1. There are \( \binom{30}{6} \) ways 6 resistors can be selected from a set of 30 resistors. In \( \binom{23}{6} \) of them, none of the resistors is defective. Then,

\[
P(\text{he/she finds at least 1 defective resistor}) = 1 - \frac{\binom{23}{6}}{\binom{30}{6}}.
\]

2. The voltage at the output resistor is \( V_o = \frac{V}{6+5} \), \( V = \frac{1}{2} V \). Hence, the power delivered to the output resistor is

\[
P_o = \frac{V_o^2}{5} = \frac{1}{5} \left( \frac{V}{2} \right)^2 = \frac{1}{20} V^2 = g(V)
\]

Note that \( g(V) \) is both monotonically decreasing and increasing. Also, \( \frac{dp_o}{dV} = \frac{dg(V)}{dV} = \frac{1}{10} V \) and \( V = \pm \sqrt{20p_0} = \sqrt[4]{g(p_0)} \). Hence, as the example in class,

\[
f_{p_0}(p_0) = \frac{f_V(V)}{g^{-1}(V)} \bigg|_{V=\sqrt{20p_0}} - \frac{f_V(V)}{g'(V)} \bigg|_{V=-\sqrt{20p_0}}
\]

\[
= \frac{1}{\sqrt{2\pi}} \frac{1}{10} V^2 \bigg|_{V=\sqrt{20p_0}} - \frac{1}{\sqrt{2\pi}} \frac{1}{2} V^2 \bigg|_{V=-\sqrt{20p_0}}
\]

\[
= \frac{10}{\sqrt{2\pi} \sqrt{20p_0}} - \frac{1}{2} (20p_0) + \frac{10}{\sqrt{2\pi} \sqrt{20p_0}} - \frac{1}{2} (20p_0)
\]

\[
= \frac{20}{\sqrt{2\pi} \sqrt{20p_0}} \epsilon - 10p_0, \quad p_0 > 0.
\]

\[
= 0, \quad p_0 < 0.
\]
or \( f_{\rho_0}(\rho_0) = \begin{cases} \sqrt{\frac{2^\rho}{\pi \rho_0}} e^{-10\rho_0}, & \rho_0 \geq 0 \\ 0, & \rho_0 < 0 \end{cases} \)

3. A parity check will not detect an error if and only if the number of bits received incorrectly is even. Thus, the desired probability is

\[
\sum_{i=1}^{\frac{4}{2}} \binom{8}{2^\rho} (1-0.9)^{2^\rho} (0.9)^{8-2^\rho} = \sum_{i=1}^{\frac{4}{2}} \binom{8}{2^\rho} (0.1)^{2^\rho} (0.9)^{8-2^\rho}
\]

4. Since we know the distribution of \( T \), we cannot use Chebyshev's inequality. Thus,

\[
P(1T - E\hat{T} \geq 2\sigma_T) = 1 - P(1T - E\hat{T} < 2\sigma_T)
\]

Now, \( P(1T - E\hat{T} < 2\sigma_T) = P(-2\sigma_T < T - E\hat{T} < 2\sigma_T) \)

Furthermore,

\[
E\hat{T}^2 = \int_{-\infty}^{\infty} f_T(t) dt = \int_{-\infty}^{\infty} t \cdot \lambda e^{-\lambda t} dt = \lambda \int_{0}^{\infty} t e^{-\lambda t} dt
\]

\[
\lambda \left[ -\frac{1}{\lambda} t e^{-\lambda t} \bigg|_{0}^{\infty} + \int_{0}^{\infty} \frac{1}{\lambda} e^{-\lambda t} dt \right]
\]

\[
= -\frac{1}{\lambda} \frac{e^{-\lambda t}}{\lambda} \bigg|_{0}^{\infty} = \frac{1}{\lambda}
\]

Therefore,

\[
P(1T - E\hat{T} \leq 2\sigma_T) = P(\frac{1}{\lambda} - 2\sigma_T < T < \frac{1}{\lambda} + 2\sigma_T)
\]

\[
\int_{\frac{1}{\lambda} - 2\sigma_T}^{\frac{1}{\lambda} + 2\sigma_T} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \bigg|_{\frac{1}{\lambda} - 2\sigma_T}^{\frac{1}{\lambda} + 2\sigma_T} = -[e^{-\lambda (\frac{1}{\lambda} + 2\sigma_T)} - e^{-\lambda (\frac{1}{\lambda} - 2\sigma_T)}]
\]

\[
= e^{1+2\lambda\sigma_T} - e^{-1-2\lambda\sigma_T} = e^{1} \left[ e^{2\lambda\sigma_T} - e^{-2\lambda\sigma_T} \right] = \frac{e^{1}}{2} \left[ e^{2\lambda\sigma_T} - e^{-2\lambda\sigma_T} \right]
\]

\[
= 2 e^{1} \left[ \frac{e^{2\lambda\sigma_T} - e^{-2\lambda\sigma_T}}{2} \right] = 2 e^{1} \sinh(2\lambda\sigma_T)
\]