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Finite element reliability analysis of bridge girders considering moment–shear interaction

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ABSTRACT

Reliability analysis is necessary in bridge design to determine which parameters have the most significant influence on the structural response to applied loadings. To support finite element reliability applications, analytical response sensitivities are derived with respect to uncertain material properties, girder dimensions, reinforcing details, and moving loads by the direct differentiation method (DDM). The resulting expressions have been implemented in the general finite element framework OpenSees which is well suited to the moving load analysis of bridges. Numerical examples verify the DDM response sensitivity equations are correct, then a first-order reliability analysis shows the effect uncertain parameters have on the interaction of negative moment and shear force near the supports of a continuous reinforced concrete bridge girder. A unique contribution is the treatment of moment–shear interaction using Lamé curves with foci calculated from MCFT equations. In addition, the analysis demonstrates non-seismic bridge engineering applications that have been developed in the OpenSees framework.

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1. Introduction

The prediction of structural performance and modeling the response of girder members under moving vehicle loads are essential in bridge design. Modeling assumptions and natural randomness in material properties, geometry, and loading make the girder response uncertain. This uncertainty is taken into account by load and resistance factors [1]; however, these aggregate factors do not indicate how the bridge response will change as a function of changes in individual parameters that may be of interest to a designer. Reliability analysis is required to assess the effect parameter variations will have on bridge response and to determine which parameters control the response. Repeated analyses with perturbed parameters lead to the response sensitivity; however, when there is a large number of parameters, this approach can be computationally intense [2].

Several researchers have used reliability methods based on Monte Carlo simulation as an assessment tool for highway bridges [3–5]. First- and second-order reliability methods (FORM and SORM) represent alternative approaches to probabilistic assessment. In these methods, it is necessary to find the most probable failure point by solving a constrained optimization problem. Several algorithms are available to solve such problems and their common characteristic is the need to compute the gradient of the

structural response, or response sensitivity, in order to find the failure point. When finite element analysis is used to evaluate the performance function for reliability methods, it is often difficult to implement the software that is necessary to compute gradients of the finite element response.

Most gradient-based finite element software instead rely on finite difference calculations where the analysis is called repeatedly for every realization of the uncertain parameters. In addition to the computational inefficiency of repeated analyses, this approach can lead to inaccurate search directions depending on the size of the parameter perturbations. A more accurate and efficient approach to evaluate gradients in reliability analysis is the direct differentiation method (DDM), which is based on the exact differentiation of the equations that govern the structural response [6]. The response sensitivity equations are implemented alongside the ordinary finite element response equations and are computed at the same precision rate without repeated analyses.

The development of the finite element software framework OpenSees [7] represents one of the first attempts to characterize all major sources of uncertainty in finite element analysis and to compute analytic response sensitivity using an object-oriented approach [8,9]. OpenSees was developed for earthquake engineering applications and several researchers have used the framework to assess the seismic response of bridges. The OpenSees framework is suitable to the repetitive nature of moving load analysis since users build and analyze models via commands added to the fully programmable Tcl scripting language [10]. As a result, OpenSees

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is suited to developing applications for moving load reliability analysis.

The objective of this paper is to use the well-established response sensitivity modules of OpenSees to assess the reliability of bridge girders subjected to moving loads. The presentation begins with a derivation of the sensitivity formulation for material properties, section dimensions, reinforcement details, and moving load parameters in bridge girders. The DDM approach for moving loads is verified by comparison with finite difference calculations and a first-order reliability analysis of bridge girder moment–shear interaction concludes the paper. In the reliability analysis, a third-order Lamé curve whose foci are determined from MCFT equations represents the limit state function for moment–shear interaction.

2. Governing response sensitivity equations

Response sensitivity calculations by the DDM consist of analytical differentiation of the equations that govern the structural response. In this study, the structural response is found by solving the equations of static equilibrium. Impact factors approximate dynamic load effects. The equilibrium equations are described in terms of the vector, Θ , which contains the uncertain material, geometric and load parameters of a structural model

$$\mathbf{P}_r(\mathbf{U}(\Theta), \Theta) = \mathbf{P}_f(\Theta) \quad (1)$$

The nodal displacement vector, $\mathbf{U}(\Theta)$, depends on the parameters, Θ , and load history. The resisting force vector, \mathbf{P}_r , which is assembled from element contributions by standard finite element procedures, depends on the structural parameters explicitly, as well as implicitly via the nodal displacements. The vector, \mathbf{P}_f , contains nodal loads, which also may depend on the parameters in Θ .

Considering the chain rule of differentiation, the derivative of Eq. (1) with respect to a single parameter, θ , in Θ , is:

$$\mathbf{K}_T \frac{\partial \mathbf{U}}{\partial \theta} + \frac{\partial \mathbf{P}_r}{\partial \theta} \Big|_{\mathbf{U}} = \frac{\partial \mathbf{P}_f}{\partial \theta} \quad (2)$$

where the tangent stiffness matrix, $\mathbf{K}_T = \partial \mathbf{P}_r / \partial \mathbf{U}$, is the partial derivative of the resisting force vector with respect to the nodal displacements. The derivative of the nodal load vector, $\partial \mathbf{P}_f / \partial \theta$, is non-zero only if the parameter, θ , represents a nodal load. The vector, $\partial \mathbf{P}_r / \partial \theta|_{\mathbf{U}}$, is the conditional derivative of the resisting force vector under the condition that the nodal displacements \mathbf{U} are held fixed. This vector is assembled from the conditional derivative of local forces, $\partial \mathbf{q} / \partial \theta|_{\mathbf{v}}$, from each element in the structural model in the same manner as the resisting force vector itself. The nodal response sensitivity is then found by solving the following system of linear equations:

$$\frac{\partial \mathbf{U}}{\partial \theta} = \mathbf{K}_T^{-1} \left(\frac{\partial \mathbf{P}_f}{\partial \theta} - \frac{\partial \mathbf{P}_r}{\partial \theta} \Big|_{\mathbf{U}} \right) \quad (3)$$

This solution is repeated for each parameter in the vector Θ , reusing the factorization of \mathbf{K}_T . Full details of the DDM equation assembly and solution procedures are given in [11], including the recovery of other response derivatives from the nodal solution in Eq. (3).

3. Bridge girder modeling approach

In a general finite element setting, the most common approach to compute the moment and shear response of bridge girders is to subdivide each span into multiple elements with nodes corresponding to critical locations. Moving loads are taken into account as statically equivalent nodal forces and the bending moment and shear force at each critical location are determined from rigid body equilibrium at the element ends.

An alternative approach is taken in this study, where each span is considered as one force-based element [12] whose integration points coincide with critical locations. Using this integration approach, it is straightforward to link bending moment and shear forces to a constitutive model rather than relying on rigid body equilibrium [13]. Furthermore, moving loads are taken into account as part of the element, rather than nodal, equilibrium equations. The force-based formulation and its associated response sensitivity are described in the remainder of this section.

3.1. Force-based element formulation

Force-based beam elements are formulated in terms of vectors, $\mathbf{q} = [M_I, M_J]^T$ and $\mathbf{v} = [\theta_I, \theta_J]^T$, that represent the end moments and end rotations, respectively, of the beam, as shown in Fig. 1. At every section along the element, there is a bending moment and shear force, $\mathbf{s}(x) = [M(x), V(x)]^T$, and the corresponding curvature and shear deformation, $\mathbf{e}(x) = [\kappa(x), \gamma(x)]^T$. Without loss of generality, axial effects are omitted.

Equilibrium between section forces, basic forces, and moving loads is satisfied in strong form:

$$\mathbf{s}(x) = \mathbf{b}(x)\mathbf{q} + \mathbf{s}_p(x) \quad (4)$$

The matrix, \mathbf{b} , contains interpolation functions for the moment and shear forces along the beam.

$$\mathbf{b}(x) = \begin{bmatrix} x/L - 1 & x/L \\ 1/L & 1/L \end{bmatrix} \quad (5)$$

The vector, \mathbf{s}_p , in Eq. (4) describes the section forces due to member loads. For the case of a moving point load, this vector is described in terms of the location and magnitude of the load in the statically determinate basic system. Since moving loads are considered part of the element equilibrium equations in the force-based formulation, they are taken into account in \mathbf{P}_r and $\partial \mathbf{P}_r / \partial \theta|_{\mathbf{U}}$ rather than \mathbf{P}_f and $\partial \mathbf{P}_f / \partial \theta$ when assembling Eqs. (1) and (3), respectively.

Based on the principle of virtual forces, the element deformations, \mathbf{v} , are obtained in terms of section deformations, \mathbf{e} , along the element.

$$\mathbf{v} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{e}_j w_j \quad (6)$$

where $\mathbf{b}_j \equiv \mathbf{b}(x_j)$ and $\mathbf{e}_j \equiv \mathbf{e}(x_j)$ are the interpolation function and the deformation evaluated at the j th section along the element, with location, x_j , and integration weight, w_j .

The element flexibility matrix is obtained by linearization of Eq. (6) with respect to basic forces:

$$\mathbf{f} = \frac{\partial \mathbf{v}}{\partial \mathbf{q}} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{f}_s \mathbf{b}_j w_j \quad (7)$$

where \mathbf{f}_s is the section flexibility matrix. The flexibility matrix in Eq. (7) is inverted to give the element stiffness matrix, $\mathbf{k} = \mathbf{f}^{-1}$, for subsequent assembly in the tangent stiffness matrix, \mathbf{K}_T , of Eq. (2). Full details of the force-based element implementation are given in [14].

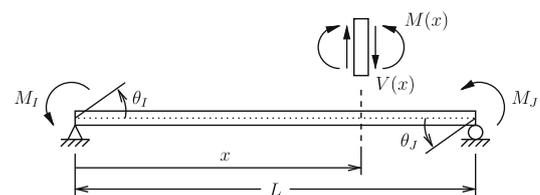


Fig. 1. Simply supported basic system for beam finite elements.

3.2. Force-based element response sensitivity

Response sensitivity equations have been derived for force-based elements using the direct differentiation method by [15,16]. Following the former derivation, the section equilibrium relationship of Eq. (4) is differentiated with respect to a single parameter, θ :

$$\frac{\partial \mathbf{s}}{\partial \theta} = \mathbf{b} \frac{\partial \mathbf{q}}{\partial \theta} + \frac{\partial \mathbf{s}_p}{\partial \theta} \quad (8)$$

The derivative of the force interpolation matrix, \mathbf{b} , is assumed to be equal to zero. Eq. (8) is expanded in terms of the derivatives $\partial \mathbf{q} / \partial \theta = \mathbf{k} \partial \mathbf{v} / \partial \theta + \partial \mathbf{q} / \partial \theta|_{\mathbf{v}}$ and $\partial \mathbf{s} / \partial \theta = \mathbf{k}_s \partial \mathbf{e} / \partial \theta + \partial \mathbf{s} / \partial \theta|_{\mathbf{e}}$ of the basic and section forces, respectively:

$$\mathbf{k}_s \frac{\partial \mathbf{e}}{\partial \theta} + \frac{\partial \mathbf{s}}{\partial \theta}|_{\mathbf{e}} = \mathbf{b} \left(\mathbf{k} \frac{\partial \mathbf{v}}{\partial \theta} + \frac{\partial \mathbf{q}}{\partial \theta}|_{\mathbf{v}} \right) + \frac{\partial \mathbf{s}_p}{\partial \theta} \quad (9)$$

where $\mathbf{k}_s = \partial \mathbf{s} / \partial \mathbf{e}$ and $\mathbf{k} = \partial \mathbf{q} / \partial \mathbf{v}$ define the section and element stiffness matrix, respectively. The conditional derivative $\partial \mathbf{q} / \partial \theta|_{\mathbf{v}}$ cannot be obtained directly from Eq. (9). To circumvent this restriction, the derivative of the element compatibility relationship in Eq. (6) is differentiated with respect to θ :

$$\frac{\partial \mathbf{v}}{\partial \theta} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \frac{\partial \mathbf{e}_j}{\partial \theta} w_j \quad (10)$$

Then, the derivative of the section deformations is obtained from Eq. (9)

$$\frac{\partial \mathbf{e}}{\partial \theta} = \mathbf{f}_s \mathbf{b} \mathbf{k} \frac{\partial \mathbf{v}}{\partial \theta} + \mathbf{f}_s \left(\mathbf{b} \frac{\partial \mathbf{q}}{\partial \theta}|_{\mathbf{v}} + \frac{\partial \mathbf{s}_p}{\partial \theta} - \frac{\partial \mathbf{s}}{\partial \theta}|_{\mathbf{e}} \right) \quad (11)$$

and combined with Eq. (10) to give the following expression:

$$\frac{\partial \mathbf{v}}{\partial \theta} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{f}_{s_j} \left(\mathbf{b}_j \mathbf{k} \frac{\partial \mathbf{v}}{\partial \theta} + \mathbf{b}_j \frac{\partial \mathbf{q}}{\partial \theta}|_{\mathbf{v}} + \frac{\partial \mathbf{s}_{p_j}}{\partial \theta} - \frac{\partial \mathbf{s}_j}{\partial \theta}|_{\mathbf{e}} \right) w_j \quad (12)$$

From the definition of the element flexibility matrix in Eq. (7), the expression $(\sum \mathbf{b}^T \mathbf{f}_s \mathbf{b} w) \mathbf{k}$ is equal to the identity, and the conditional derivative of the basic forces can be reduced to

$$\frac{\partial \mathbf{q}}{\partial \theta}|_{\mathbf{v}} = \mathbf{k} \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{f}_{s_j} \left(\frac{\partial \mathbf{s}_j}{\partial \theta}|_{\mathbf{e}_j} - \frac{\partial \mathbf{s}_{p_j}}{\partial \theta} \right) w_j \quad (13)$$

The section force gradient, $\partial \mathbf{s}_p / \partial \theta$, is non-zero when θ corresponds to either the load magnitude or location, e.g., for uncertain axle spacing. These derivatives of moment and shear are obtained by analytic differentiation of the internal moment and shear functions of a simply supported beam with a point load. The conditional derivative, $\partial \mathbf{s} / \partial \theta|_{\mathbf{e}}$, depends on how the section forces are computed, as described in the following section.

4. Section response sensitivity

There are a variety of approaches to compute the forces at each girder cross-section: elastic constants, closed-form solutions for a particular reinforcing pattern, and fiber discretizations. To facilitate DDM computations for a wide array of longitudinal reinforcing details and material properties, a fiber discretization is employed to compute the section bending moment. The numerical integral is evaluated over a user-defined number of fibers, N_f , with area, A_k , and distance, y_k , from a reference axis. The fiber stress, σ_k , is computed from the fiber strain, which is a function of the section curvature, $\varepsilon_k = y_k \kappa$. The section shear force is assumed to be linear-elastic, as described by the shear modulus, G , and shear area, F . The bending moment and shear force are combined to form the section force vector:

$$\mathbf{s} = \begin{bmatrix} M(x) \\ V(x) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N_f} y_k \sigma_k A_k \\ G(x) F(x) \gamma(x) \end{bmatrix} \quad (14)$$

The aggregation of section forces in Eq. (14) highlights the ease with which shear deformation is included in force-based elements [17]. Approximate displacement fields are not necessary, as is the case with a Timoshenko formulation of combined flexural and shear response.

It can easily be shown that the derivative of Eq. (14) under the condition of fixed section deformations is equal to:

$$\frac{\partial \mathbf{s}}{\partial \theta}|_{\mathbf{e}} = \begin{bmatrix} \sum_{k=1}^{N_f} \left[\frac{\partial y_k}{\partial \theta} \sigma_k A_k + y_k \frac{\partial \sigma_k}{\partial \theta} A_k + y_k \sigma_k \frac{\partial A_k}{\partial \theta} \right] \\ \left(\frac{\partial G}{\partial \theta} F + G \frac{\partial F}{\partial \theta} \right) \gamma \end{bmatrix} \quad (15)$$

For a single parameter, θ , most derivatives in Eq. (15) will be equal to zero. For example, when θ corresponds to the width or depth of a section, $\partial y_k / \partial \theta$, $\partial A_k / \partial \theta$, and $\partial F / \partial \theta$ will be non-zero while all other derivatives with respect to θ in Eq. (15) will be zero. Similar conclusions are drawn for the case where θ corresponds to reinforcing details and material properties.

5. Numerical examples

To demonstrate the application of response sensitivity analysis in assessing uncertainty, numerical examples are presented for an interior girder of the McKenzie River Bridge, which carries the northbound lanes of Interstate-5 just north of Eugene, OR. Each span of this reinforced concrete deck girder (RCDG) bridge is 15.2 m long, as shown in Fig. 2. The depth, d , of the interior girder is uniform at 122 cm while the girder width, b , is tapered from 33 cm at quarter spans to 50 cm at the continuous supports.

For the two-dimensional analyses presented herein, each span is described as a single force-based element with integration points that correspond to critical locations at midspan and at distances d , $2d$, and $3d$ from the supports:

$$x = \{1.22, 2.44, 3.66, 7.60, 11.6, 12.8, 14.0\} \text{ m} \quad (16)$$

These locations dictate where section forces, \mathbf{s} , and the corresponding response sensitivity, $\partial \mathbf{s} / \partial \theta|_{\mathbf{e}}$, are evaluated during the analysis. The associated integration weights are computed from the average distance between adjacent integration points:

$$w = \{1.83, 1.22, 2.58, 4.00, 2.58, 1.22, 1.83\} \text{ m} \quad (17)$$

There is little advantage to using high order integration methods, such as Gauss-Lobatto, in moving load analysis of force-based elements [13]. Accordingly, this low order approach is sufficient for the following set of numerical examples. Section dimensions, reinforcing details, and material properties for the critical sections are listed in Fig. 3 where the labels, ij , indicate the span number, i , and the section number, j , as shown in Fig. 2.

5.1. Verification of DDM equations

To verify the DDM equations for section force response sensitivity, finite difference calculations are carried out with successively smaller parameter perturbations. As the perturbation decreases, the finite difference approximation should converge to the analytic derivative:

$$\lim_{\varepsilon \rightarrow 0} \frac{\mathbf{s}(\theta + \varepsilon) - \mathbf{s}(\theta)}{\varepsilon} = \frac{\partial \mathbf{s}}{\partial \theta} \quad (18)$$

For the DDM solution, $\partial \mathbf{s} / \partial \theta$ is recovered from terms on the right-hand side of Eq. (9), each of which is known after the DDM equations have been solved during the analysis.

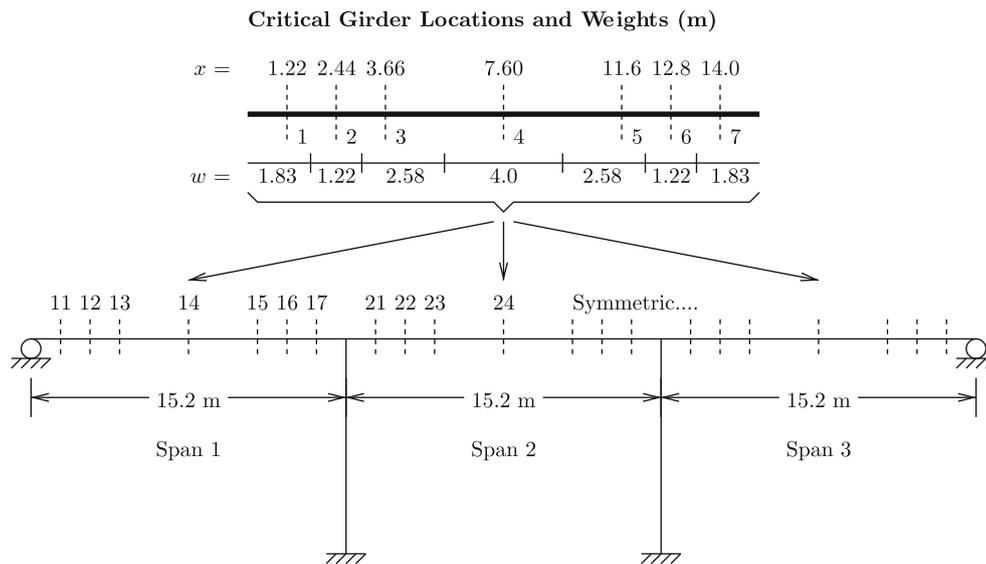


Fig. 2. Span lengths and critical locations for an interior girder of the McKenzie River Bridge.

The three axle AASHTO HS-20 design truck [18] shown in Fig. 4a moves across the bridge in 100 load increments. The derivatives of bending moment at Section 17 (on span 1 closest to the interior support) with respect to concrete material properties, section dimensions, area of reinforcement, axle spacing and load magnitudes are presented in Fig. 5. As anticipated, the results obtained by the FDM converge to those obtained by the DDM at every load increment of the analysis.

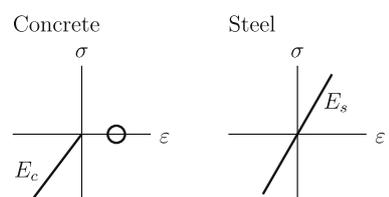
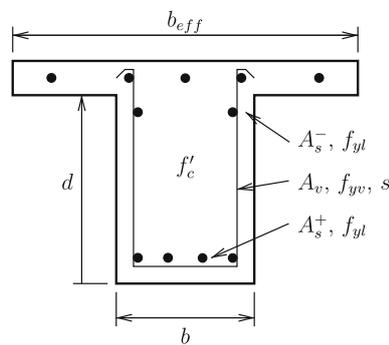
The sensitivities are multiplied by the initial value of the corresponding parameter such that absolute changes in response can be estimated from Fig. 5 for a relative (%) parameter change. The moment of Section 17 is much more sensitive to the load parameters than to the material and geometric parameters; however, the section moment is nearly as sensitive to relative changes in section depth as it is to the axle load. Similar analyses indicate that the shear force response is much less sensitive to the chosen parameters than that for bending moment.

5.2. First-order reliability analysis

A first-order reliability (FORM) analysis is carried out to assess the effect of uncertain girder properties and moving loads on the interaction of negative moment and shear force at the critical location investigated in the verification example. While conservative in design, treating moment and shear separately can lead to non-conservative estimates of reliability. The approach taken herein is to define the limit state as a smooth Lamé curve

$$g = 1 - \left| \frac{M}{M_n} \right|^3 - \left| \frac{V}{V_n} \right|^3 \tag{19}$$

where $M = IF * DF_M * M_{FE}$ is the bending moment from the finite element analysis, M_{FE} , modified by the impact factor, IF , and moment distribution factor, DF_M . Similarly, the shear, $V = IF * DF_V * V_{FE}$, is obtained from the finite element analysis, impact factor, and shear distribution factor, DF_V . The impact and moment and



Section Label	d (cm)	b (cm)	A _s ⁻ (cm ²)	A _s ⁺ (cm ²)	s (cm)
11	122	33	2.58	46.1	30.5
12	122	33	2.58	60.4	48.3
13,14	122	33	2.58	80.5	48.3
15	122	33	20.3	47.4	23.0
16	122	39	40.3	30.2	23.0
17,21	122	45	60.4	30.2	23.0
22	122	39	55.0	30.2	23.0
23	122	33	40.3	30.2	30.5
24	122	33	2.58	46.6	48.3

- f'_c = 22.8 MPa
- f_{yt} = 276 MPa
- f_{yv} = 276 MPa
- A_v = 2.58 cm²
- E_c = 22.6 GPa
- E_s = 200 GPa

Fig. 3. Typical reinforcing layout and details for an interior girder of the McKenzie River Bridge.

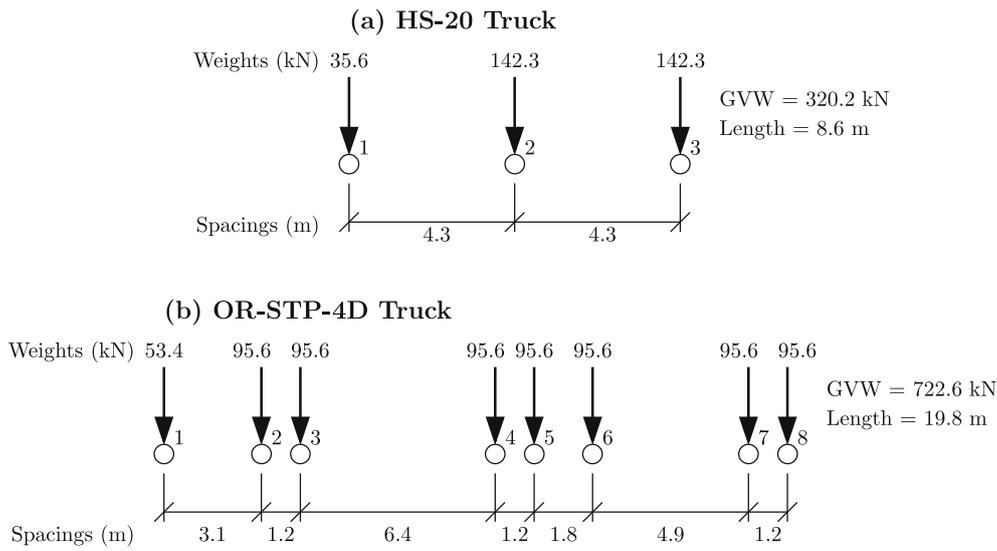


Fig. 4. Axle weights and spacings of vehicles used in sensitivity and reliability analyses: (a) HS-20 truck; (b) OR-STP-4D single-trip permit truck.

shear distribution factors are treated as random variables with mean values 1.1, 0.854, and 0.884, respectively, obtained from LRFR specifications [19]. Field testing and three-dimensional analysis of the McKenzie River Bridge [20] offer more realistic estimates of the impact and distribution factors; however, the use of LRFR values does not affect the analysis methodology presented herein.

The nominal capacities, M_n^- and V_n , that describe the shape of the Lamé curve are given by peak values from MCFT analysis:

$$M_n^- = A_s^- f_{yl} \left(d - \frac{a}{2} \right), \quad a = \frac{A_s^- f_{yl}}{0.85 f_c b} \quad (20a)$$

$$V_n = \beta \sqrt{f_c} b d + \frac{A_v f_{yv} d}{s \tan \theta} \quad (20b)$$

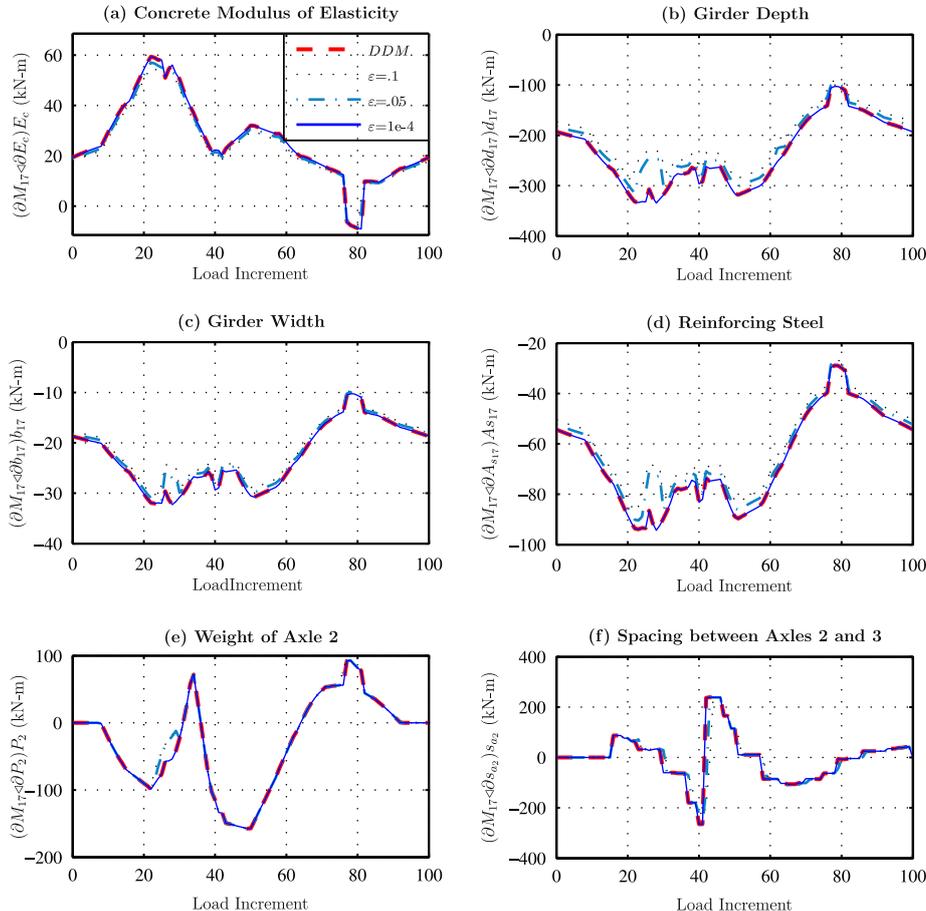


Fig. 5. Verification of DDM response sensitivity computations for moving load analysis of an interior girder of the McKenzie River Bridge.

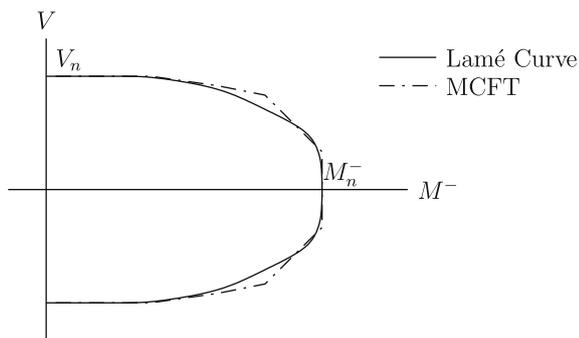


Fig. 6. Performance functions for the interaction of negative moment and shear at a girder cross-section.

Table 1
Random variable descriptions for the finite element reliability analysis of an interior girder of the McKenzie River Bridge.

RV no.	Parameter	Distribution	Mean	COV
1	d , Section depth	Normal	122 cm	0.015
2	b , Section width	Normal	45 cm	0.015
3	A_s^- , Neg. r/f steel area	Normal	60.4 cm ²	0.024
4	A_s^+ , Pos. r/f steel area	Normal	30.2 cm ²	0.024
5	E_c , Concrete modulus	Lognormal	22.6 GPa	0.08
6	G , Concrete shear modulus	Lognormal	9.41 GPa	0.08
7	E_s , Steel modulus	Lognormal	200 GPa	0.06
8	f'_c , Concrete strength	Normal	22.8 MPa	0.15
9	A_v , Shear r/f area	Normal	2.58 cm ²	0.024
10	f_{yv} , Transverse yield stress	Lognormal	276 MPa	0.12
11	s , Stirrup spacing	Normal	23 cm	0.10
12	f_{yt} , Long. yield Stress	Lognormal	276 MPa	0.12
13	IF , Impact factor	Normal	1.10	0.08
14	DF_M , Moment distr. factor	Normal	0.854	0.10
15	DF_V , Shear distr. factor	Normal	0.884	0.10
16	P_1 , Axle Load	Lognormal	53 kN	0.20
17–23	P_2 – P_8 , Axle Load	Lognormal	96 kN	0.20

As shown in Fig. 6, the smooth Lamé curve closely approximates the general shape of a multi-linear MCFT interaction surface. The exponent in Eq. (19) can be adjusted to change the shape of the curve.

All parameters that define M_n^- and V_n at Section 17 are treated as random variables with distribution properties shown in Table 1. The material properties and dimensions are taken as random variables with descriptors based on available literature [21,22]. This example constitutes a component reliability analysis of Section 17; however, for a system reliability analysis for all critical sections, the material properties and dimensions at all sections would be correlated. For the mean values of material properties and reinforcing details, the MCFT parameters β and θ in Eq. (20b) are found to be 1.8 and 42°, respectively. These parameters remain deterministic during the analysis; however, the moment and shear capaci-

Table 2
Ranking of random variables with importance measure exceeding 0.1 in the McKenzie Bridge girder example.

RV no.	Parameter	γ -Value
13	IF	0.698
15	DF_V	0.363
10	f_{yv}	-0.236
1	d	-0.145
19	P_4	0.132
21	P_6	0.131
20	P_5	0.130
12	f_{yt}	-0.110

ties, and accordingly the shape of the Lamé curve, will change during the analysis for every realization of the uncertain parameters in Eq. (20).

An eight-axle single-trip permit truck, type OR-STP-4D [23] shown in Fig. 4b, moves across the bridge in 100 load increments. All axle weights are assumed to be correlated lognormal random variables, with descriptors based on WIM specifications [24]. The correlation between all axle weights is 0.4, while the correlation between axles in the tandem and triple groups (axles 2-3, 4-5-6, and 7-8) is 0.8 [25]. Coefficients of variation for the impact and distribution factors available in the literature [26,27] are used in the analysis.

The improved HLRF algorithm [28] is employed to find the design point during the moving load reliability analysis in OpenSees. This algorithm requires the gradient of the performance function with respect to the uncertain parameters

$$\frac{\partial g}{\partial \mathbf{Y}} = \frac{\partial g}{\partial \mathbf{s}} \frac{\partial \mathbf{s}}{\partial \boldsymbol{\Theta}} \mathbf{J}_{\boldsymbol{\Theta}, \mathbf{Y}} \quad (21)$$

where $\partial g / \partial \mathbf{s}$ is the gradient of the performance function with respect to the section forces, $\partial \mathbf{s} / \partial \boldsymbol{\Theta}$ is obtained by the previously verified DDM procedures, and $\mathbf{J}_{\boldsymbol{\Theta}, \mathbf{Y}}$ is the Jacobian matrix of the transformation of random variables to standard normal space [29].

Using the DDM to evaluate $\partial \mathbf{s} / \partial \boldsymbol{\Theta}$, the iHLRF algorithm requires five iterations and six g -function evaluations to converge to the minimum reliability index, $\beta_{cr} = 2.39$, shown in Fig. 7a. This is in contrast to the 126 g -function evaluations required to reach the same failure point using finite differences of the girder response to compute $\partial \mathbf{s} / \partial \boldsymbol{\Theta}$. This critical reliability index corresponds to a 0.84% probability of failure.

For the critical load position, the ranking of the RVs with importance measures (γ -values presented by [30]) greater than 0.1 is shown in Table 2. The random variables that characterize the impact factor and shear distribution factor rank highest in importance, which reflects the large amount of epistemic uncertainty in estimating dynamic effects and three-dimensional load distribution in this simplified 2D girder analysis. The transverse steel yield

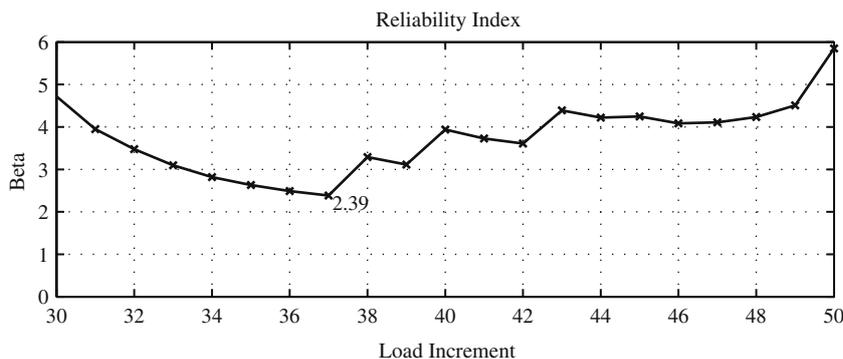


Fig. 7. First-order reliability indices for moment-shear interaction at Section 17 for load increment 30–50 on an interior girder of the McKenzie River Bridge.

stress, f_{yt} , is the most important resistance variable in the analysis, followed by the section depth, d , and the longitudinal yield stress, f_{yl} . Although changes in section depth have little effect on finite element demand calculations, the high importance of this variable shown in Table 2 is due to its presence in the capacity equations (Eq. (20)) for moment and shear. The high ranking of both transverse and longitudinal yield stress highlights the interaction of moment and shear at the failure point. Due to the strong correlation assigned to axle loads 4 through 6, each random variable assigned to these axles has approximately the same importance factor.

6. Conclusions

The direct differentiation method (DDM) of response sensitivity has been applied in the reliability analysis of continuous bridge girders subjected to moving loads. Sensitivity analyses indicated how girder response will change as a function of parameters that are of interest in bridge design. In addition to design considerations, the response sensitivity was used in a first-order reliability analysis of moment–shear interaction at a critical girder cross-section. The interaction of girder bending moment and shear force was directly formulated in the limit state function using a smooth curve that approximated the features of an MCFT interaction surface. Ranking of importance measures for the random variables indicated the presence of moment–shear interaction in the most probable failure state. The implementation of DDM response sensitivity analysis in the OpenSees finite element framework make it an ideal platform for other gradient-based applications (optimization, system identification, and damage detection), the assessment of epistemic uncertainty associated with bridge modeling, and the evaluation of bridge reliability under combined seismic and live load hazards. Furthermore, the first-order reliability analyses presented in this paper are an essential component to system reliability and downcrossing analysis of time-variant girder moment–shear interaction via finite element methods.

References

- [1] Minervino C, Sivakumar B, Moses F, Mertz D, Eddberg W. New AASHTO guide manual for load and resistance factor rating of highway bridges. *J Bridge Eng* 2004;9(1):43–54.
- [2] Patrick MD, Huo XS, Puckett JA, Jablin M, Mertz D. Sensitivity of live load distribution factors to vehicle spacing. *J Bridge Eng* 2006;11(1):131–4.
- [3] Nowak AS, Zhou J. System reliability models for bridges. *Struct Safety* 1990;7(2–4):247–54.
- [4] Stewart MG, Rosowsky DV, Val D. Reliability-based bridge assessment using risk-ranking decision analysis. *Struct Safety* 2001;23:397–405.
- [5] Akgül F, Frangopol DM. Rating and reliability of existing bridges in a network. *J Bridge Eng* 2003;8(6):383–93.
- [6] Kleiber M, Antunez H, Hien TD, Kowalczyk P. Parameter sensitivity in nonlinear mechanics. John Wiley & Sons; 1997.
- [7] McKenna F, Fenves GL, Scott MH. Open system for earthquake engineering simulation. Berkeley, CA: University of California; 2000. Available from: <<http://opensees.berkeley.edu>>.
- [8] Der Kiureghian A, Haukaas T, Fujimara K. Structural reliability software at the University of California, Berkeley. *Struct Safety* 2006;28:44–67.
- [9] Haukaas T, Der Kiureghian A. Methods and object-oriented software for FE reliability and sensitivity analysis with application to a bridge structure. *J Comput Civil Eng* 2007;21(3):151–63.
- [10] Ousterhout JK. Tcl and the Tk toolkit. Reading, MA: Addison-Wesley; 1994.
- [11] Zhang Y, Der Kiureghian A. Dynamic response sensitivity of inelastic structures. *Comput Methods Appl Mech Eng* 1993;108(1–2):23–36.
- [12] Spacone E, Ciampi V, Filippou FC. Mixed formulation of nonlinear beam finite element. *Comput Struct* 1996;58(1):71–83.
- [13] Kidarsa A, Scott MH, Higgins CC. Analysis of moving loads using force-based finite elements. *Finite Elem Anal Des* 2008;44(4):214–24.
- [14] Neuenhofer A, Filippou FC. Evaluation of nonlinear frame finite-element models. *J Struct Eng* 1997;123(7):958–66.
- [15] Scott MH, Franchin P, Fenves GL, Filippou FC. Response sensitivity for nonlinear beam-column elements. *J Struct Eng* 2004;130(9):1281–8.
- [16] Conte JP, Barbato M, Spacone E. Finite element response sensitivity analysis using force-based frame models. *Int J Numer Methods Eng* 2004;59:1781–820.
- [17] Marini A, Spacone E. Analysis of reinforced concrete elements including shear effects. *ACI Struct J* 2006;103(5):645–55.
- [18] AASHTO. LRFD bridge design specifications. 2nd ed. Washington, DC: American Association of State Highway and Transportation Officials; 2003.
- [19] AASHTO. Manual for condition evaluation and load and resistance factor rating (LRFR) of highway bridges. Washington, DC: American Association of State Highway and Transportation Officials; 2003.
- [20] Potisuk T, Higgins C. Field testing and analysis of CRC deck girder bridges. *J Bridge Eng* 2007;12(1):53–63.
- [21] Ellingwood B, Galambos TV, MacGregor JG, Cornell CA. Development of a probability based load criterion for American National Standard A58. Tech. Rep. SP577. Washington, DC: National Bureau of Standards; 1980.
- [22] Nowak AS, Yamani A, Tabsh SW. Probabilistic models for resistance of concrete bridge girders. *ACI Struct J* 1994;91(3):269–76.
- [23] ODOT. Bridge design and drafting manual (BDDM). Salem, OR: Oregon Department of Transportation; 2004.
- [24] ASTM. Standard specifications for highway weight-in-motion (WIM) systems with user requirements and test method (ASTM E 1318-02). Philadelphia, PA: American Society for Testing and Materials; 2002.
- [25] Katz AJ, Rakha H. Field and modeling framework and case study of truck weigh station operations. Tech. Rep. A research project report for the Center for ITS Implementation Research in US. DOT University Transportation Center, Virginia Tech Transportation Institute, Blacksburg, VA; 2002.
- [26] Hwang ES, Nowak AS. Simulation of dynamic load for bridges. *J Struct Eng* 1991;117(5):1413–34.
- [27] Kim S, Nowak AS. Load distribution and impact factors for I-Girder bridges. *J Bridge Eng* 1997;2(3):97–104.
- [28] Liu PL, Der Kiureghian A. Optimization algorithms for structural reliability. *Struct Safety* 1991;9(3):161–77.
- [29] Liu PL, Der Kiureghian A. Multivariate distribution models with prescribed marginals and covariances. *Probabil Eng Mech* 1986;1(2):105–12.
- [30] Haukaas T, Der Kiureghian A. Parameter sensitivity and importance measures in nonlinear finite element reliability analysis. *J Eng Mech* 2005;131(10):1013–26.