Modeling the World as a Mesh of Springs

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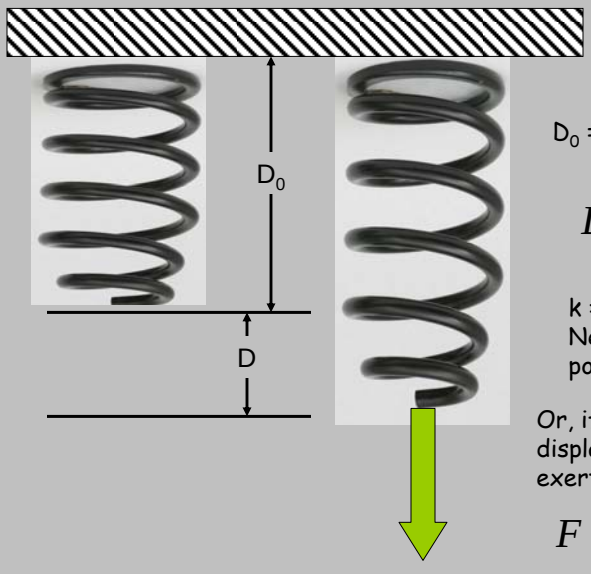
How do Springs Work?



D_0 = unloaded spring length

$$D = D_0 + \frac{F}{k}$$

k = spring stiffness in Newtons/meter or pounds/inch

Or, if you know the displacement, the force exerted by the spring must be:

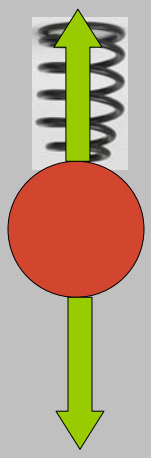
$$F = k(D - D_0)$$

Solving Motion where there is a Spring

$$F_{spring} = k(D - D_0)$$

$$v(y,t) = \int \frac{\sum F}{m} dt$$

$$v(y,t) = \int \frac{W - ky}{m} dt$$



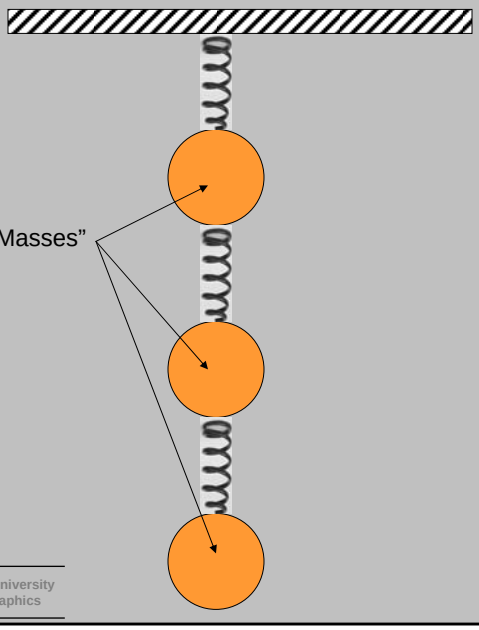
Weight

```
void
GetVelAcc( float t, float y, float vy, float *v, float *a )
{
        *v = vy;
        *a = ( W – K*y  ) /  MASS;
}
```

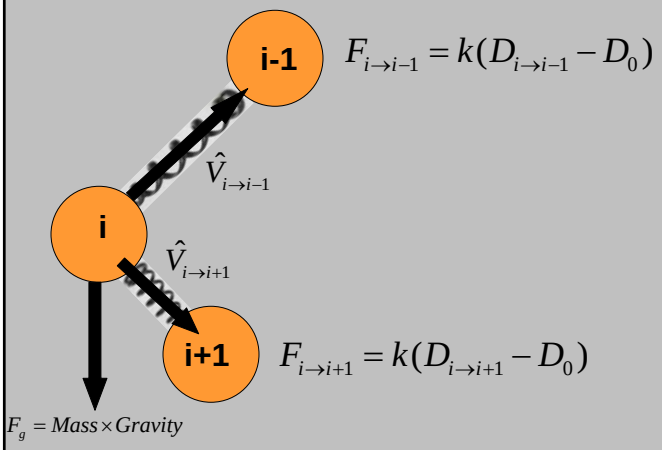
Note that, instead of using weight, we could write that last line with mass:

```
*a = ( MASS*GRAVITY – K*y  ) /  MASS;
```

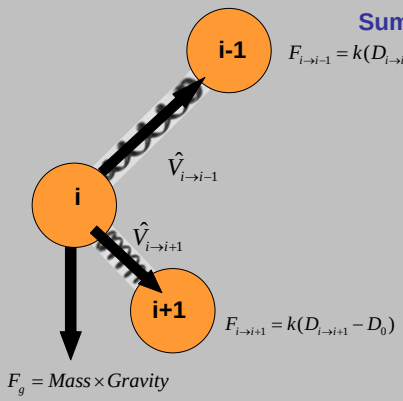

Modeling a String as a Group of Masses Connected by Springs



Computing Forces



Summing Forces



$$\sum F_{i,x} = \hat{V}_{i\to i-1,x} \cdot F_{i\to i-1} + \hat{V}_{i\to i+1,x} \cdot F_{i\to i+1} = Mass \cdot \ddot{x}$$

$$\sum F_{i,y} = \hat{V}_{i\to i-1,y} \cdot F_{i\to i-1} + \hat{V}_{i\to i+1,y} \cdot F_{i\to i+1} + F_g = Mass \cdot \ddot{y}$$

Solve for Each State as a Whole, not as Individual Links: Do it this Way

Second Order solution:

```
for( int i = 0; i < NUMLINKS; i++ )
{
        GetVelAccel( i,  &vx1[i], &vy1[i], &ax1[i], &ay1[i] );
}
for( int i = 0; i < NUMLINKS; i++ )
{
        vxtmp[i] = Links[i].vx + DT * ax1[i];
        vytmp[i] = Links[i].vy + DT * ay1[i];
        xtmp[i]  = Links[i].x  + DT * vx1[i];
        ytmp[i]  = Links[i].y  + DT * vy1[i];
}

for( int i = 0; i < NUMLINKS; i++ )
{
        GetVelAccel( i,  &vx2[i], &vy2[i], &ax2[i], &ay2[i] );
}
for( int i = 0; i < NUMLINKS; i++ )
{
        Links[i].vx = Links[i].vx  + DT * ( ax1[i] + ax2[i] ) / 2.;
        Links[i].vy = Links[i].vy  + DT * ( ay1[i] + ay2[i] ) / 2.;
        Links[i].x  = Links[i].x   + DT * ( vx1[i] + vx2[i] ) / 2.;
        Links[i].y  = Links[i].y   + DT * ( vy1[i] + vy2[i] ) / 2.;
}
```

Get *all* the velocities and accelerations

Apply *all* the velocities and accelerations

Get *all* the velocities and accelerations

Apply *all* the velocities and accelerations

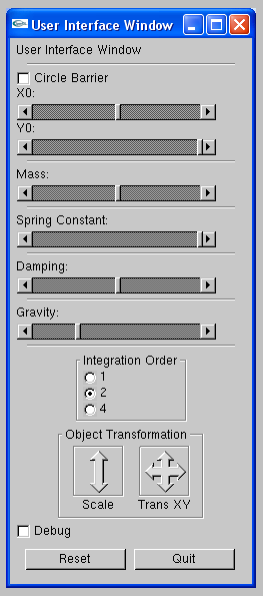
Don't do it this Way!

Second Order solution:

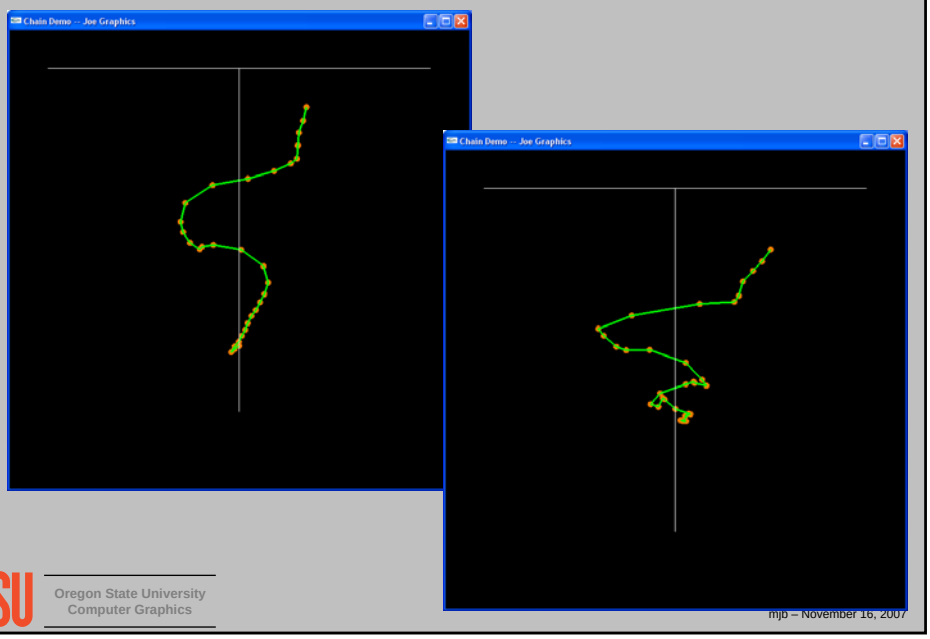
```
for( int i = 0; i < NUMLINKS; i++ )
{
        GetVelAccel( i,  &vx1[i], &vy1[i], &ax1[i], &ay1[i] );
        vxtmp[i] = Links[i].vx + DT * ax1[i];
        vytmp[i] = Links[i].vy + DT * ay1[i];
        xtmp[i]  = Links[i].x  + DT * vx1[i];
        ytmp[i]  = Links[i].y  + DT * vy1[i];
}

for( int i = 0; i < NUMLINKS; i++ )
{
        GetVelAccel( i,  &vx2[i], &vy2[i], &ax2[i], &ay2[i] );
        Links[i].vx = Links[i].vx  + DT * ( ax1[i] + ax2[i] ) / 2.;
        Links[i].vy = Links[i].vy  + DT * ( ay1[i] + ay2[i] ) / 2.;
        Links[i].x  = Links[i].x   + DT * ( vx1[i] + vx2[i] ) / 2.;
        Links[i].y  = Links[i].y   + DT * ( vy1[i] + vy2[i] ) / 2.;
}
```

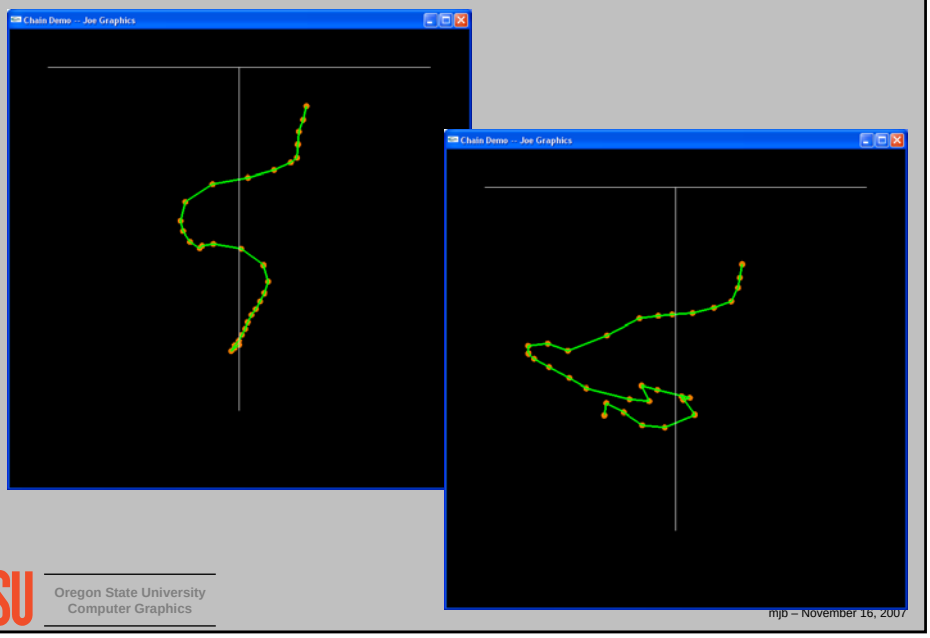
Changing Variables on-the-fly



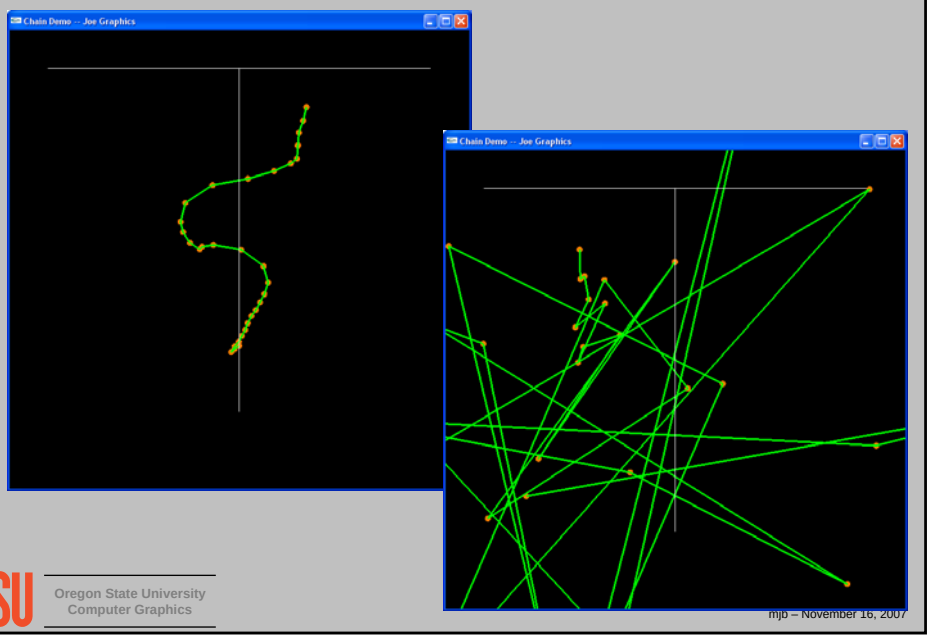
Simulating a String



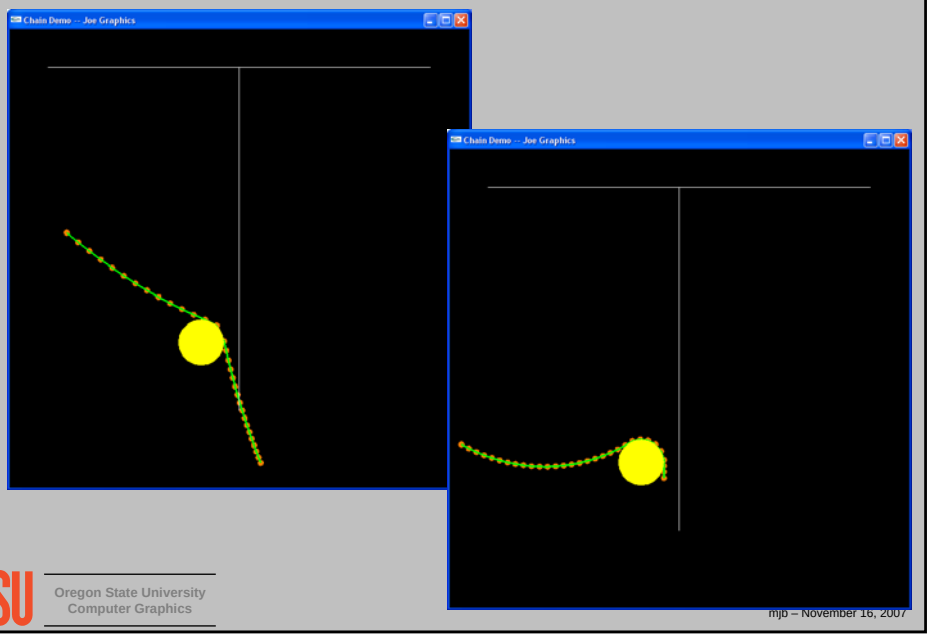
Less Damping



First Order Instability



Placing a Physical Barrier in the Scene



Placing a Physical Barrier in the Scene

```
if( DoCircle )
{
	for( int i = 0; i < NUMLINKS; i++ )
	{
		float dx = Links[i].x - CIRCX;
		float dy = Links[i].y - CIRCY;
		float rsqd = dx*dx + dy*dy;
		if( rsqd < CIRCR*CIRCR )
		{
			float r = sqrt( rsqd );
			dx /= r;
			dy /= r;
			Links[i].x = CIRCX + CIRCR * dx;
			Links[i].y = CIRCY + CIRCR * dy;
			Links[i].vx *=  dy;
			Links[i].vy *= -dx;
		}
	}
}
```

Vector from circle center to the lumped mass

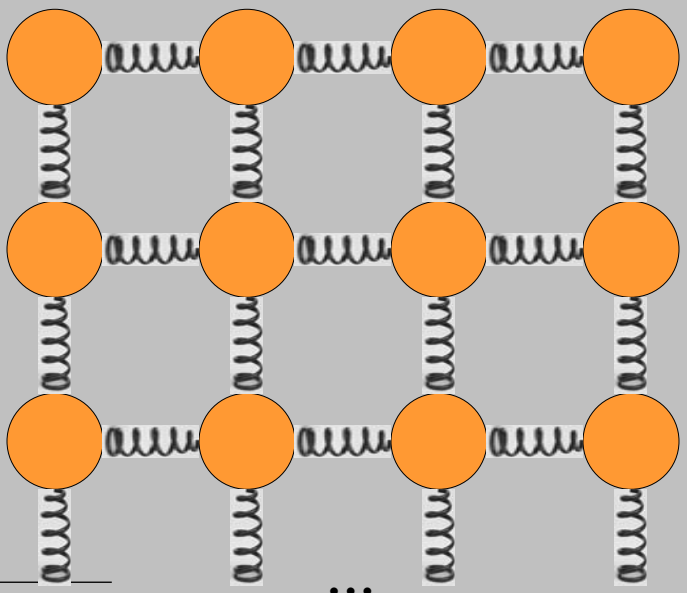
If the lumped mass is inside the circle …

Unit vector from circle center to the lumped mass

Push the lumped mass from inside the circle to the circle's surface

Keep just the tangential velocity

Modeling Cloth



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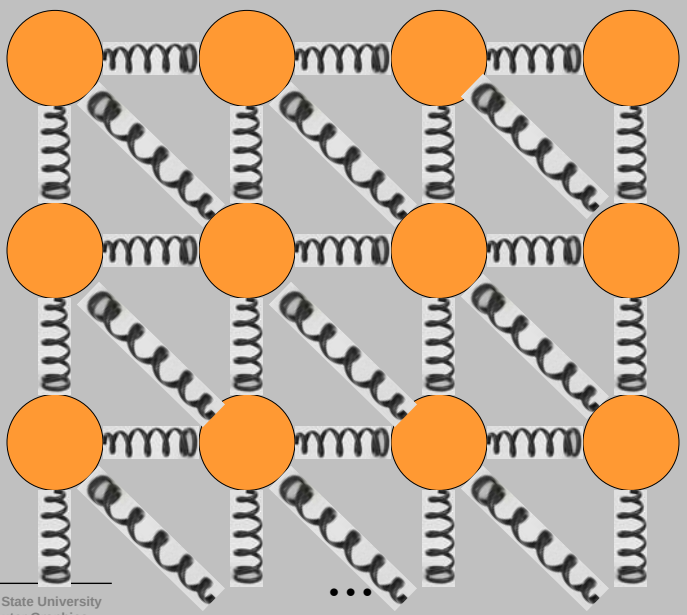
Modeling Cloth



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Examples





David E. Breen Donald H. House, Michael J. Wozny: *Predicting the Drape of Woven Cloth Using Interacting Particles Using Interacting Particles*

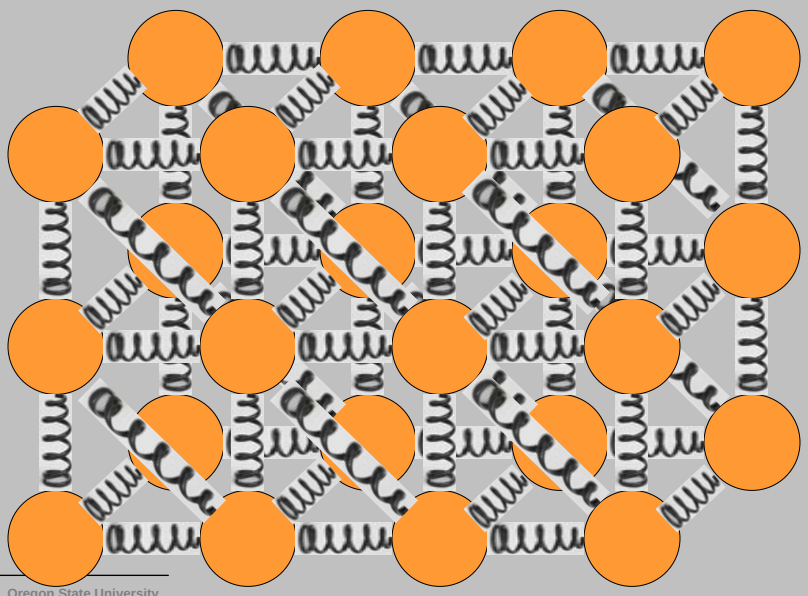
Examples

MiraLab, University of Geneva

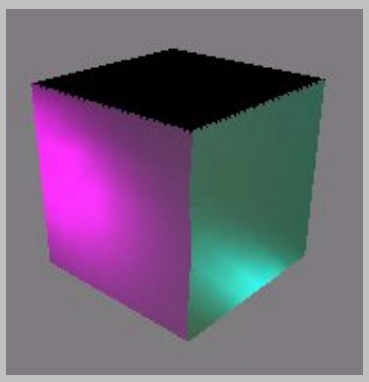
Examples

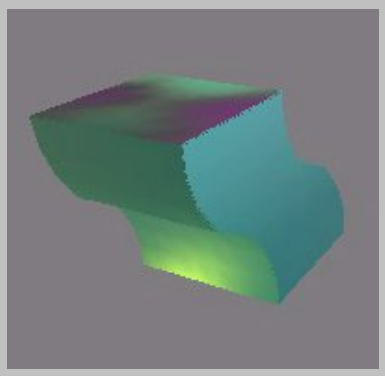
Pixar

Modeling Jello



Examples





MIT

We can also use this same Method to Model Rigid Objects

California Department of Transportation

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