Collisions

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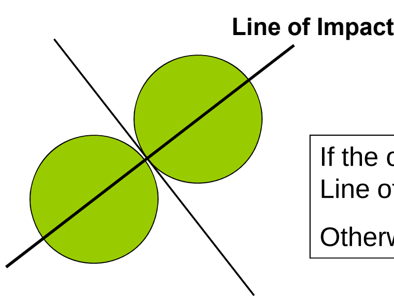
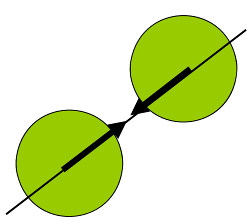
Collisions -- Definitions

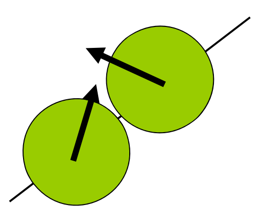


If the objects' velocities are parallel to the Line of Impact, this is a ***Direct Impact***

Otherwise it is called an ***Oblique Impact***



Direct Impact



Oblique Impact

Collisions – Fundamental Quantities

The ***momentum*** of an object is defined as its mass multiplied by its velocity:

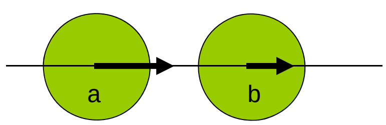
$$Momentum = mv$$

The ***energy*** of an object is defined as one half of its mass multiplied by its velocity squared:

$$Energy = \frac{1}{2} mv^2$$

Collisions – Conservation of Momentum

In a collision, the total momentum after the impact is equal to the total momentum before the impact:



$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

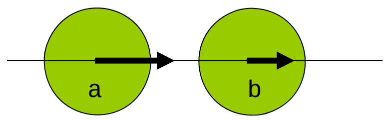
where the primes **′** refer to velocities after the impact

This is referred to as the *Conservation of Momentum Law*

Collisions – Coefficient of Restitution

In a collision, energy is conserved in the entire system, but not necessarily in the form of velocities. (It can become permanent deformation, heat, light, etc..)

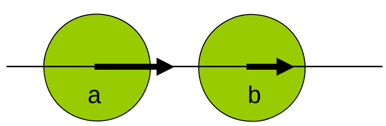
This loss of velocity is expressed as the ***Coefficient of Restitution*** (COR). The COR, e, is how much less the relative velocities of the objects are after impact than they were before impact:



$$v_b' - v_a' = -e(v_b - v_a)$$

(the negative sign is there to indicate the “bounce”)

Collisions – Combining Momentum and Restitution Laws



Starting with these two equations:

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$v_b' - v_a' = -e(v_b - v_a)$$

Treat the two initial velocities as inputs and solve for the two resulting velocities. This gives:

$$v_a' = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

$$v_b' = \frac{m_a v_a + m_b v_b - e m_a (v_b - v_a)}{m_a + m_b}$$

Collisions with Immoveable Objects

To treat the case of mass b being an immoveable object, such as the ground or a solid wall, solve for the resulting velocities taking the limit: $\lim_{m_b \to \infty}$

$$\lim_{m_b \to \infty} v'_a = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

$$= \lim_{m_b \to \infty} \left[\frac{m_a v_a}{m_a + m_b} + \frac{m_b v_b}{m_a + m_b} + \frac{e m_b (v_b - v_a)}{m_a + m_b} \right]$$

$$= \left[0 + v_b + e(v_b - v_a) \right]$$

Since mass b is immoveable, its velocity is zero, so that a's post-collision velocity is:

$$\boxed{v'_a = -e v_a}$$

Collisions – Experimentally Determining the Coefficient of Restitution

Velocities are hard to measure live, but distances are not.

So, drop the object from a height ***h***, and measure its bounce to a height **h'** :

Before the bounce:

$$v_2{}^2 = 0^2 + 2gh$$

$$v = \sqrt{2gh}$$

After the bounce:

$$0^2 = v'^2 - 2gh'$$

$$v' = \sqrt{2gh'}$$

$$|v'| = e|v|$$

$$\boxed{e = \frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}}}$$

Collisions – Some Coefficients of Restitution of Balls Bounced on a Concrete Surface

Ball Material	CoR
range golf ball	0.858
tennis ball	0.712
billiard ball	0.804
hand ball	0.752
wooden ball	0.603
steel ball bearing	0.597
glass marble	0.658
ball of rubber bands	0.828
hollow, hard plastic ball	0.688

http://hypertextbook.com/facts/2006/restitution.shtml

Collisions – Totally Plastic Collisions

$$v'_a = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

$$v'_b = \frac{m_a v_a + m_b v_b - e m_a (v_b - v_a)}{m_a + m_b}$$

If e=0, then the two objects stick together and end up with the same resulting velocity:

$$v'_a = v'_b = \frac{m_a v_a + m_b v_b}{m_a + m_b}$$

Collisions – Totally Elastic Collisions

What happens when e=1? The two fundamental equations are

$$m_b v'_b + m_a v'_a = m_b v_b + m_a v_a$$
$$v'_b - v'_a = -e(v_b - v_a) = (v_a - v_b)$$

Rearranging:

$$m_a \left(v'_a - v_a\right) = m_b \left(v_b - v'_b\right)$$
$$v'_a + v_a = v_b + v'_b$$

Collisions – Elastic Collisions

Then, multiplying the two together gives:

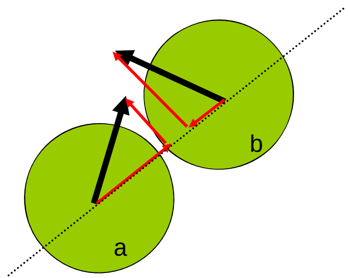
$$m_a v'^2_a - m_a v_a{}^2 = m_b v_b{}^2 - m_b v'^2_b$$

Or:

$$\boxed{\frac{1}{2} m_b v'^2_b + \frac{1}{2} m_a v'^2_a = \frac{1}{2} m_b v_b{}^2 + \frac{1}{2} m_a v_a{}^2}$$

which shows that energy is conserved when the Coefficient of Restitution is 1.0

Collisions – Oblique Impacts



Oblique Impacts are then handled by using vector math to determine the velocity components parallel and perpendicular to the Line of Impact.

The parallel components are changed using the equations we just derived.

The perpendicular components are left unchanged. (This assumes no friction.)

The new components are then combined to produce the resulting velocity vectors.