Digital communications set themselves apart from analog communications in that they possess a great deal of flexibility in terms of message signal representation and their ability to detect and/or correct transmission errors. Depending on the frequency band used to transmit the signal through, there are two classes of digital communications, namely, baseband and passband.
Baseband (Lowpass) Communications:

Def. In Pulse Amplitude Modulation (PAM), the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values \( m(nT_s) \) of a continuous-time message signal \( m(t) \). The pulses can be of rectangular form or have some other shape.

Mathematically, PAM is represented by

\[
s_{PAM}(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s),
\]

where

\[
T_s = \text{the sampling period},
\]

\[
m(nT_s) = m(t) \bigg|_{t=nT_s}
\]

\[h(t)\] is the pulse shape (impulse response) whose duration is less than or equal to \( T_s \).

If we use rectangular pulses, then

\[
h(t) = \begin{cases} 
1, & 0 < t < T_d \leq T_s \\
1/2 & t = 0, T_d \\
0, & \text{otherwise}
\end{cases}
\]

\[T_d = \text{duration of rectangular pulse}\]
Def. If the samples $m(nT_s)$ of the message signal are used to vary the duration of the individual pulses of the pulse train that represents the message signal, then we obtain Pulse Duration Modulation (PDM), also known as Pulse width Modulation (PWM).

Def. If the values of $m(nT_s)$ are used to vary the position of a pulse relative to its unmodulated time of occurrence, then we obtain Pulse Position Modulation (PPM).
Pulse train representations of an analog waveform
All three forms of pulse modulation (PAM, PDM, PPM) are of the analog type (in reality, a form between analog and digital). PAM is an intermediate form of modulation used in ASK, PSK, QAM, and PCM. PDM is seldom used in commercial communications applications. PPM, on the other hand, is heavily used in new ultra wideband communications applications.

Let us now consider Pulse Code Modulation (PCM). It is a form of digital source encoding where the pulses that represent a binary word have fixed length and amplitude. In order to generate a digital signal and prior to encoding, the message signal $m(t)$ has to be sampled and quantized.

Discretization process of an analog signal
In the process of quantization, the range of definition of \( m(t) \) is divided into a finite set of values (determined by the number of quantization levels).

Let \( \mathcal{M} = \{ m_k: k = 1, \ldots, L \} \) be the set of decision levels at the input of the quantizer and let \( \mathcal{V} = \{ v_k: k = 1, \ldots, L \} \) be the set of reconstruction values at the output of the quantizer. Then the quantization operation is a memoryless operation described by

\[
v = g(m), \text{ where } g \text{ is a many to 1 transformation (non unique)}
\]

that is, \( v_k \) represents all values of \( m \in I_k \equiv [m_k, m_{k+1}] \). The following figures show two uniform quantizers and a uniformly quantized waveform.

Let \( m = M(\xi) \) with \( \mathbb{E}\{M(\xi)\} = 0 \) and \( q = m - v \). Let the quantizer be symmetric, then \( \mathbb{E}\{V\} = 0 \) and \( Q = M-V \) is called the quantization noise and is such that \( \mathbb{E}\{Q\} = 0 \).

Let \( m \in (-m_{\max}, m_{\max}) \), then for an \( L \)-level uniform quantizer the step size is \( \Delta = 2m_{\max}/L \) and \( q \in [-\Delta/2, \Delta/2] \). Furthermore, if \( L \) is sufficiently large, \( Q \) has a uniform distribution in the interval \( (-\Delta/2, \Delta/2) \).
Uniform quantizers

(a) midtread

(b) midrise
Mathematically,

\[ f_q(q) = \begin{cases} \frac{1}{\Delta}, & q \in \left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right) \\ 0, & \text{otherwise} \end{cases} \]

Hence, \( Var(Q) = E\{Q^2\} = \int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}} q^2 f_q(q) dq = \int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}} q^2 \cdot \frac{1}{\Delta} dq = \frac{1}{\Delta} \frac{q^3}{3} \bigg|_{q=-\frac{\Delta}{2}}^{q=\frac{\Delta}{2}} = \frac{\Delta^2}{12} \).

Let \( R \) be the number of bits/sample or \( R = \log_2(L) \), since \( L = 2^R \). Then \( \Delta = 2m_{\text{max}}/2^R \) and

\[
\text{var}(Q) = \left(\frac{2m_{\text{max}}}{2^R}\right)^2 \left(\frac{1}{12}\right) = \frac{2^{-2R}}{3} \cdot m_{\text{max}}^2 \quad \text{and the output signal-to-noise ratio of the quantizer is}
\]

\[
\text{SNR}_{O,Q} = \frac{\text{average signal power}}{\text{average q-noise power}} = \frac{P_m}{\text{Var}(Q)} = \left(\frac{3P_m}{m_{\text{max}}^2}\right) \cdot 2^{2R}.
\]
Quantized waveform and quantization error
**Example:** Design a 16-level uniform quantizer for an input signal with a dynamic range of ±10 volts and find the quantizer output value and quantizer error for an input signal of amplitude 1.2 volts.

**Solution:** Let us assume a midrise quantizer. For no overload condition, let $16\Delta = 20$, then $\Delta = 1.25$ volts. An input sample of 1.2 volts falls in the range of 0 to 1.25 $\Rightarrow$ the output of the midrise uniform quantizer is $v = \Delta/2 = 0.625$ volts and the quantizer error is $q = m - v = 1.2 - 0.625 = 0.575$ volts.

Once the signal has been quantized, it needs to be encoded for efficient transmission. The most robust form of encoding is binary encoding, that is, the quantized amplitudes are represented using a binary word $R$-bits long.

The next figure shows 5 possible baseband binary encoding strategies (line codes), namely, 5 different ways to represent a binary sequence. The two figures after that show the power spectra of these 5 baseband binary encoding strategies.
Five popular line codes
Power spectra of first 4 line codes
Power spectrum of Manchester line code
A typical baseband PCM system is shown below.

\[ m(t) \rightarrow \text{LPF} \rightarrow m(nT) \rightarrow \text{quantizer} \rightarrow v(nT) \rightarrow \text{encoder} \rightarrow s_{\text{PCM}}(nT) \]

\[ s_{\text{PCM}}(nT) \rightarrow \text{Channel} \rightarrow \tilde{s}_{\text{PCM}}(nT) \rightarrow \text{Regeneration circuit} \rightarrow \text{Decoder} \rightarrow \text{Reconstruction filter} \rightarrow \hat{m}(t) \]

PCM-based baseband digital communication system

where \( \tilde{s}_{\text{PCM}}(nT) \) is the distorted version of \( s_{\text{PCM}}(nT) \) modified by the channel and \( \hat{m}(t) \) is the estimate of \( m(t) \). This configuration assumes that the transmitted pulses have rectangular shape, which may not be the optimum one, since theoretically, they occupy an infinite bandwidth.
Let $f_{s,\text{min}} = 2W_m$ be the minimum sampling rate applied to the message signal $m(t)$. Then $T_{s,\text{max}} = 1/(2W_m)$ and if $R$ bits are used to represent a code word, then the maximum duration of a bit is $T_{b,\text{max}} = T_{s,\text{max}}/R = 1/(2RW_m)$. Now, the required bandwidth for the transmission of the bit stream is $B_{T,\text{min}} = k/(T_{b,\text{max}}) = 2kRW_m$ Hz, $k > 0$. This means that the required transmission bandwidth increases with the number of bits $R$. This is because the larger the $R$, the shorter the $T_{b,\text{max}}$, which implies a larger $B_{T,\text{min}}$. However, this conflicts with improvements in quantization signal-to-noise ratio, since

$$\text{SNR}_{O,Q} = \left(\frac{3P_m}{m^2_{\text{max}}}\right) \cdot 2^{2R}. $$

Therefore, a compromise needs to be made between transmission bandwidth and signal representation accuracy.
**Intersymbol Interference**

Let the transmitted pulse be described by $g(t)$ and let us model a time-dispersive channel by the impulse response $h_c(t)$. Let the baseband communication system be modeled by

$$
\sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b)
$$

Transmission through a band-limited channel and intersymbol interference
where \( a_l = \begin{cases} 1, & \text{if bit } b_l = "1" \\ -1, & \text{if bit } b_l = "0" \end{cases} \), \( g(t) \) is the impulse response of the pulse-shaping filter, \( W(t) \) is the additive thermal (usually white and Gaussian) noise, and \( h_r(t) \) is the impulse response of the receiver filter. At the output of the pulse-shaping filter,

\[
\begin{align*}
 s(t) &= \left( \sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b) \right) * g(t) = \sum_{l=-\infty}^{\infty} a_l g(t - lT_b). \\

\end{align*}
\]

At the input of the receiver filter, the signal is described by

\[
\begin{align*}
 x(t) &= h_c(t) * s(t) + w(t) = x_{\text{ISI}}(t) + w(t) \\
 &= h_c(t) * \sum_{l=-\infty}^{\infty} a_l g(t - lT_b) + w(t) = \sum_{l=-\infty}^{\infty} a_l h_c(t) * g(t - lT_b) + w(t). \\

\end{align*}
\]
At the output of the receiver filter,

\[ y(t) = h_r(t) * x(t) = h_r(t) * x_{ISI}(t) + h_r(t) * w(t) \]

\[ = h_r(t) * \left( \sum_{l=-\infty}^{\infty} a_l h_c(t) * g(t - lT_b) \right) + h_r(t) * w(t) \]

\[ = \sum_{l=-\infty}^{\infty} a_l h_r(t) * h_c(t) * g(t - lT_b) + h_r(t) * w(t) \]

\[ = \mu \sum_{l=-\infty}^{\infty} a_l p(t - lT_b) + n(t) \]

where \( \mu \) is a scale factor such that \( \mu p(t) = h_r(t) * h_c(t) * g(t) \) and \( n(t) = h_r(t) * w(t) \) is colored noise (no longer white).
At the output of the ideal sampler,

\[ y(mT_b) = y(t) \big|_{t=mT_b} = \mu \sum_{l=-\infty}^{\infty} a_l p(mT_b - lT_b) + n(mT_b) \]

\[ = \mu a_m p(0) + \mu \sum_{l=-\infty}^{\infty} a_l p((m-l)T_b) + n(mT_b) \cdot \]

\[ \text{ISI} \]

Let \( y_m = y(mT_b) \), \( p_m = p(mT_b) \) and \( n_m = n(mT_b) \). Then

\[ y_m = \mu \sum_{l=-\infty}^{\infty} a_l p_{m-l} + n_m = \mu \sum_{l=-\infty}^{\infty} p_l a_{m-l} + n_m. \]

Clearly, for \( m \neq l \), \( p_l \) represents the coefficient of intersymbol interference, i.e. the contribution due to the other pulses (bits).
Suppose for the moment that $p_l \begin{cases} = 0, & l < 0 \text{ and } l > v \\ \neq 0, & 0 \leq l \leq v \end{cases}$, then $y_m = \mu \sum_{l=0}^{v} p_l a_{m-l} + n_m$.

Let $u_m = \mu \sum_{l=0}^{v} p_l a_{m-l}$, then $y_m = u_m + n_m$, where $u_m$ is the output of an FIR filter with input sequence $\{a_m\}$. Hence, the time-dispersive channel may be modeled as a discrete-time channel, i.e. in block diagram form,

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Discrete equivalent ISI channel
Let the noise be zero for the moment, then \( y_m = u_m \), or

\[
y_m = \mu \sum_{l=0}^{\nu} p_l a_{m-l} = \mu p_0 a_m + \mu \sum_{l=1}^{\nu} p_l a_{m-l} = u_m.
\]

Therefore, the \( m \)th bit in the sequence is

\[
a_m = \frac{1}{\mu p_0} \left[ u_m - \mu \sum_{l=1}^{\nu} p_l a_{m-l} \right].
\]

When noise is present in the system, define \( y'_m \equiv y_m - \mu \sum_{l=1}^{\nu} p_l \hat{a}_{m-l} \), where \( \hat{a}_{m-l}, l = 1, 2, \ldots, \nu \), are estimates of the previous \( \nu \) bits in the sequence. Explicitly, \( y'_m \) can be written as follows:

\[
y'_m = \mu p_0 a_m + \mu \sum_{l=1}^{\nu} p_l a_{m-l} + n_m - \mu \sum_{l=1}^{\nu} p_l \hat{a}_{m-l} = \mu p_0 a_m + n_m + \mu \sum_{l=1}^{\nu} p_l \left[ a_{m-l} - \hat{a}_{m-l} \right].
\]

If the estimates \( \hat{a}_{m-l}, l = 1, 2, \ldots, \nu \) are correct, then \( y'_m = \mu p_0 a_m + n_m \) and the problem is reduced to decoding the single bit \( a_m \). A negative consequence of this approach is that an erroneous decision can cause a sequence of errors.
Recall that the bit stream \( \{b_k\} \) can be obtained by decoding the instantaneous pulse sequence \( \{a_k\} \). Moreover, under no noise conditions,

\[
y(mT_b) = \mu a_m p(0) + \mu \sum_{l=-\infty}^{\infty} a_l p((m-l)T_b), \; \forall m
\]

Ideally, we would want \( y(mT_b) = \mu p(0) a_m, \; \forall m \). Again, under no noise conditions and assuming an infinite number of interfering pulses,

\[
y(mT_b) = \mu \sum_{l=-\infty}^{\infty} a_l p((m-l)T_b) = \mu \sum_{l=-\infty}^{\infty} p(lT_b) a_{m-l}, \; \forall m,
\]

which implies that the pulse \( p(t) \) must be periodically sampled at \( t = lT_b \), or

\[
p(t) \xleftarrow{t = lT_b} p(lT_b) = p^*(t)
\]

Ideal sampling system

where \( p^*(t) \) is the starred transform of \( p(t) \).
In the frequency domain,

\[ P^*(f) = \frac{1}{T_b} \sum_{l=-\infty}^{\infty} P(f - lf_s) = R_b \sum_{l=-\infty}^{\infty} P(f - lR_b), \quad f_s = \frac{1}{T_b} = R_b, \]

where \( R_b \) is the bit rate in bits/s.

From the sampling theorem,

\[ P^*(f) = \frac{1}{T_b} \sum_{l=-\infty}^{\infty} P(f - lf_s) = \sum_{l=-\infty}^{\infty} p(lT_b) e^{-jl(2\pi f T_b)}. \]

If we design to produce ISI-free communication, then it must be necessary that

\[ p(lT_b) = \begin{cases} 
  p(0), & l = 0 \\
  0, & l \neq 0
\end{cases}. \]

This implies that

\[ P^*(f) = \frac{1}{T_b} \sum_{l=-\infty}^{\infty} P(f - lR_b) = p(0). \]

Equivalently,

\[ \sum_{l=-\infty}^{\infty} P(f - lR_b) = p(0)T_b. \]
If \( p(0) = 1 \), then the condition for zero ISI is

\[
\sum_{l=-\infty}^{\infty} P( f - l R_b ) = T_b, \quad \forall |f| \leq \frac{R_b}{2} \quad \otimes.
\]

**Def.** If equation \( \otimes \) holds, then we say that the Nyquist criterion for distortionless baseband transmission in the absence of noise is satisfied.

Suppose the channel has bandwidth \( W \), then the transfer function of the channel is such that \( H_c(f) = 0, \, |f| > W \) and \( P(f) = 0, \, |f| > W \). Let

\[
Z(f) \equiv \sum_{l=-\infty}^{\infty} P( f - l R_b ).
\]

Consider the following cases:

1. \( R_b > 2W \). In this case, \( Z(f) \) consists of nonoverlapping replicas of \( P(f) \), separated by \( R_b \). Graphically,
Clearly, for all $|f| \leq R_b/2$, there is no $P(f)$ such that $Z(f) = T_b$ and we cannot design an ISI-free communication system.

2. $R_b = 2W$. $Z(f)$ consists of replicas of $P(f)$ that are about to overlap, i.e.
In this case, $Z(f) = T_b$ if and only if $P(f) = \begin{cases} T_b, & |f| \leq W \\ 0, & |f| > W \end{cases}$, or

$$p(t) = \text{sinc}(2Wt) = \text{sinc}(R_b t).$$

For this to happen, $\min \{T_b\} = \frac{1}{2W}$ for zero ISI. The only problem is that this $p(t)$ is noncausal (not physically realizable) and the tails of $p(t)$ decay at the rate of $1/t$. Thus, a mistiming error in sampling $y(t)$ can result in nonzero ISI.

3. $R_b < 2W$. $Z(f)$ consists of overlapping replicas of $P(f)$ separated by $R_b$.

Clearly, when $R_b < 2W$ there exists an infinite number of choices for $P(f)$ such that $Z(f) = T_b$. 
A particular pulse spectrum that satisfies $R_b < 2W$ and has desirable spectral properties (and it is widely used in practice) is the raised cosine spectrum. Mathematically it is described by

$$P(f) = \begin{cases} 
\frac{1}{2W}, & |f| < f_1 \\
\frac{1}{4W} \left[ 1 - \sin \left( \frac{\pi (|f| - W)}{2W - 2f_1} \right) \right], & f_1 \le |f| < 2W - f_1 \\
0, & |f| \ge 2W - f_1
\end{cases}$$

Alternatively, if we want to control the rate of decay of the spectral components beyond a certain band of frequencies, we can introduce a rolloff factor $\beta$, i.e.

$$P(f) = \begin{cases} 
\frac{1}{2W}, & |f| < (1 - \beta)W \\
\frac{1}{4W} \left[ 1 - \sin \left( \frac{\pi (|f| - W)}{2\beta W} \right) \right], & (1 - \beta)W \le |f| < (1 + \beta)W \\
0, & |f| \ge (1 + \beta)W
\end{cases}$$
where $\beta$ is the rolloff factor, $\beta \in [0, 1]$. $\beta$ determines the excess bandwidth over the ideal solution $W$ and the steepness of the rolloff of the pulse spectrum. The maximum pulse bandwidth is $B_p = (1 + \beta)W$.

Raised cosine spectra for several values of $\beta$. 

Raised cosine spectra for several values of $\beta$
The pulse shape in the time domain is \( p(t) = \text{sinc}(2Wt) \cdot \left[ \frac{\cos(2\pi\beta Wt)}{1 - 16\beta^2 W^2 t^2} \right] \). Note that the tails of this \( p(t) \) decay at the rate of \( 1/|t|^3 \) and the mistiming errors are considerably smaller than in the case of a pure sinc pulse.

Time domain pulses generated by raised cosine spectra for several values of \( \beta \)
Example: What is the required absolute bandwidth to transmit an information rate of 8 kbits/s using 64-level baseband signaling over a raised cosine channel with a rolloff factor $\beta$ of 40%?

Solution: Let $N_b$ be the number of bits per symbol and $R_s$ be the number of symbols/s. Then $R_b = N_b \cdot R_s$ and $R_s = R_b / N_b$, i.e.

$$R_s = \frac{8 \times 10^3 \text{ (bits/s)}}{\log_2 (64) \text{ (bits/symbol)}} = 1333.3 \text{ symbols/s}.$$
Therefore, $B_T = (1 + \beta) \left( \frac{R_s}{2} \right) = (1 + 0.4) \left( \frac{1333.3}{2} \right) = 933.1 \text{Hz.}$

**PARTIAL RESPONSE SIGNALING**

Consider the following communication system transmitter

![Diagram of Baseband communications system with partial response signaling]

Baseband communications system with partial response signaling
The block diagram variables are described by

\[ v_i(t) = \sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b) \]

\[ s(t) = v_i(t) + v_i(t - T_b) = \sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b) + \sum_{l=-\infty}^{\infty} a_l \delta(t - T_b - lT_b) \]

\[ = \sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b) + \sum_{k=-\infty}^{\infty} a_{k-1} \delta(t - kT_b) = \sum_{l=-\infty}^{\infty} [a_l + a_{l-1}] \delta(t - lT_b) \]

Now,

\[ H_T(f) = 1 + e^{-j2\pi fT_b} = e^{-j\pi fT_b} \left[ \frac{e^{j\pi fT_b} + e^{-j\pi fT_b}}{2} \right] = 2e^{-j\pi fT_b} \cos(\pi fT_b). \]

Finally,

\[ H_I(f) = H_{Nyquist}(f)H_T(f) = 2H_{Nyquist}(f) \cdot \cos(\pi fT_b)e^{-j\pi fT_b}. \]
The transfer function $H_I(f)$ is called a partial response filter because its impulse response covers more than 1 bit.

Let $H_{Nyquist}(f)$ be ideal with bandwidth $W = R_b/2 = 1/(2T_b)$, namely,

$$H_{Nyquist}(f) = \begin{cases} 
1, & |f| \leq R_b / 2 \\
0, & \text{otherwise} 
\end{cases}.$$ 

Then $H_I(f) = \begin{cases} 
2\cos(\pi fT_b)e^{-j\pi fT_b}, & |f| \leq R_b / 2 \\
0, & \text{otherwise} 
\end{cases}.$

The magnitude and phase responses of $H_I(f)$ is shown in the next figure. Note that the filter has linear phase.
Magnitude response of $H_I(f)$
Phase response of $H_I(f)$
The resulting system is called a type I system. Its impulse response is described by

\[
h_I(t) = \mathcal{F}^{-1}\{H_I(f)\} = \int_{-R_b/2}^{R_b/2} \left[ 1 + e^{-2\pi f T_b} \right] e^{j2\pi ft} df
\]

\[
= \frac{1}{j2\pi t} e^{j2\pi f t} \left| \begin{array}{c}
f = R_b / 2 \\
f = -R_b / 2
\end{array} \right. + \frac{1}{j2\pi} e^{j2\pi f(t-T_b)} \left| \begin{array}{c}
f = R_b / 2 \\
f = -R_b / 2
\end{array} \right.
\]

\[
= \frac{\sin \left( \frac{\pi t}{T_b} \right)}{\pi t} + \frac{\sin \left( \frac{\pi (t-T_b)}{T_b} \right)}{\pi (t-T_b)} \quad - \frac{\sin \left( \frac{\pi t}{T_b} \right)}{\pi t} - \frac{\sin \left( \frac{\pi (t-T_b)}{T_b} \right)}{\pi (t-T_b)}
\]

\[
= \frac{\sin \left( \frac{\pi t}{T_b} \right)}{\pi} \left[ \frac{1}{t} - \frac{1}{t-T_b} \right] = \frac{\sin \left( \frac{\pi t}{T_b} \right)}{\pi t} \left[ - \frac{T_b}{t(t-T_b)} \right] = T_b \frac{\sin \left( \frac{\pi t}{T_b} \right)}{\pi t(T_b-t)}
\]
Plot of $h_I(t)$
**Remark:** It is clear that the response to an input pulse is spread over more than one bit period. Hence the response in any signaling interval is only “partial”.

Note that this partial response pulse has tails that decay at the rate of $1/|t|^2$, which is faster than a regular sinc pulse.

Now, $c(t) = \left( \sum_{l=-\infty}^{\infty} a_l \delta(t - lT_b) \right) * h_l(t) = \sum_{l=-\infty}^{\infty} a_l h_l(t - lT_b)$.

At the sampling instant $t = kT_b$,

$$c(kT_b) = c(t) \bigg|_{t=kT_b} = \sum_{l=-\infty}^{\infty} a_l h_l(kT_b - lT_b) = \sum_{l=-\infty}^{\infty} a_l h_l((k-l)T_b) = \sum_{l=-\infty}^{\infty} a_{k-l} h_l(lT_b)$$

$$= a_k h_l(0) + a_{k-1} h_l(T_b) = \frac{1}{T_b} [a_k + a_{k-1}]$$

Clearly, if we knew $a_{k-1}$, then $a_k = T_b c(kT_b) - a_{k-1}$.
Note that this is similar to decision feedback without noise. Because of this similarity, a decision error can cause a sequence of errors.

To avoid dependency on previous decision values, consider the following precoding of the input bit sequence \( \{b_k\} \):

Partial response signaling with precoding
The precoder operation is described by

\[ d_k = b_k \oplus d_{k-1} = \begin{cases} 
"1", & \text{if either } b_k \text{ or } d_{k-1} \text{ is } "1", \text{ but not both} \\
"0", & \text{otherwise}
\end{cases} \]

The ideal PAM maps a “0” to -1 and a “1” to 1. As before, \( c(kT_b) = \frac{1}{T_b} [a_k + a_{k-1}] \).

Let us now analyze the effect of the precoder on the decoding process.

If \( d_{k-1} = "0", \) or \( a_{k-1} = -1 \), then \( d_k = \begin{cases} 
"1" \oplus "0" = "1", & \text{or } a_k = 1 \\
"0" \oplus "0" = "0", & \text{or } a_k = -1
\end{cases} \) and

\[
c(kT_b) = \begin{cases} 
\frac{1}{T_b} [1-1] = 0, & \text{or } b_k = "1" \\
\frac{1}{T_b} [-1-1] = \frac{-2}{T_b}, & \text{or } b_k = "0"
\end{cases}.
\]
If \( d_{k-1} = "1" \) or \( a_{k-1} = 1 \), then \( d_k = \begin{cases} "1" \oplus "1" = "0", & \text{or } a_k = -1 \\ "0" \oplus "1" = "1", & \text{or } a_k = 1 \end{cases} \)
and

\[
c(kT_b) = \begin{cases} \frac{1}{T_b}[-1+1] = 0, & \text{or } b_k = "1" \\ \frac{1}{T_b}[1+1] = \frac{2}{T_b}, & \text{or } b_k = "0" \end{cases}.
\]

Hence, \( c(kT_b) = \begin{cases} 0, & \text{if } b_k = "1" \\ \pm \frac{2}{T_b}, & \text{if } b_k = "0" \end{cases} \).

A simple decoding strategy that does not depend on prior decisions is

If \( |T_b c(kT_b)| < 1 \), then \( b_k = "1" \) was transmitted

If \( |T_b c(kT_b)| > 1 \), then \( b_k = "0" \) was transmitted

If \( |T_b c(kT_b)| = 1 \), then randomly choose either \( b_k = "1" \) or \( b_k = "0" \).
The only drawback with the selected \( H_i(f) \) is that it contains a nonzero dc component. Other modifications of this partial signaling scheme exist.

**Eye diagram.**

An important parameter in the design of modulation scheme is the “eye diagram”. It is a tool that can be used to study the effects of channel imperfections and timing errors on a pulse train. An eye diagram is generated by the superposition of the possible \( T_b \)-second transitions of a long random sequence of pulses. It can be visualized in a scope by applying a baseband signal to the vertical input of the scope and triggering the time base of the scope at the same sampling rate \( 1/T_b \) as that of the incoming pulses. This way a sweep that lasts exactly \( T_b \) seconds is achieved (the same as the symbol duration). What appears on the scope is the input signal to the vertical channel cut up every \( T_b \) seconds and then superimposed on top of one another. The resulting pattern looks like a human eye, hence its name.
The information born by the eye diagram is

1) The width of the eye opening defines the time interval over which the received signal can be sampled without error from intersymbol interference.
2) The sensitivity of the communication system to timing errors is determined by the rate of closure of the eye as the sampling is varied.
3) The height of the eye opening, at a specified sampling time, defines the noise margin of the system.
Part (a) of the second figure shows the eye diagram of a Nyquist raised cosine pulse (RC) for the case of no channel impairments. Part (b) shows the effect of timing errors at the transmitter (the receiver can still make correct decisions (the eye is still open). Part (c) shows the effect of adding WGN to the RC pulse.
Example of a real life eye diagram