HOMEWORK #7 SOLUTIONS
ECE 464/564

4.23 The poles are given by $p_\ell = e^{j\pi(5+2\ell)/2N}$, $1 \le \ell \le 6$. Hence,

$p_1 = e^{j(7\pi/12)} = -0.2588 + j0.9659$, $p_2 = e^{j(9\pi/12)} = -0.7071 + j0.7071$,

$p_3 = e^{j(9\pi/12)} = -0.9659 + j0.2588$, $p_4 = e^{j(11\pi/12)} = p_3^* = -0.9659 - j0.2588$,

$p_5 = e^{j(13\pi/12)} = p_2^* = -0.7071 - j0.7071$, $p_6 = e^{j(15\pi/12)} = p_1^* = -0.2588 - j0.9659$.

The poles can also be determined in MATLAB using the statement `[z,p,k]=buttap(6)` which yields

```
p =
  -0.2588 + 0.9659i
  -0.2588 - 0.9659i
  -0.7071 + 0.7071i
  -0.7071 - 0.7071i
  -0.9659 + 0.2588i
  -0.9659 - 0.2588i
```

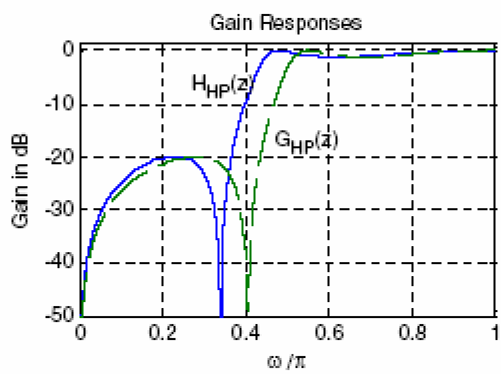
4.25 From the solution of Problem 4.22, we have $\frac{1}{k} = 4$ and $\frac{1}{k_1} = 72.9381$. Hence,

$N = \frac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)} = 2.4151$. We choose the filter order as $N = 3$.

The filter order obtained using the MATLAB statement `[N,Wn]=cheb1ord(2*pi*1500,2*pi*6000,0.25, 25, 's')` results in `N=3`.

9.25 $\hat{\omega}_p = 0.45\pi$, and $\omega_p = 0.52\pi$. $\lambda = \frac{\sin\left(\frac{0.52\pi - 0.45\pi}{2}\right)}{\sin\left(\frac{0.52\pi + 0.45\pi}{2}\right)} = 0.1099.$

$$H_{HP}(z) = G_{HP}(z)\Big|_{z=\frac{\hat{z}^{-1}-0.1099}{1-0.1099\hat{z}^{-1}}} = \frac{2.873 - 5.622\hat{z}^{-1} + 5.629\hat{z}^{-2} - 2.874\hat{z}^{-3}}{9.7623 - 1.5359\hat{z}^{-1} + 6.711\hat{z}^{-2} + \hat{z}^{-3}}.$$



9.27 $\omega_0 = 2\pi\left(\frac{80}{400}\right) = 0.4\pi$, $B_\omega = 2\pi\left(\frac{5}{400}\right) = 0.025\pi$. $\alpha = \frac{1-\tan\left(\frac{B_\omega}{2}\right)}{1+\tan\left(\frac{B_\omega}{2}\right)} = 0.9244$

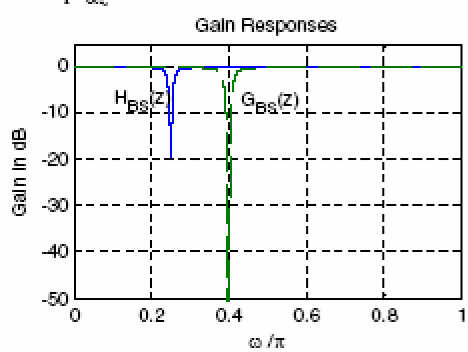
$\beta = \cos(\omega_0) = 0.3090.$

$$G(z) = \frac{1}{2}\frac{(1+\alpha)-2\beta(1+\alpha)z^{-1}+(1+\alpha)z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} = \frac{0.5\left(1.9244-1.1893z^{-1}+1.9244z^{-2}\right)}{1-0.5946z^{-1}+0.9244z^{-2}}$$

$$= \frac{0.9622-0.59465z^{-1}+0.9622z^{-2}}{1-0.5946z^{-1}+0.9244z^{-2}}.$$

$$\alpha = \frac{\sin\left(\frac{\omega_0-\hat{\omega}_0}{2}\right)}{\sin\left(\frac{\omega_0+\hat{\omega}_0}{2}\right)} = 0.2738. \quad \hat{\omega}_0 = \left(\frac{50}{400}\right)2\pi = 0.25\pi.$$

$$H_{BS}(z) = G_{BS}(z)\Big|_{z^{-1}=\frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}} = \frac{1.0301-1.45679\hat{z}^{-1}+1.0301\hat{z}^{-2}}{1.06017-1.45674\hat{z}^{-1}+\hat{z}^{-2}}.$$



M9.1 $F_T = 100\text{ kHz}, \alpha_p = 0.4\text{ dB}, F_p = 10\text{ kHz}, \alpha_s = 50\text{ dB}, \& F_s = 30\text{ kHz},$

$$\omega_p = \frac{2\pi F_p}{F_T} = 0.628, \quad \omega_s = \frac{2\pi F_s}{F_T} = 1.885.$$

Let $T = 2$. $\Omega_p = \tan\left(\frac{\omega_p}{2}\right) = 1.376$, and $\Omega_s = \tan\left(\frac{\omega_s}{2}\right) = 1.376$. Therefore,

$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 4.235$. Now, $20\log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right) = -0.4$. Hence, $\varepsilon^2 = 0.096$.

From $20\log_{10}\left(\frac{1}{A}\right) = -50$ we obtain $A^2 = 100{,}000$. Therefore,

$k_1 = 9.798\times 10^{-4}$, or $\frac{1}{k_1} = 1020.62$. As a result,

$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = 4.80 \to 5$. Next, solving $\left(\frac{\Omega_s}{\Omega_c}\right)^{10} = A^2 - 1 = 99{,}999$ we get

$\Omega_c = \frac{\Omega_s}{99{,}999^{1/10}} = 0.316\Omega_s = 0.435$

Using the M-file `buttap`, we determine the normalized analog Butterworth transfer function of 5[th] order with a 3-dB cutoff frequency at $\Omega_c = 1$, which is:

$$H_{an}(s) = \frac{1}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}.$$

Denormalize $H_{an}(s)$ to move Ω_c to 0.316:

$$H_a(s) = H_{an}\left(\frac{s}{0.435}\right) = \frac{1}{(2.30s+1)(5.28s^2+1.42s+1)(5.28s^2+3.72s+1)}$$

$$= \frac{1}{1+7.44s+27.66s^2+63.58s^3+90.30s^4+64.12s^5}.$$

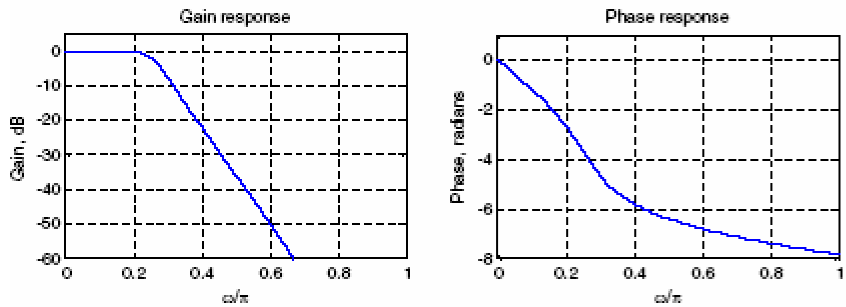
$$G(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.0039+0.0197z^{-1}+0.0394z^{-2}+0.0394z^{-3}+0.0197z^{-4}+0.0039z^{-5}}{1-2.3617z^{-1}+2.6139z^{-2}-1.5492z^{-3}+0.4865z^{-4}-0.0637z^{-5}}.$$

Matlab code is as follows:

```
% Program M9.01
N = 5;
[z, p, k] = buttap(N);
[num, den] = zp2tf(z, p, k);
% s -> s/0.435
den = [64.12 90.30 63.58 27.66 7.44 1];
num = [0 0 0 0 0 1];
% compute z, p, and k
[z, p, k] = tf2zp(num, den);
% perform bilinear transformation with T = 2;
[zd, pd, kd] = bilinear(z, p, k, 1/2);
% get the digital transfer function
[n2, d2] = zp2tf(zd, pd, kd);
% get the frequency response
[h, w] = freqz(n2, d2, 512);
figure(1);
plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, dB');
```

```
title('Gain response');
figure(2);
plot(w/pi, unwrap(angle(h))); grid;
axis([0 1 -8 1]);
xlabel('\omega/\pi'); ylabel('Phase, radians');
title('Phase response');
```

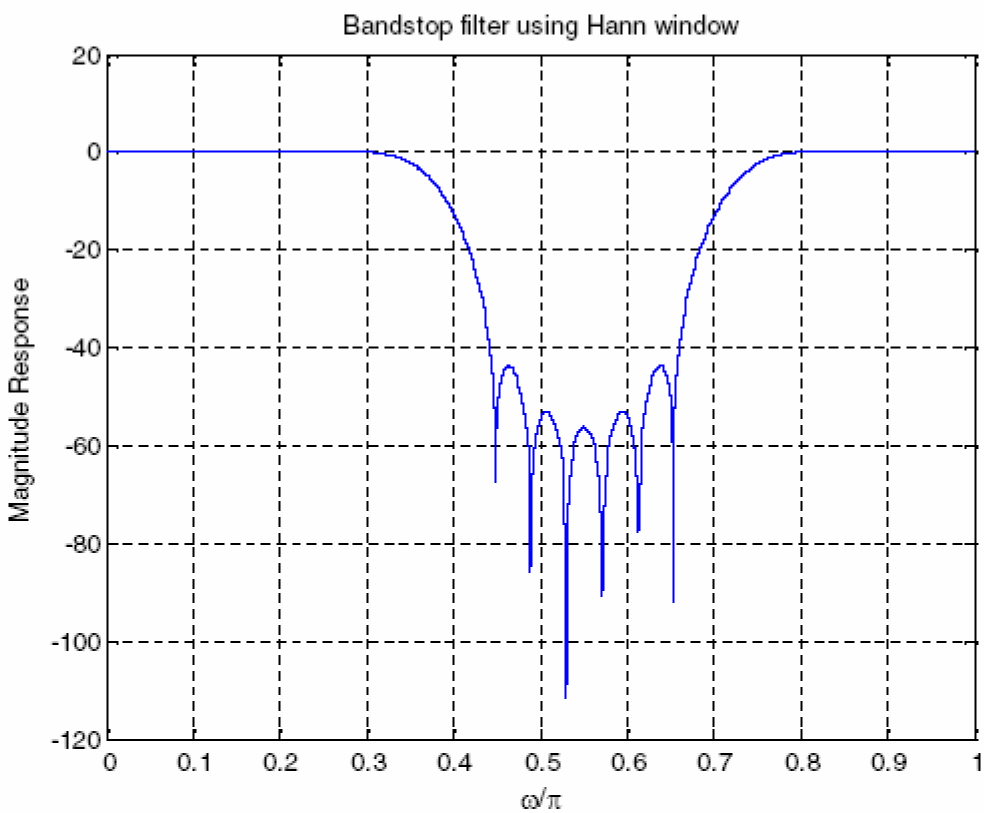


10.17 $\omega_{p1} = 0.3\pi$, $\omega_{p2} = 0.8\pi$, $\omega_{s1} = 0.45\pi$, $\omega_{s2} = 0.65\pi$, $\Delta\omega_1 = \Delta\omega_2 = 0.15\pi$,

$\delta_{p1} = 0.05$, $\delta_{p2} = 0.009$, $\delta_s = 0.02$,

$\alpha_s = -20\log_{10}\delta_s = 34$ dB

From Table 10.2, we see that the Hann window will have minimum length and meet the minimum stopband attenuation.



M10.10 $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$, $\alpha_s = 42$ dB, $\omega_c = 0.5\pi$, $\Delta\omega = 0.2\pi$

We will use the Hann window since it meets the requirements and has the lowest order from Table 10.2.

$$M = \frac{3.11\pi}{\Delta\omega} = 15.55 \rightarrow 16 \Rightarrow N = 32.$$

```
% Problem M10.10
n = -16:16;
lp = 0.5*sinc(0.5*n);wh = hanning(33);
b = lp.*wh';
figure(1);
k=0:2*n;stem(k,b);
title('Impulse Response Coefficients');
xlabel('Time index n');ylabel('Amplitude');
figure(2);
[h, w] = freqz(b,1,512);
plot(w/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi');ylabel('Gain, in dB');
title('Lowpass filter designed using Hann window');
axis([0 1 -80 10]);
```