ECE 464/564 - Digital Signal Processing: Midterm Exam
(Open book and open notes)
February 15, 2007

Problem 1 – An LTI system with impulse response $h_1[n] = (\frac{1}{3})^n \mu[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$

Determine $h_2[n]$.

Problem 2 – Consider the two sequences

$$x[n] = \cos(\tfrac{2\pi n}{N}) \quad \text{and} \quad y[n] = \sin(\tfrac{2\pi n}{N}), \quad 0 \le n \le N-1.$$

(a) Determine the N-point circular convolution $w[n] = x[n] \circledN y[n]$.
(b) Determine the N-point circular cross-correlation between the two sequences defined as $v[n] = x[n] \circledN y^*[\langle -n \rangle_N]$.

Problem 3 – The z-transform of a sequence $x[n]$ is given by

$$X(z) = \frac{z^{20}}{(z - \frac{1}{2})(z-2)^5 (z + \frac{5}{2})^2 (z+3)}.$$

Furthermore, it is known that $X(z)$ converges for $|z| = 1$.
(a) Determine the ROC of $X(z)$.
(b) Determine $x[n]$ at $n = -18$.

Problem 4 – When the input to an LTI system is

$$x[n] = (\tfrac{1}{3})^n \mu[n] + 2^n \mu[-n-1],$$

the corresponding output is

$$y[n] = 5(\tfrac{1}{3})^n \mu[n] - 5(\tfrac{2}{3})^n \mu[n].$$

(a) Find the transfer function $H(z)$ of the system. Plot its pole-zero plot and indicate the ROC.
(b) Find the impulse response $h[n]$ of the system.
(c) Is the system BIBO stable? Is it causal?