H(s)$
$\Rightarrow$ output
$Y(s) = H(s)X(s)$

Design Problem: Given an input and a desired output
"invent" a circuit that gives the desired input-output relation $\Rightarrow H(s)$

$Y(s) = H(s)X(s)$ or $H(s) = \frac{Y(s)}{X(s)}$

These relationships hold only in the $s$-domain.

$\downarrow$ \hspace{1cm} $\downarrow$ \hspace{1cm} $\uparrow$

$y(t)$ \hspace{1cm} $h(t)$ \hspace{1cm} $x(t)$

$y(t) \neq h(t)x(t)$ \hspace{1cm} $h(t) \neq \frac{y(t)}{x(t)}$

$y(t) = h(t)x(t) \quad \text{Convolution}$

\[ y(t) = x(t) * h(t) \Rightarrow \int_{0}^{t} h(t-z) x(z) \, dz \]

Example: $h(t) = 2e^{-t} u(t)$; $x(t) = tu(t)$

$y(t) = \int_{0}^{t} 2e^{-(t-z)} u(t-z) \overbrace{zu(z)} \, dz$

$Y(s) = H(s)X(s) = \frac{2}{s+1} \frac{1}{s^2}$

Suppose $x(t)$ is available as measured data, then convolution is the method to use.

$x(t) = \overbrace{zu(t)}$

$h(t)$

$h(-z) \downarrow \text{reflection}$

$2e^{-z} u(z)$

$h(z)$

$2$

$2$
Revisit the transfer function
\[ H(s) = \frac{Y(s)}{X(s)} \]
\[ Y(s) = H(s) X(s) = \frac{N_1(s)}{D_1(s)} \frac{N_2(s)}{D_2(s)} \]
\[ = \frac{N_1(s)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \frac{N_2(s)}{(s-p_{x1})(s-p_{x2})\cdots(s-p_{xm})} \]

Assume a proper rational function and no repeated poles

Partial Fraction Expansion
\[ Y(s) = \frac{K_1}{s-p_1} + \frac{K_2}{s-p_2} + \cdots + \frac{K_n}{s-p_n} + \frac{Kx_1}{(s-p_{x1})} + \frac{Kx_2}{(s-p_{x2})} + \cdots + \frac{Kx_m}{(s-p_{xm})} \]

\[ Y(t) = \left[ \sum_{j=1}^{n} K_j e^{p_j t} + \sum_{l=1}^{m} \frac{Kx_l}{s-p_{x_l}} e^{p_{x_l} t} \right] u(t) \]

Stability of a Circuit

A linear circuit is stable if its natural response decays to zero as \( t \to \infty \)
\[ \lim_{t \to \infty} \sum_{j=1}^{n} K_j e^{p_j t} = 0 \Rightarrow \lim_{t \to \infty} K_j e^{p_j t} = 0 \]
for all \( j \)
In general, \( p_j \) is a complex \( p_j = \alpha_j + j\beta_j \)
\[
\lim_{t \to \infty} e^{(\alpha_j + j\beta_j)t} \to 0 \quad \text{if} \quad \alpha_j < 0
\]
\[\text{i.e. if } \Re(p_j) < 0\]

A linear circuit is stable if all its natural poles are in the left half s-plane.
\[\text{i.e. the poles of } H(s) \text{ (transfer function)}\]

\[
\begin{align*}
H_1(s) &= \frac{1}{s+1} & \text{pole } s = -1, \text{ stable} \\
H_2(s) &= \frac{1}{s-2} & \text{pole } s = 2, \text{ unstable} \\
H_3(s) &= \frac{1}{s} & \text{pole } s = 0, \text{ unstable}
\end{align*}
\]

**Impulse Response**

\[
[x(t) = \delta(t)] \quad \text{Impulse response}
\]

The zero-state response resulting from a unit impulse applied at the input at \( t=0 \)
\[
Y(s) = H(s) \times (s) = H(s)
\]
\[
y(t) = \mathcal{Z}^{-1}[Y(s)] = \mathcal{Z}^{-1}[H(s)] = h(t)
\]

\[
h(t) = \mathcal{Z}^{-1}[H(s)] = \text{Impulse response}
\]

\[
H(s) = \frac{N_1(s)}{(s-p_1)(s-p_2) \cdots (s-p_n)}
\]
\[
= \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \cdots + \frac{k_n}{s-p_n}
\]

\[
h(t) = \left[ k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_n e^{p_n t} \right] u(t)
\]

For a stable circuit: \( h(t) \to 0 \) as \( t \to \infty \)
\[h(t) = e^t u(t) \quad \text{not stable}\]