ENGR 203                      Spring 2018

Test 1 (04/26/2018)

Total # Pages: 5
Total # Problems: 5

Name  SOLUTION

1. (25 points)  
2. (20 points)  
3. (10 points)  
4. (5 points)   
5. (20 points)  

Total (80 points)  

GOOD LUCK
1. For the functions $f(t)$ shown determine the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$. (25 points).

a). $f(t) = tu(t-2) = (t-2+2)u(t-2) = (t-2)u(t-2) + 2u(t-2)$

\[ \therefore F(s) = \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s} \]

b). $f(t) = e^{-(t-2)}\delta(t-1) = e^{-(t-2)}\delta(t-1) = e\delta(t-1)$

\[ \therefore F(s) = e e^{-s} \]

c). $f(t) = -\gamma(t-1) + 2u(t-2) + \gamma(t-3)$

\[ \therefore F(s) = -\frac{e^{-s}}{s^2} + \frac{2e^{-2s}}{s} + \frac{e^{-3s}}{s^2} \]

d). Using the time integration property of the Laplace transform show that $\mathcal{L}\{r(t)\} = 1/s^2$. Recall $r(t)$ is the time integration of the unit step function $u(t)$.

\[ \mathcal{L}\{u(t)\} = \frac{1}{s} \]

\[ \therefore \mathcal{L}\{r(t)\} = \mathcal{L}\left[\frac{u(t)}{s}\right] = \frac{1}{s^2} = \frac{1}{s^2} \]
2. For the functions $F(s)$ shown find the inverse Laplace transform $f(t)$. (20 points).

a). $F(s) = \frac{s}{(s+1)^2 + 1} = \frac{s+1 - 1}{(s+1)^2 + 1} = \frac{s+1}{(s+1)^2 + 1} - \frac{1}{(s+1)^2 + 1}$

\[ f(t) = e^{-t} \cos t \, u(t) - e^{-t} \sin t \, u(t) \]

b). $F(s) = \frac{s+2}{s^2(s+1)} = \frac{K_1}{s+1} + \frac{K_{21}}{s} + \frac{K_{22}}{s^2}$

\[ K_1 = \left. \frac{s+2}{s^2} \right|_{s=-1} = \frac{-1+2}{(-1)^2} = 1 \]

\[ K_{22} = \left. \frac{s+2}{s+1} \right|_{s=0} = 2 \]

\[ K_{21} = \left. \frac{d}{ds} \left( \frac{s+2}{s+1} \right) \right|_{s=0} = \left. \frac{d}{ds} \left( 1 + \frac{1}{s+1} \right) \right|_{s=0} = -\left. \frac{1}{(s+1)^2} \right|_{s=0} = -1 \]

\[ F(s) = \frac{1}{s+1} - \frac{1}{s} + \frac{2}{s^2} \]

\[ f(t) = e^{-t} u(t) - u(t) + 2t u(t) \]
3. For the circuit shown there is no energy stored (zero initial conditions). Derive the expression for the ratio \( V_2(s)/V_1(s) \). (10 points).

\[
V_2(s) = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} V_1(s)
\]

\[
Z_1(s) = s + 1
\]

\[
Z_2(s) = \frac{1}{s} \cdot 1 = \frac{s}{s+1} = \frac{1}{s+1}
\]

\[
V_2(s) = \frac{1}{s+1} \frac{1}{s+1} = \frac{1}{s^2 + 2s + 2}.
\]

4. Indicate the poles and zeros of \( F(s) = \frac{s^2 + 1}{s(s+1)} \) on the pole zero diagram. (5 points).

zeros: \( s = \pm j \)

poles: \( s = 0, s = -1 \)
5. For the circuit shown, $i_L(0^-) = 2$ A (all inductors) and $v_C(0^-) = 4$ V. The reference directions are as shown.

a). **Draw** the circuit in the transform domain from which the node voltages $V_1(s)$ ($\mathcal{L}\{v_1(t)\}$) and $V_2(s)$ ($\mathcal{L}\{v_2(t)\}$) can be determined. **Label** each component. **(10 points)**

![Circuit Diagram]

b). **Write** the node voltage equations that would allow you to solve for $V_1(s)$ and $V_2(s)$. **Do not solve or simplify** any equation. **(10 points)**

\[
\frac{v_1(s) - 7}{s} + \frac{v_1(s)}{2} + \frac{v_1(s) - v_2(s)}{s} + 4 + \frac{2}{s} = 0
\]

\[
\frac{v_2(s) - v_1(s)}{s} + \frac{v_2(s)}{1} + \frac{v_2(s)}{2s} + \frac{2}{s} - \frac{2}{s} = 0
\]