1. (50 pts.) Let a message signal \( m(t) \) have the following spectral characteristics:

Consider the following modulator

\[ m(t) \]

\[ \cos(2\pi f_c t) \]

\[ \cos(2\pi (f_b + f_c) t) \]

Both high pass and low pass filters have the same cutoff frequency \( f_c \) and \( f_c > f_b \).

a) Mathematically describe the modulator output signal \( s_f(t) \) and sketch its magnitude spectrum.

b) Based on the modulator, construct a demodulator that will recover the message signal \( m(t) \).

2. (50 pts.) Let the received signal in an FM system be described by

\[ s_r(t) = a(t) \cos(\omega_r t + \phi(t)) \]

\[ \phi(t) = k_\omega \int_0^t m(\tau) \, d\tau \]

Let \( a(t) \) vary \textbf{very slowly} compared with \( \phi(t) \). If we use an ideal frequency discriminator to demodulate the received signal, what is its output?