1. For the circuit shown **derive** the transfer function \( H(s) = \frac{V_1(s)}{I_1(s)} \) (10 points).

2. For \( H(s) = \frac{s^2 + s + 3}{s^2 + 2^2} \) **find** the impulse response \( h(t) \). (10 points).

3. When the input to a circuit is \( tu(t) \), the output response is \( te^{-3t} u(t) \). **Determine** the transfer function \( H(s) \) of the circuit. (5 points).
<table>
<thead>
<tr>
<th>Type</th>
<th>( f(t) ) ( (t &gt; 0^+) )</th>
<th>( F(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(impulse)</td>
<td>( \delta(t) )</td>
<td>1</td>
</tr>
<tr>
<td>(step)</td>
<td>( u(t) )</td>
<td>( \frac{1}{s} )</td>
</tr>
<tr>
<td>(ramp)</td>
<td>( t )</td>
<td>( \frac{1}{s^2} )</td>
</tr>
<tr>
<td>(exponential)</td>
<td>( e^{-at} )</td>
<td>( \frac{1}{s + a} )</td>
</tr>
<tr>
<td>(sine)</td>
<td>( \sin \omega t )</td>
<td>( \frac{\omega}{s^2 + \omega^2} )</td>
</tr>
<tr>
<td>(cosine)</td>
<td>( \cos \omega t )</td>
<td>( \frac{s}{s^2 + \omega^2} )</td>
</tr>
<tr>
<td>(damped ramp)</td>
<td>( te^{-at} )</td>
<td>( \frac{1}{(s + a)^2} )</td>
</tr>
<tr>
<td>(damped sine)</td>
<td>( e^{-at} \sin \omega t )</td>
<td>( \frac{\omega}{(s + a)^2 + \omega^2} )</td>
</tr>
<tr>
<td>(damped cosine)</td>
<td>( e^{-at} \cos \omega t )</td>
<td>( \frac{s + a}{(s + a)^2 + \omega^2} )</td>
</tr>
</tbody>
</table>

**Ohm’s law in the s-domain** \( V = ZI \)

\( Z \) is the s-domain impedance of the element. A resistor has an impedance of \( R \) ohms, an inductor has an impedance of \( sL \) ohms, and a capacitor has an impedance of \( 1/sC \) ohms.

The admittance is the reciprocal of the impedance \( Y = 1/Z \)

**Transfer function** \( H(s) \) is the ratio of the output (\( Y(s) \)) to the input (\( X(s) \)) for zero initial conditions.

\[
H(s) = \frac{Y(s)}{X(s)} \quad \text{zero initial conditions}
\]

or \( Y(s) = H(s)X(s) \)

**Impulse response** is the output response for a unit impulse (\( \delta(t) \)) as the input.

\[
X(s) = 1 \Rightarrow Y(s) = H(s) \Rightarrow H(s) \text{ is the impulse response transform (s-domain)}
\]

\( h(t) = \mathcal{L}^{-1}[H(s)] \) is the impulse response waveform (t-domain)

**Step response** is the output response for a unit step (\( u(t) \)) as the input.

\[
X(s) = \frac{1}{s} \Rightarrow Y(s) = G(s) = H(s)\frac{1}{s} \quad \text{where } G(s) \text{ is the step response transform (s-domain)}
\]

\( g(t) = \mathcal{L}^{-1}[G(s)] \) is the step response waveform (t-domain)

**Relation between Step and Impulse responses**

\[
G(s) = \frac{H(s)}{s} \quad \text{(s-domain)} \\
\quad \quad \quad g(t) = \frac{1}{0} h(\tau) d\tau \quad \text{(t-domain)}
\]