ECE 353 Final Exam  
Spring 2006  

Instructions: Please make sure to justify your answers in order to receive any credit for your work. This is a closed-notes, closed-book exam and no electronic aids of any kind are permitted.

1. Consider a double sideband amplitude modulated signal described by  
   \[ X(t) = M(t) \cos(\omega_c t + \Theta), \]
   where \( \omega_c \) is a known constant frequency, \( \Theta \) is a uniformly distributed random variable in the range \((0, 2\pi)\), and the signal \( M(t) \) is a zero mean random process with autocorrelation
   \[ R_M(t_1, t_2) = E\{M(t_1)M(t_2)\} = \frac{\sin^2(10\pi(t_2 - t_1))}{(10\pi(t_2 - t_1))^2}. \]
   Furthermore, at any time \( t \), \( M(t) \) and \( \Theta \) are two statistically independent random variables.
   a) (15 pts.) What is the autocorrelation function of \( X(t) \), i.e., \( R_X(t_1, t_2) = E\{X(t_1)X(t_2)\} \)?
   b) (10 pts.) Is the random process \( X(t) \) wide-sense stationary (WSS)?

2. (25 pts.) Let \( Z = X + Y \), where both \( X \) and \( Y \) are two statistically independent random variables which are uniformly distributed in the interval \((-2, 2)\). What is the probability density function of the random variable \( Z \)?

3. (25 pts.) Consider the following circuit

   ![Circuit Diagram]

   where the resistance \( R \) is a uniformly distributed random variable in the range \((9, 11)\). What is the probability that the current \( I \) is greater than 1 Amp, i.e., what is \( P(I > 1) \)?

4. A discrete random variable \( X \) has probability mass function (PMF) described by
   \[ p_X(x) = \begin{cases} 
   c \left( \frac{1}{2} \right)^x, & x = 0, 1, 2, 3 \\
   0, & \text{otherwise} 
   \end{cases} \]
   a) (15 pts.) What should \( c \) be for \( p_X(x) \) to be a valid PMF?
   b) (10 pts.) What is the mean value of \( X \)?