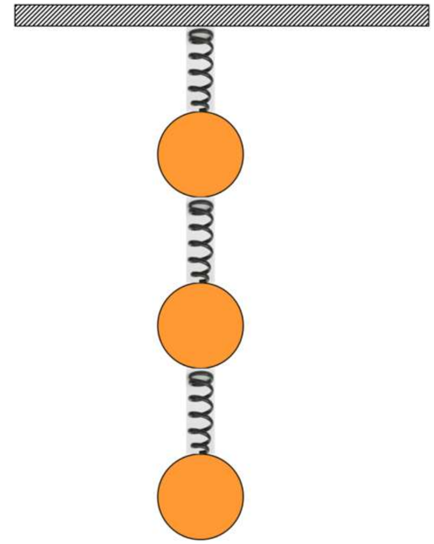
Modeling the World as a Mesh of Springs



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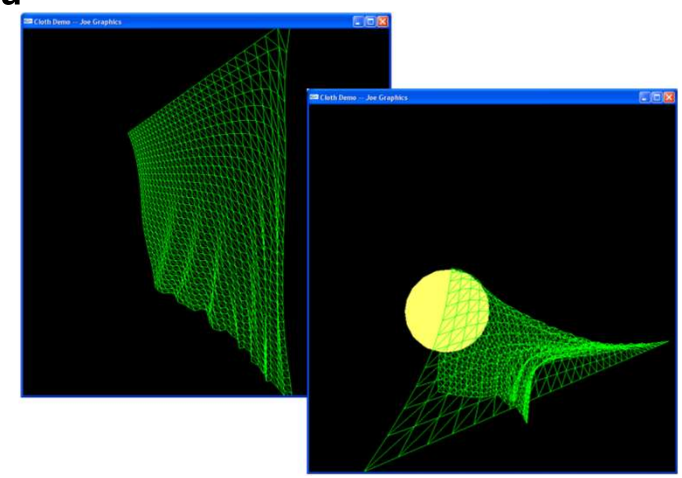
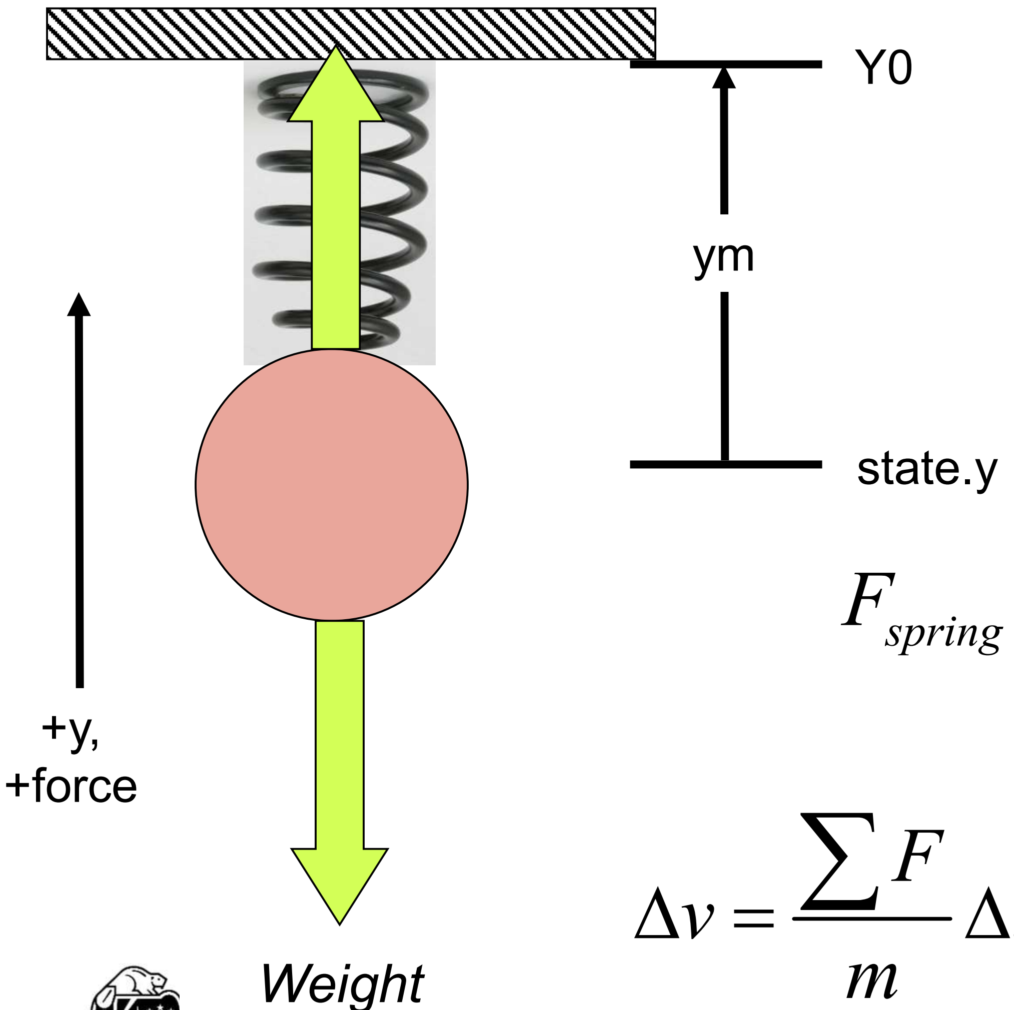


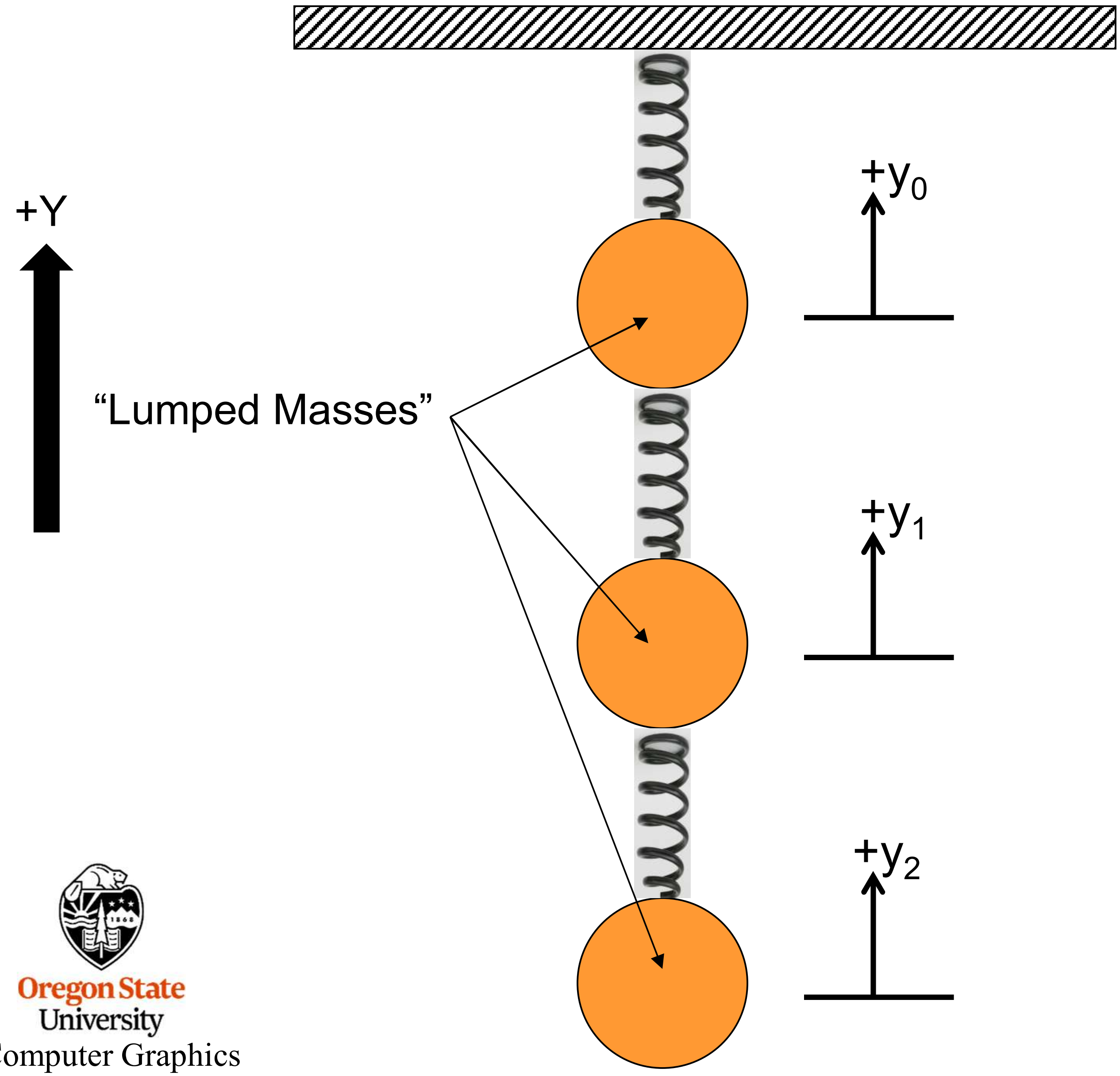
Solving for Motion where there is a Spring



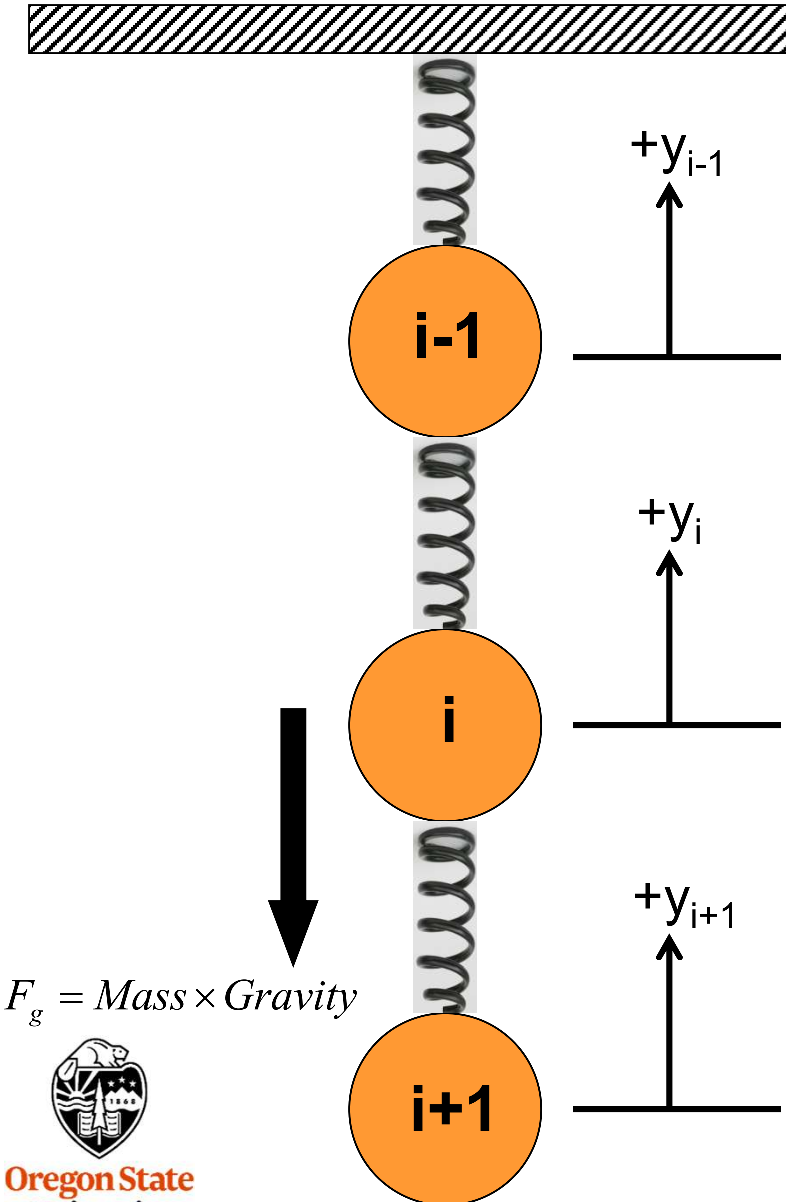
$$F_{spring} = -k(y - D_0)$$

$$\Delta v = \frac{\sum F}{m}\Delta t = \frac{-W - k(y - D_0)}{m}\Delta t$$

Modeling a String as a Group of Masses Connected by Springs



Computing Forces in 1D



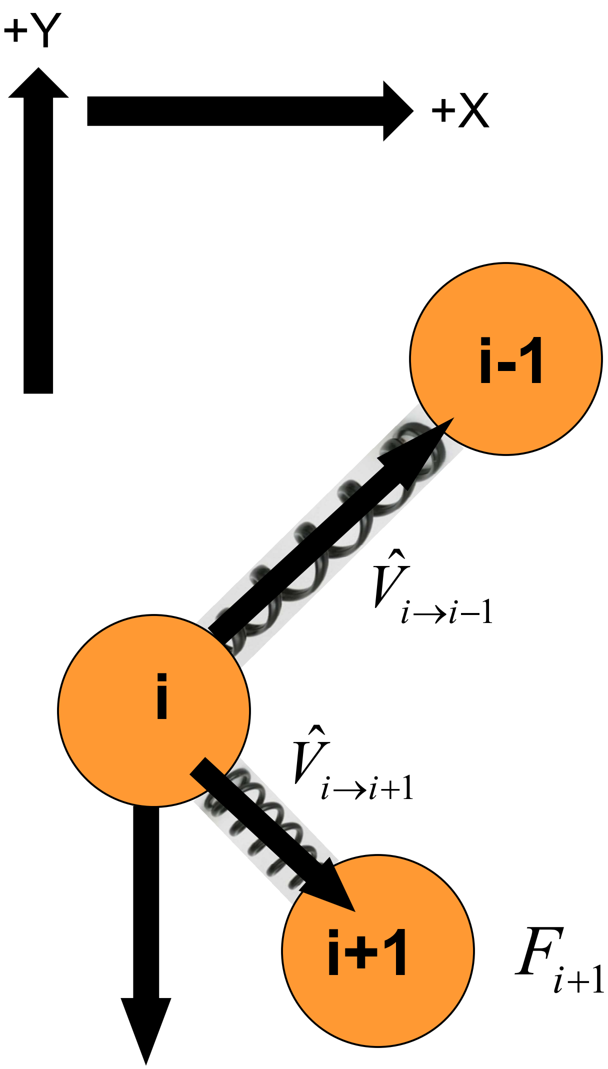
$$F_g = Mass \times Gravity$$

Generalize by using indices:

$$F_{i-1 \to i} = k(Y_{i-1} - Y_i - D_0)$$

$$F_{i+1 \to i} = k(Y_{i+1} - Y_i - D_0)$$

Computing Forces in 2D



$$F_{i-1\to i} = k(D_{i\to i-1} - D_0)$$

X: $F_{i-1\to i,x} = F_{i-1\to i}\cos\theta_{i\to i-1} = F_{i-1\to i}\dfrac{x_{i-1}-x_i}{D_{i\to i-1}}$

Y: $F_{i-1\to i,y} = F_{i-1\to i}\sin\theta_{i\to i-1} = F_{i-1\to i}\dfrac{y_{i-1}-y_i}{D_{i\to i-1}}$

$$F_{i+1\to i} = k(D_{i\to i+1} - D_0)$$

X: $F_{i+1\to i,x} = F_{i+1\to i}\cos\theta_{i\to i+1} = F_{i+1\to i}\dfrac{x_{i+1}-x_i}{D_{i\to i+1}}$

Y: $F_{i+1\to i,y} = F_{i+1\to i}\sin\theta_{i\to i+1} = F_{i+1\to i}\dfrac{y_{i+1}-y_i}{D_{i\to i+1}}$

$F_g = Mass \times Gravity$

From the *Physics* Notes: What does a Second Order solution look like in a Program?

```
void
AdvanceOneTimeStep( )
{
        GetDerivs( state, &derivs1);
        state2.t    = state.t   + Δt;
        state2.x    = state.x   + derivs1->vx * Δt;
        state2.vx = state.vx + derivs1->ax * Δt;

        GetDerivs( state2, &derivs2 );
        float aavg = ( derivs1->ax + derivs2->.ax) / 2.;
        float vavg = ( derivs1->vx + derivs2->vx) / 2.;

        state.x   = state.x   + vavg * Δt;
        sate.vx  = state.vx + aavg * Δt;
        state.t    = state.t   + Δt ;
}
```

Solve for Each State as a Whole, not as Individual Links: *Do it this Way*

Correct Second Order solution:

```
for( int i = 0; i < NUMLINKS; i++ )
{
        GetOneBodysDerivs( Links, i,  &vx1[i], &vy1[i], &ax1[i], &ay1[i] );
}
for( int i = 0; i < NUMLINKS; i++ )
{
        TmpLinks[i].vx = Links[i].vx  + DT * ax1[i];
        TmpLinks[i].vy = Links[i].vy  + DT * ay1[i];
        TmpLinks[i].x   = Links[i].x    + DT * vx1[i];
        TmpLinks[i].y   = Links[i].y    + DT * vy1[i];
}

for( int i = 0; i < NUMLINKS; i++ )
{
        GetOneBodysDerivs( TmpLinks, i,  &vx2[i], &vy2[i], &ax2[i], &ay2[i] );
}
for( int i = 0; i < NUMLINKS; i++ )
{
        Links[i].vx = Links[i].vx  +  DT * ( ax1[i] + ax2[i] ) / 2.;
        Links[i].vy = Links[i].vy  +  DT * ( ay1[i] + ay2[i] ) / 2.;
        Links[i].x   = Links[i].x    +  DT * ( vx1[i] + vx2[i] ) / 2.;
        Links[i].y   = Links[i].y    +  DT * ( vy1[i] + vy2[i] ) / 2.;
}
```

Get *all* the velocities and accelerations first

Apply *all* the velocities and accelerations

Get *all* the velocities and accelerations first

Apply *all* the velocities and accelerations

Solve for Each State as a Whole, not as Individual Links:
Do it this Way

C Extensions for Array Notion (CEAN) makes it look cleaner, and possibly more efficient:

```
for( int i = 0; i < NUMLINKS; i++ )
{
        GetOneBodysDerivs( Links, i,  &vx1[i], &vy1[i], &ax1[i], &ay1[i] );
}

TmpLinks[ 0 : NUMLINKS ].vx = Links[ 0 : NUMLINKS ].vx  + DT  * ax1[ 0 : NUMLINKS ];
TmpLinks[ 0 : NUMLINKS ].vy = Links[ 0 : NUMLINKS i].vy  + DT * ay1[ 0 : NUMLINKS ];
TmpLinks[ 0 : NUMLINKS ].x   = Links[ 0 : NUMLINKS ].x    + DT * vx1[ 0 : NUMLINKS ];
TmpLinks[ 0 : NUMLINKS ].y   = Links[ 0 : NUMLINKS ].y    + DT * vy1[ 0 : NUMLINKS ];

for( int i = 0; i < NUMLINKS; i++ )
{
        GetOneBodysDerivs( TmpLinks, i,  &vx2[i], &vy2[i], &ax2[i], &ay2[i] );
}

Links[ 0 : NUMLINKS ].vx = Links[ 0 : NUMLINKS ].vx  +  DT * ( ax1[ 0 : NUMLINKS ] + ax2[ 0 : NUMLINKS ] ) / 2.;
Links[ 0 : NUMLINKS ].vy = Links[ 0 : NUMLINKS ].vy  +  DT * ( ay1[ 0 : NUMLINKS ] + ay2[ 0 : NUMLINKS ] ) / 2.;
Links[ 0 : NUMLINKS ].x   = Links[ 0 : NUMLINKS ].x    +  DT * ( vx1[ 0 : NUMLINKS ] + vx2[ 0 : NUMLINKS ] ) / 2.;
Links[ 0 : NUMLINKS ].y   = Links[ 0 : NUMLINKS ].y    +  DT * ( vy1[ 0 : NUMLINKS ] + vy2[ 0 : NUMLINKS ] ) / 2.;
```

```
void
GetOneBodysDerivs(  array, node,  float *vxi, float *vyi, float *axi, float *ayi )
{
        float xm, ym;       // vector from node to previous node (up the chain)
        float xp,  yp;      // vector from node to next node (down the chain)

        float sumfx = 0.;
        float sumfy = -Weight;

        if( node == 0 )
        {
                xm = X0 - array[node].x;            // (X0,Y0) is the top of the chain
                ym = Y0 - array[node].y;
        }
        else
        {
                xm = array[node-1].x - array [node].x;
                ym = array[node-1].y - array [node].y;
         }

        float length = sqrt( xm*xm + ym*ym );       // length of the spring
        xm /= length;                               // normalize the vector
        ym /= length;
        float stretch = length - D0;                // amount the spring is stretched
        float force = K * stretch;
        sumfx += force * xm;
        sumfy += force * ym;
```

GetLinkVelAcc( ), II


```
		if( node < NUMLINKS-1 )
		{
			xp = array [node+1].x - array[node].x;
			yp = array [node+1].y - array [node].y;
			length = sqrt( xp*xp + yp*yp );
			xp /= length;
			yp /= length;
			stretch = length - D0;
			force = K * stretch;
			sumfx += force * xp;
			sumfy += force * yp;
		}

		sumfx -= Cd * array[node].vx;		// damping
		sumfy -= Cd * array[node].vy;

		*vx = array[node].vx;
		*vy = array[node].vy;
		*ax = sumfx / Mass;
		*ay = sumfy / Mass;
}
```

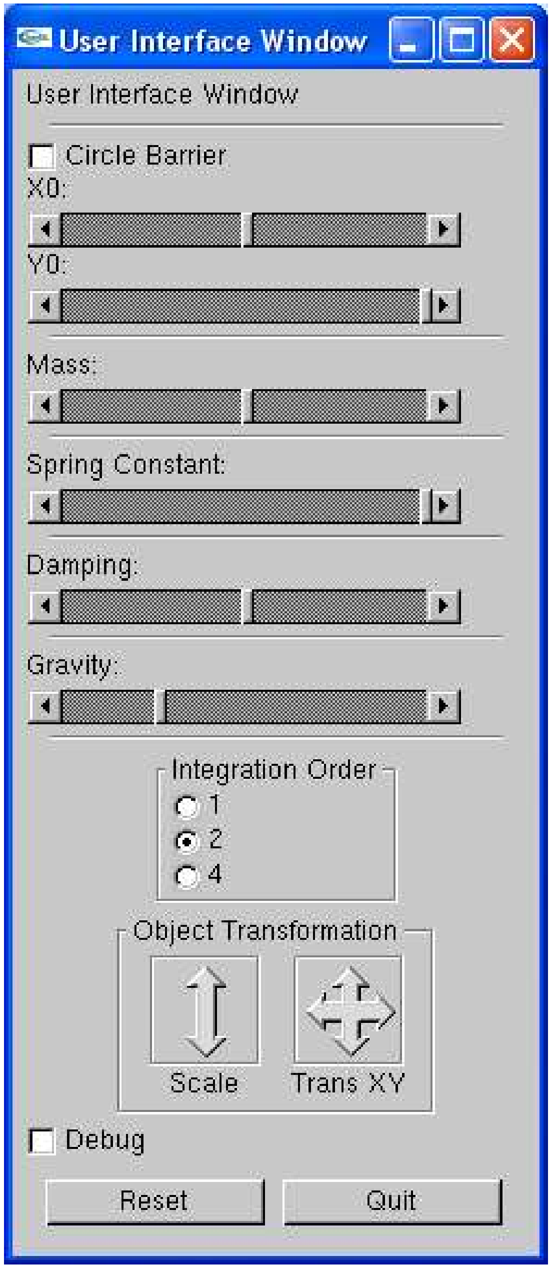
Don't do it this Way!

Incorrect Second Order solution:

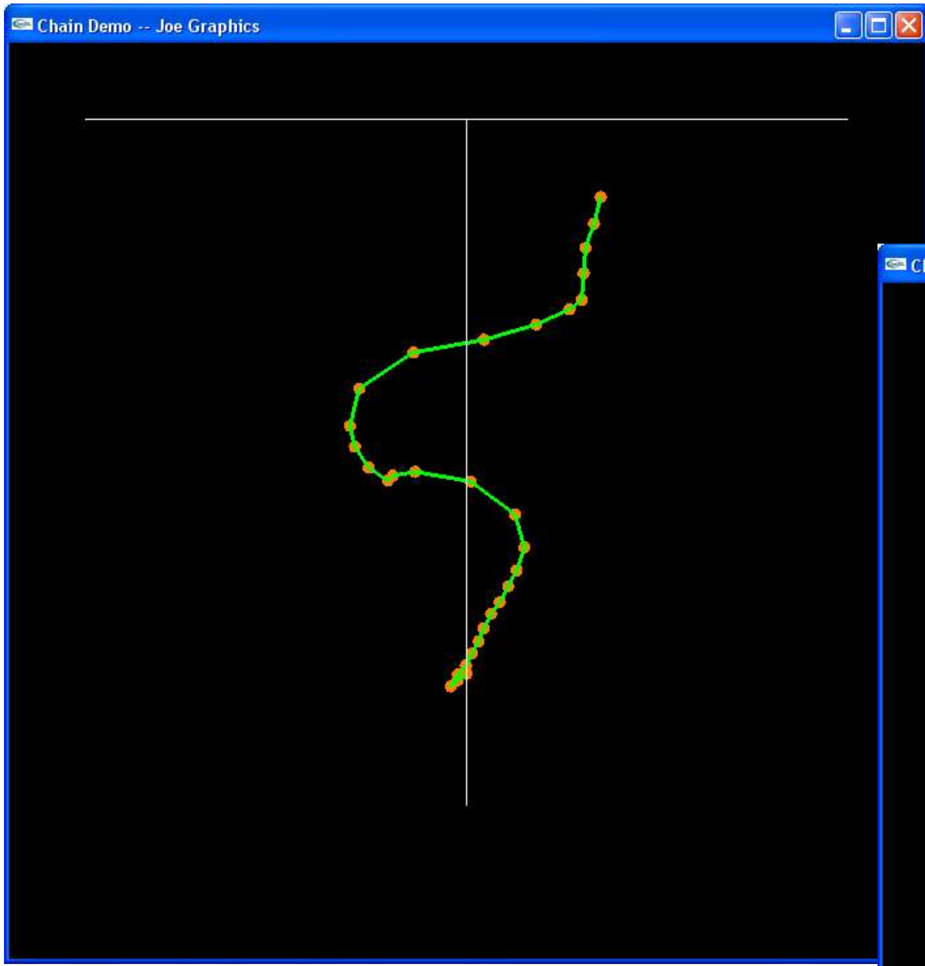
```
for( int i = 0; i < NUMLINKS; i++ )
{
		GetOneBodysDerivs( Links, i,  &vx1[i], &vy1[i], &ax1[i], &ay1[i] );
		TmpLinks[i].vx = Links[i].vx  + DT * ax1[i];
		TmpLinks[i].vy = Links[i].vy  + DT * ay1[i];
		TmpLinks[i].x   = Links[i].x    + DT * vx1[i];
		TmpLinks[i].y   = Links[i].y    + DT * vy1[i];
}

for( int i = 0; i < NUMLINKS; i++ )
{
		GetOneBodysDerivs( TmpLinks, i,  &vx2[i], &vy2[i], &ax2[i], &ay2[i] );
		Links[i].vx = Links[i].vx  +  DT * ( ax1[i] + ax2[i] ) / 2.;
		Links[i].vy = Links[i].vy  +  DT * ( ay1[i] + ay2[i] ) / 2.;
		Links[i].x   = Links[i].x    +  DT * ( vx1[i] + vx2[i] ) / 2.;
		Links[i].y   = Links[i].y    +  DT * ( vy1[i] + vy2[i] ) / 2.;
}
```

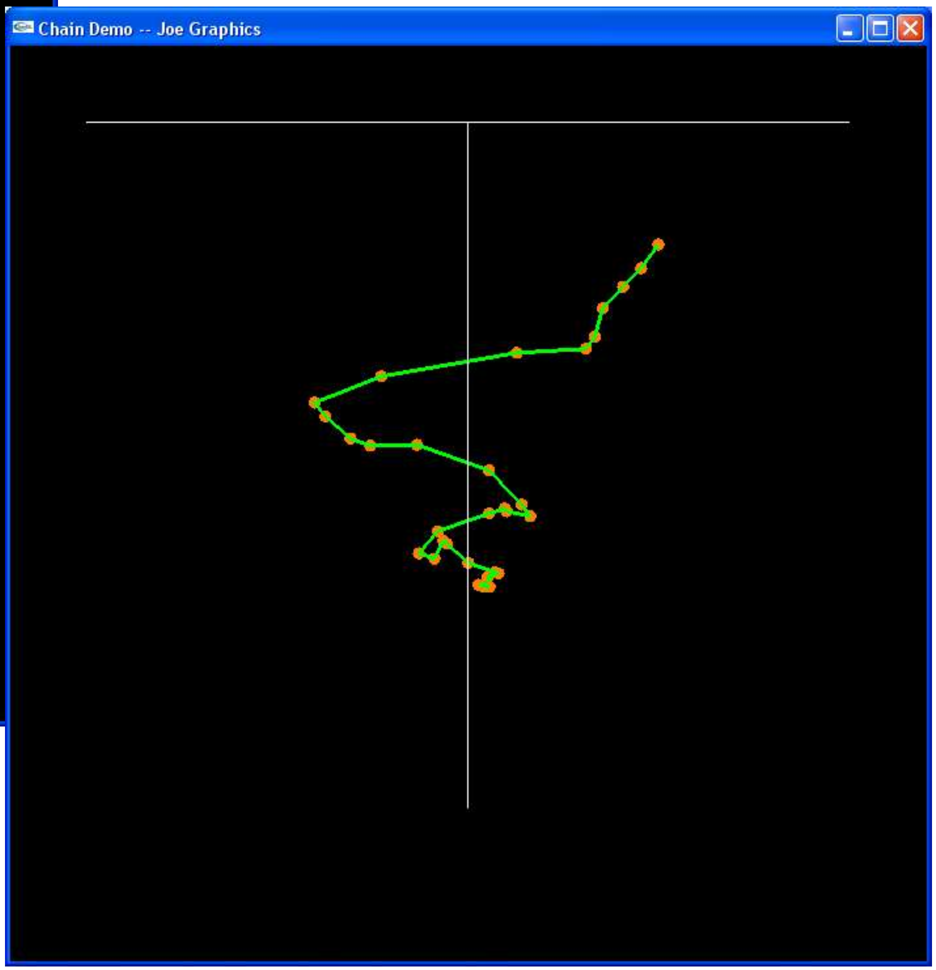
Changes the state before we are done getting the derivatives !

Changing Variables on-the-fly in the String Project

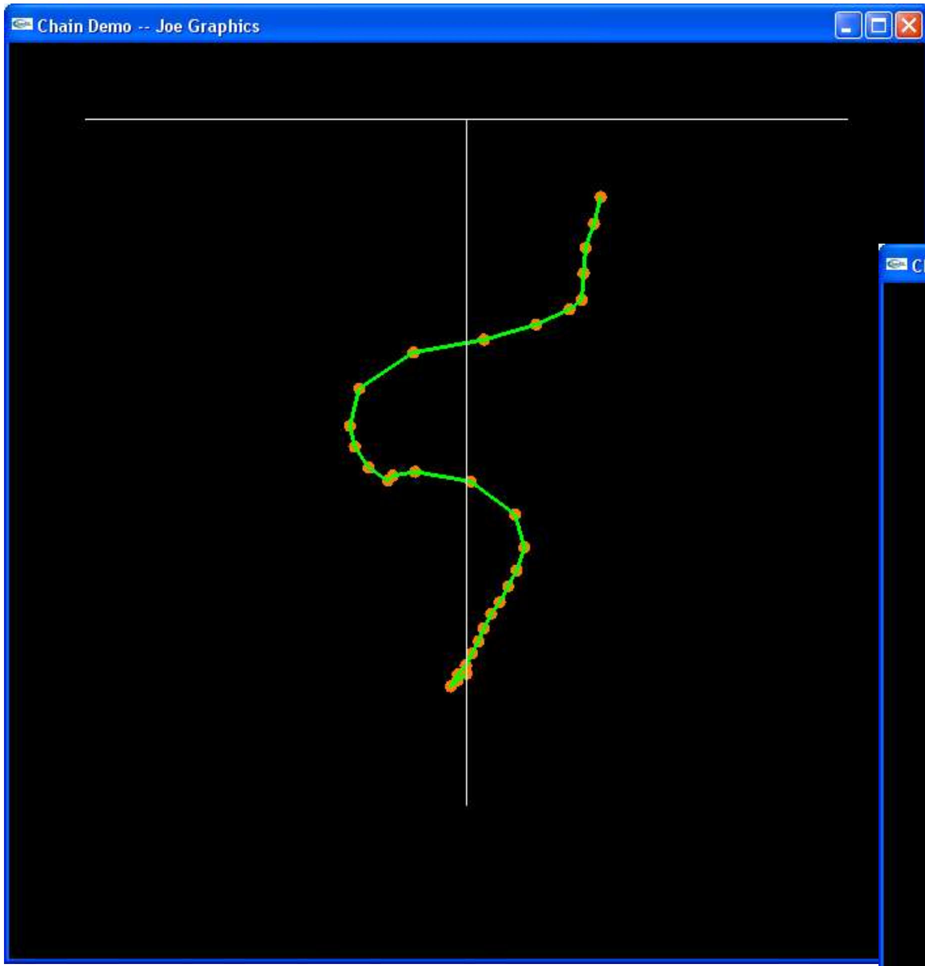


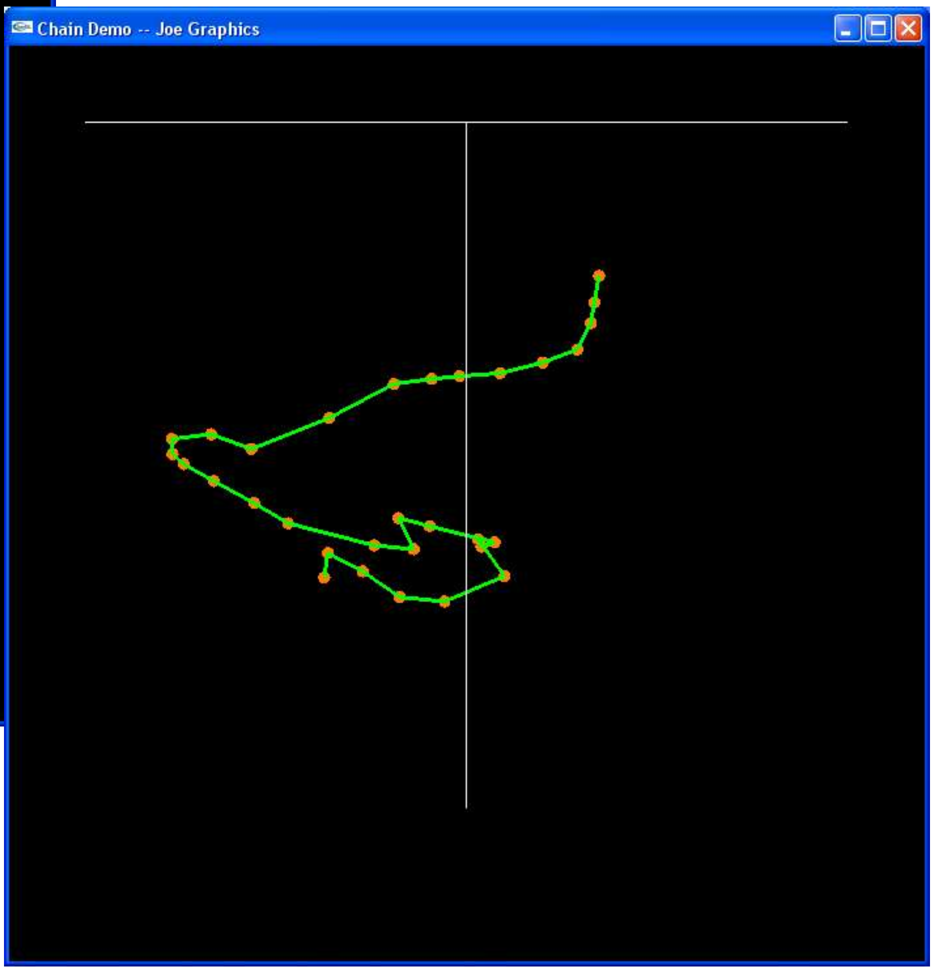
Simulating a String



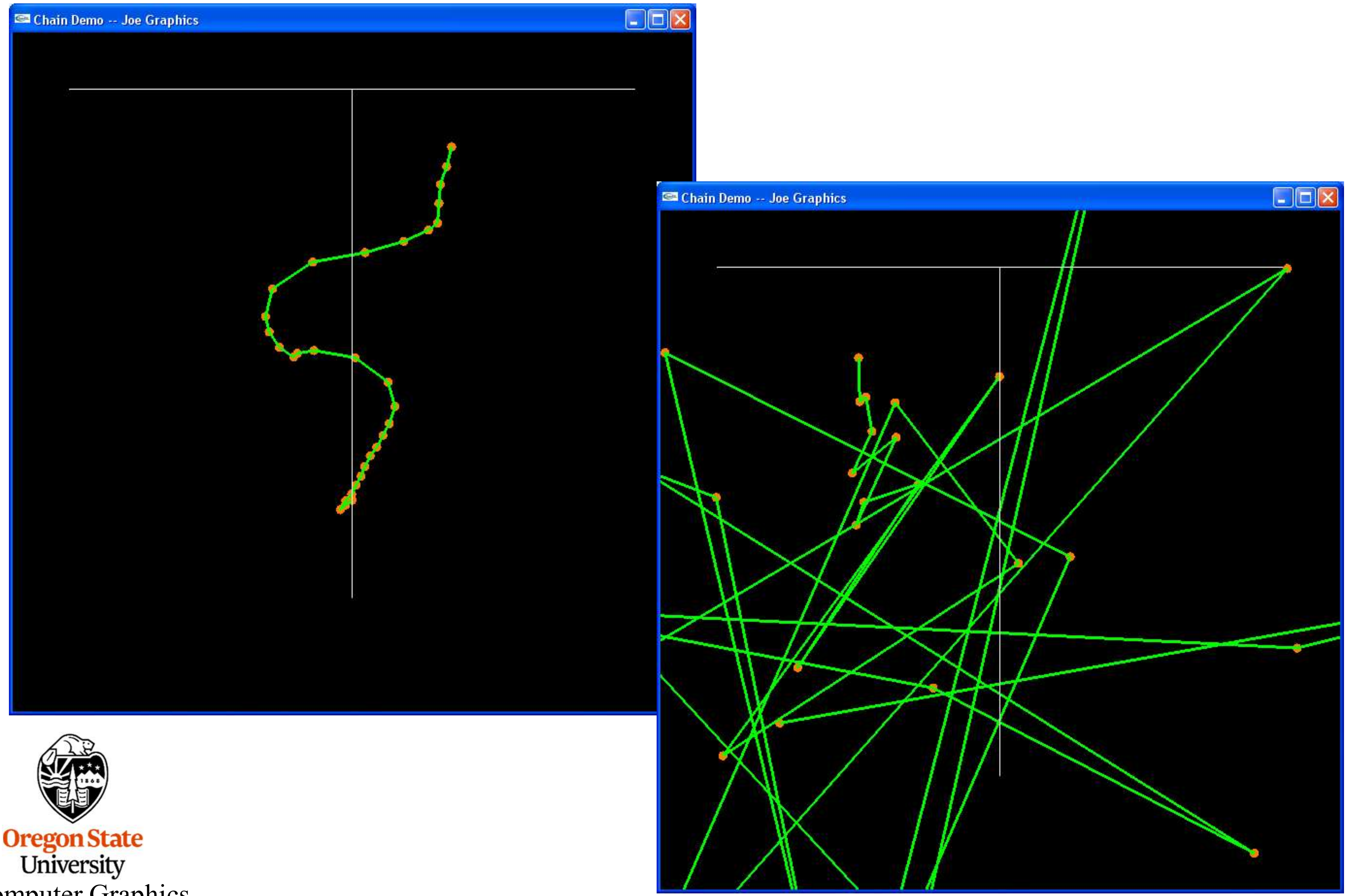


Less Damping

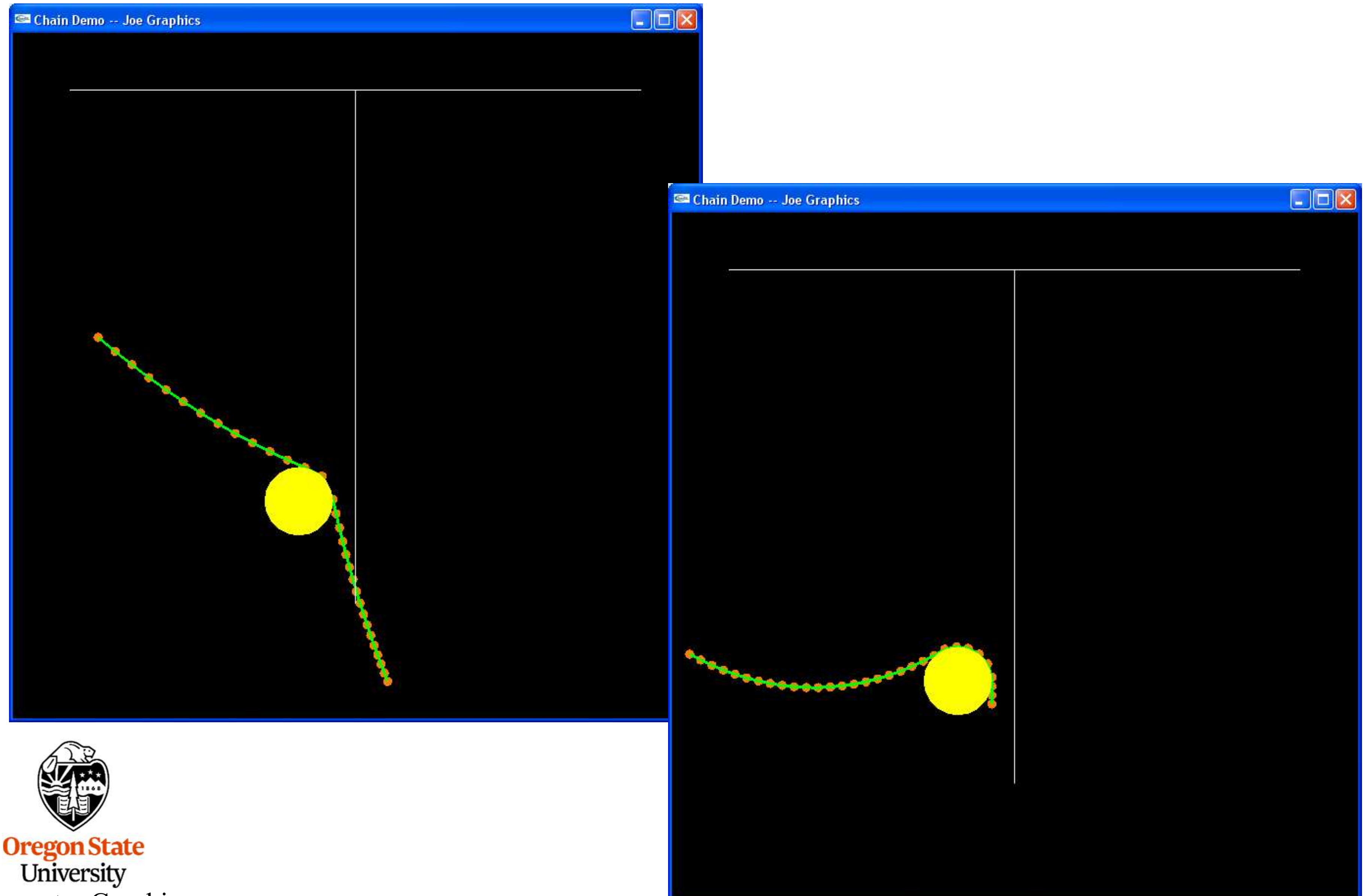




First Order Instability



Placing a Physical Barrier in the Scene



Placing a Physical Barrier in the Scene

```
if( DoCircle )
{
	for( int i = 0; i < NUMLINKS; i++ )
	{
		float dx = Links[i].x - CIRCX;
		float dy = Links[i].y - CIRCY;
		float rsqd = dx*dx + dy*dy;
		if( rsqd < CIRCR*CIRCR )
		{
			float r = sqrt( rsqd );
			dx /= r;
			dy /= r;
			Links[i].x = CIRCX + CIRCR * dx;
			Links[i].y = CIRCY + CIRCR * dy;
			Links[i].vx *=  dy;
			Links[i].vy *= -dx;
		}
	}
}
```

Vector from circle center to the lumped mass

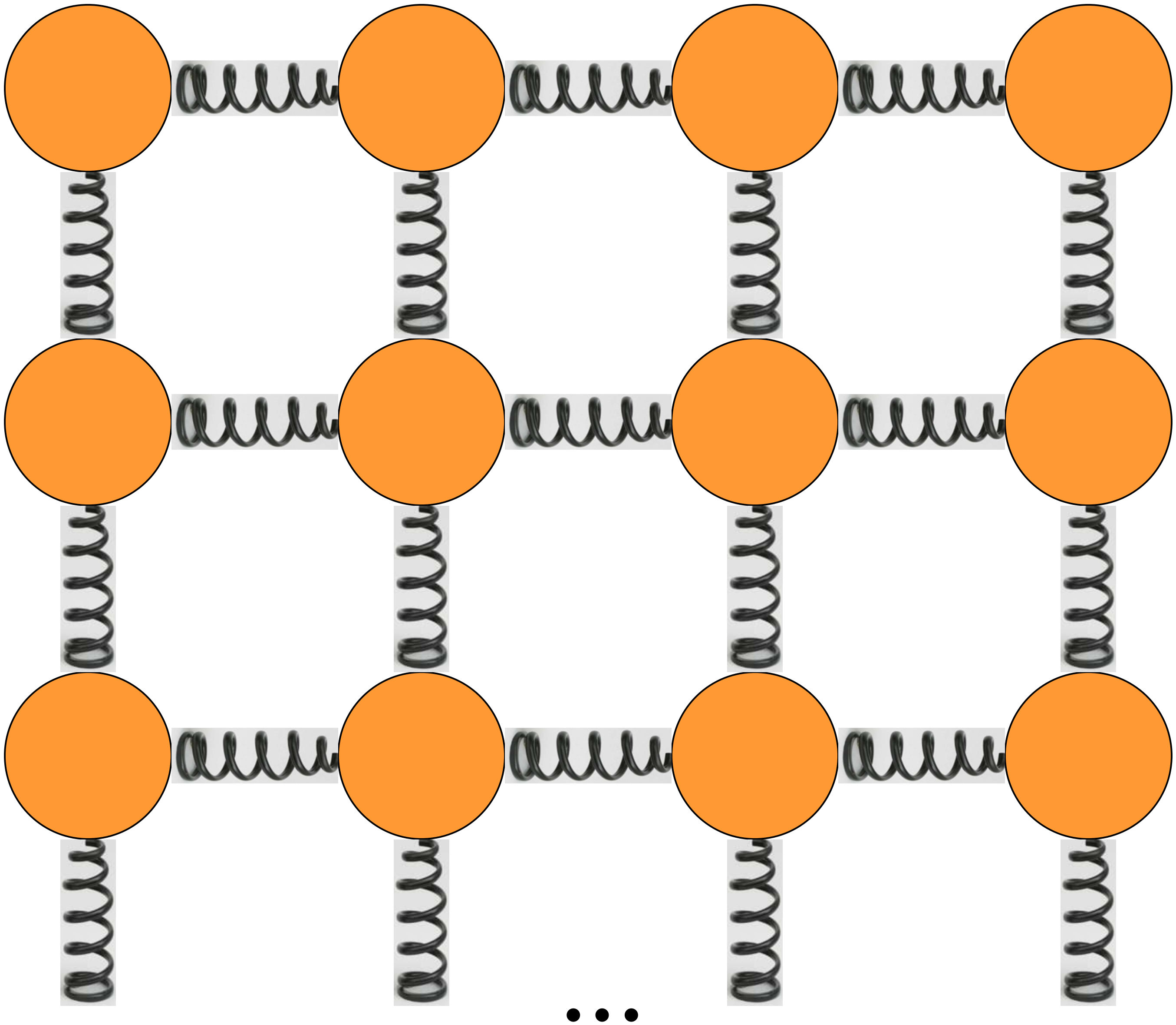
If the lumped mass is inside the circle …

Unit vector from circle center to the lumped mass.
dx = cosΘ
dy = sinΘ

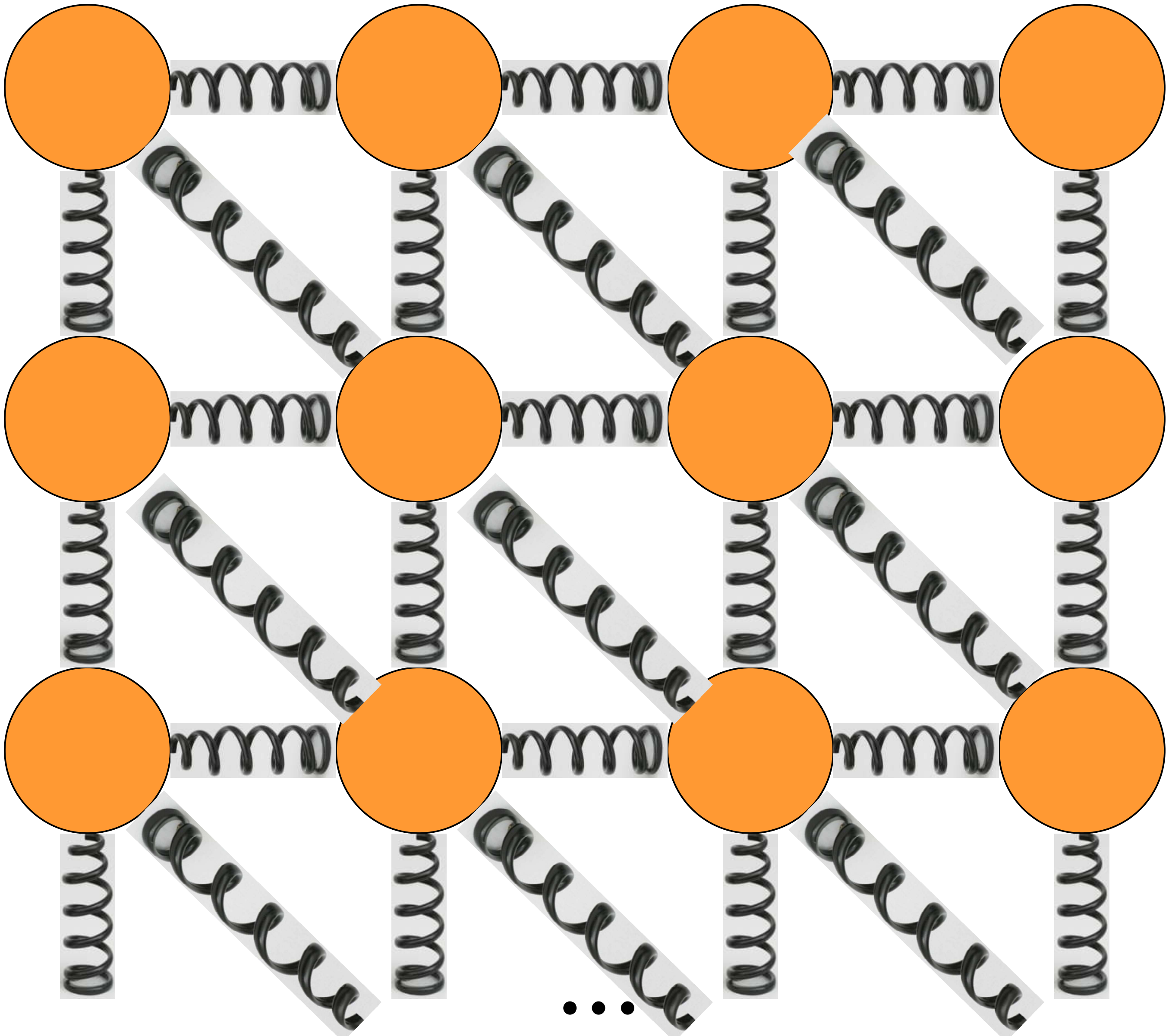
Push the lumped mass from inside the circle to the circle's surface

Keep just the tangential velocity

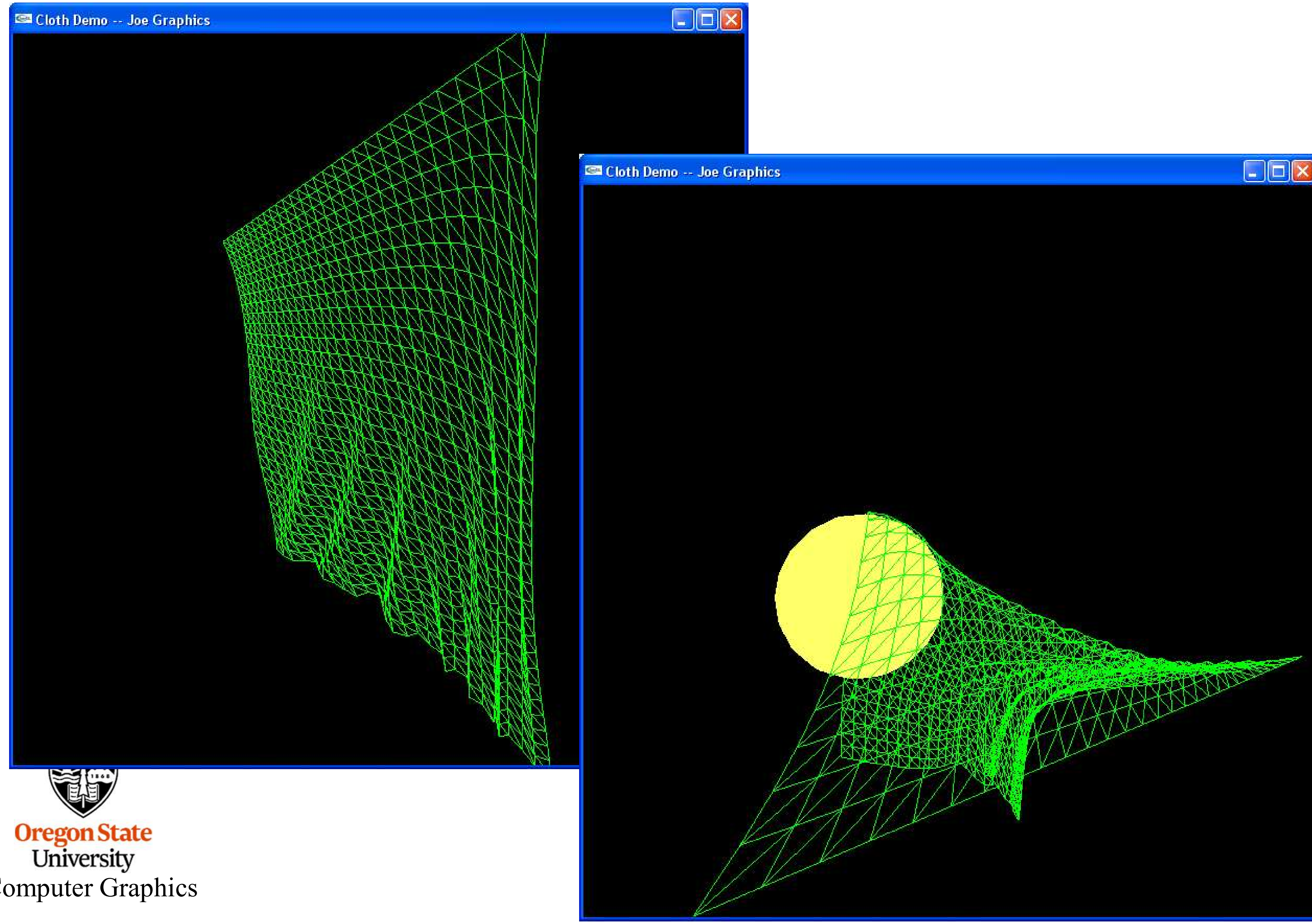
Modeling Cloth



Modeling Cloth



Cloth Example



Cloth Examples

David E. Breen Donald H. House, Michael J. Wozny: *Predicting the Drape of Woven Cloth Using Interacting Particles*

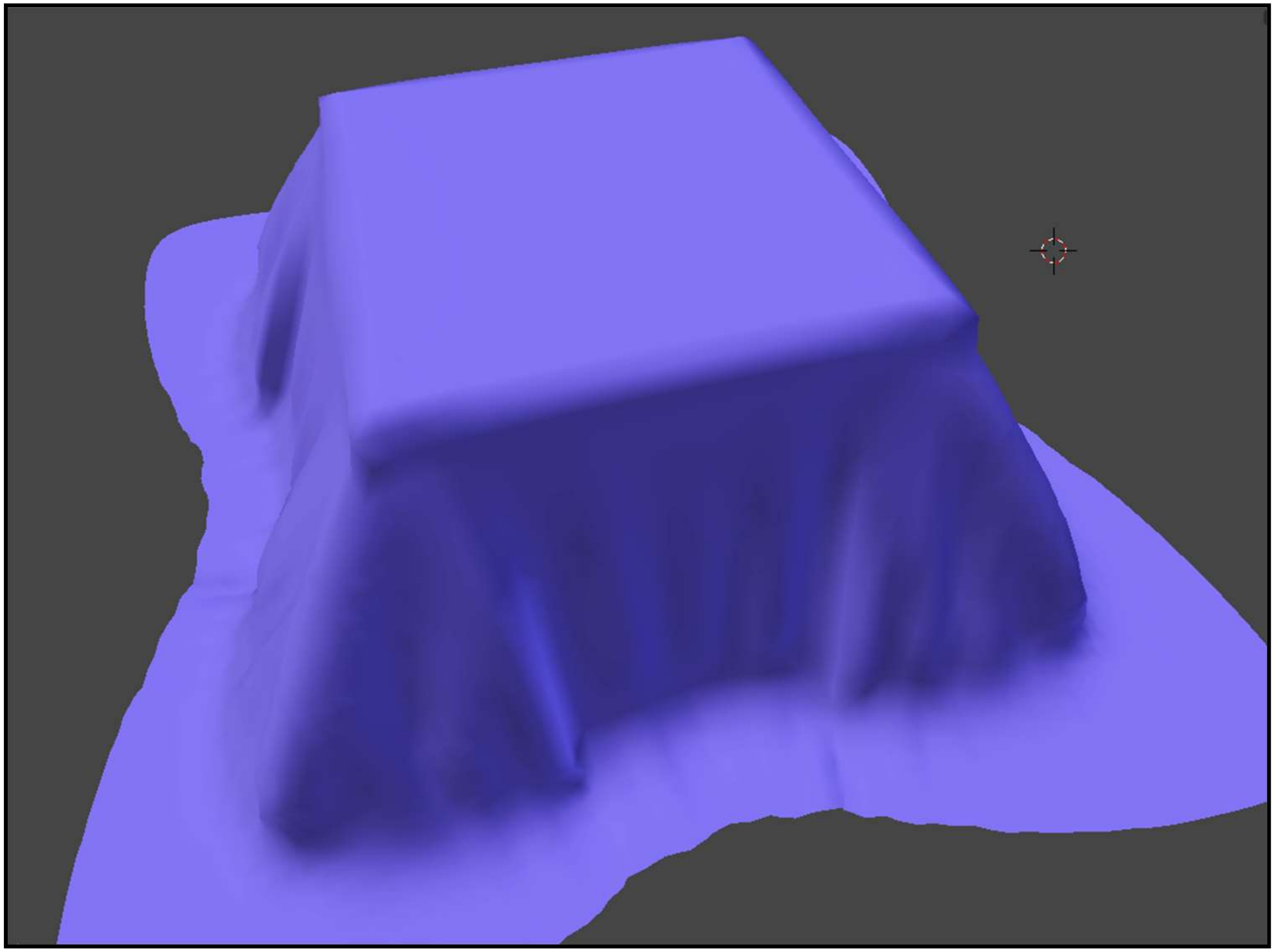
Cloth Examples

MiraLab, University of Geneva

Cloth Example

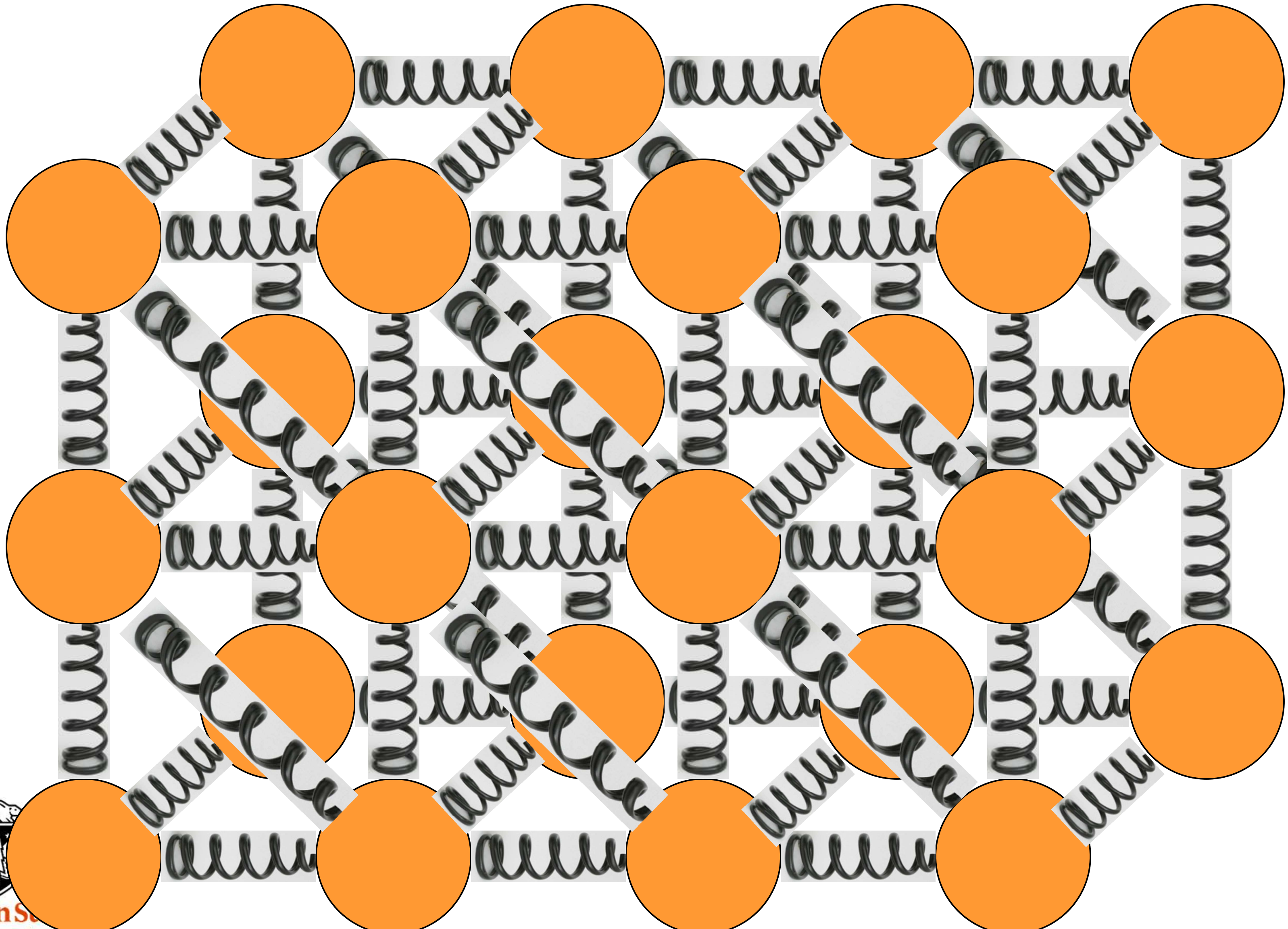
Pixar

Cloth Example with Blender

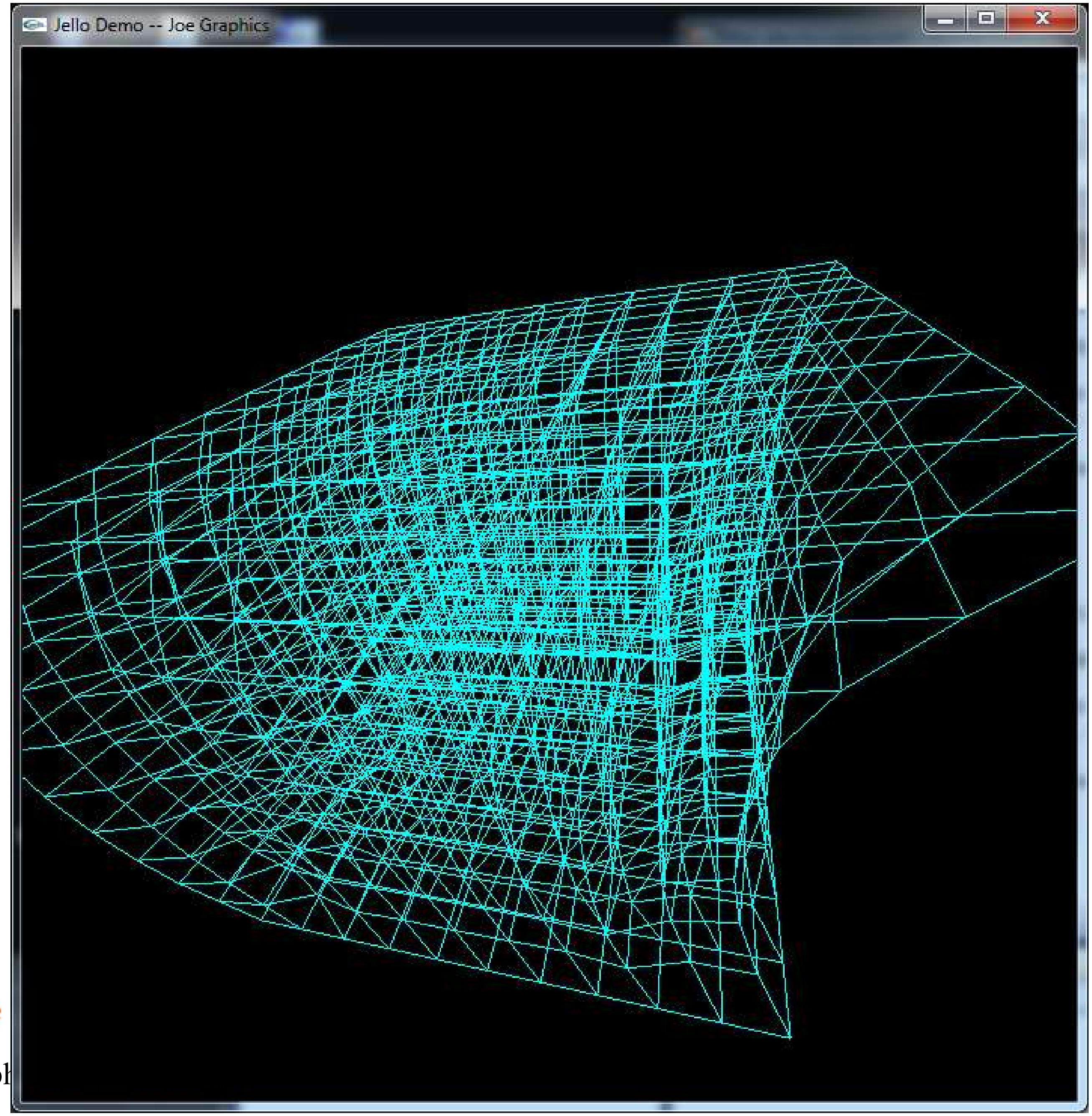


Oregon State University
Computer Graphics

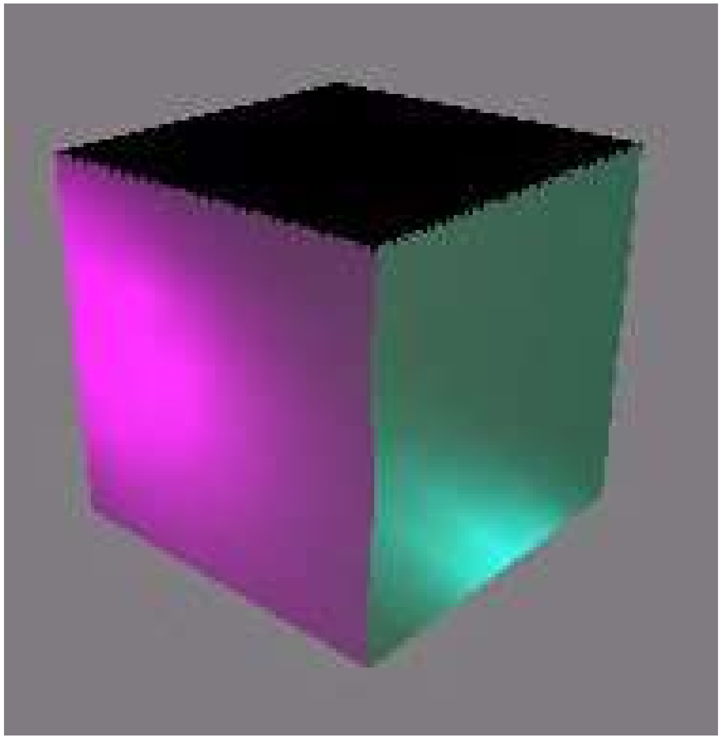
Modeling Jello

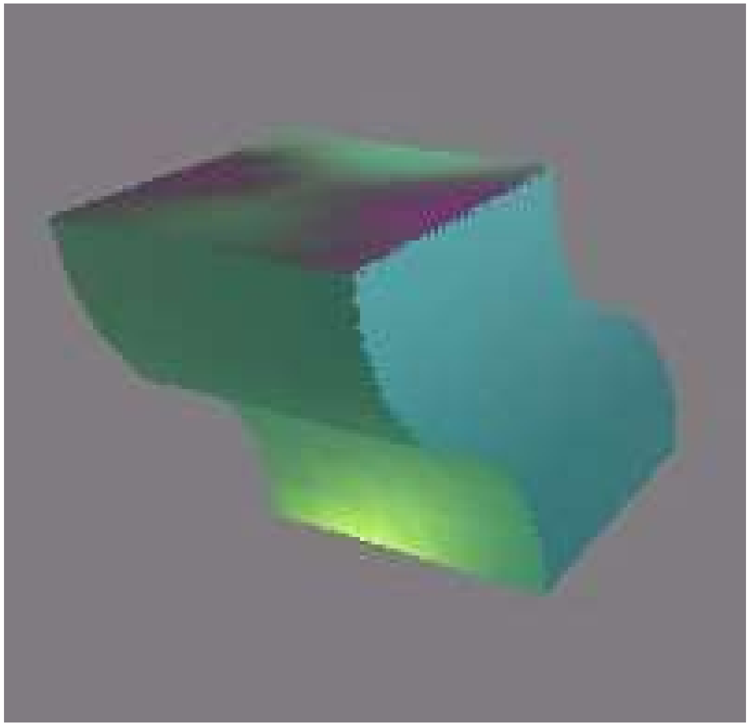


Modeling Jello



Modeling Jello





MIT

We Can Also use this Same Method to Model and Analyze Rigid Objects

California Department of Transportation

We Can Also use this Same Method to Model and Analyze Rigid Objects

OSU Johnson Hall

A Bridge on Top of Loose Soil