

# Solving a Nonlinear Equation: Newton's Method

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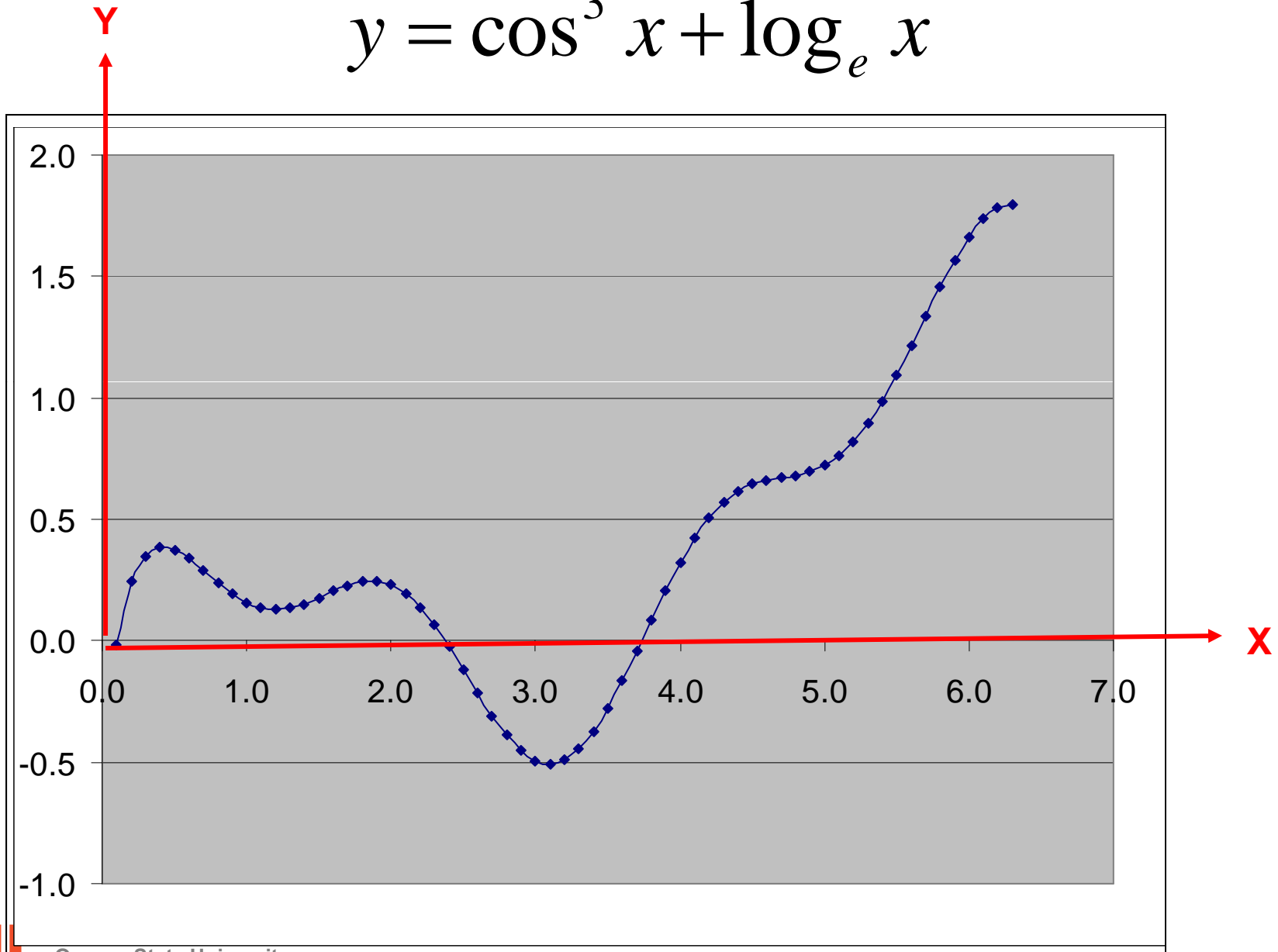
## Newton's Method for Solving a Nonlinear Equation

**Scenario: Have an equation  $y(x) = 0$ , but it is too messy to solve directly. You do have an initial guess at the correct value of  $x$ . It is close, but wrong. For example, solve:**

$$y(x) = \cos^3 x + \log_e x = 0$$

**Starting with  $x=6$**

$$y = \cos^3 x + \log_e x$$



## Newton's Method for Solving a Nonlinear Equation

You can take the  $x$  you have and plug it into the equation to produce  $y_{\text{have}}$ . Now what?

From calculus, we know that:

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} \quad \text{or} \quad \Delta y = \frac{dy}{dx} \Delta x$$

The required  $\Delta y$  is:

$$\Delta y = y_{\text{want}} - y_{\text{have}} = 0. - y_{\text{have}}$$

So that: 
$$\frac{dy}{dx} \Delta x = y_{\text{want}} - y_{\text{have}} = 0. - y_{\text{have}}$$

Or: 
$$\Delta x = \frac{-y_{\text{have}}}{\frac{dy}{dx}}$$

We will use that to find the next value of  $x$  to try, and then repeat the process:


$$x_{\text{have}} = x_{\text{have}} + \Delta x$$

$$y_{\text{have}} = y(x_{\text{have}})$$

## Watching Newton's Method Work

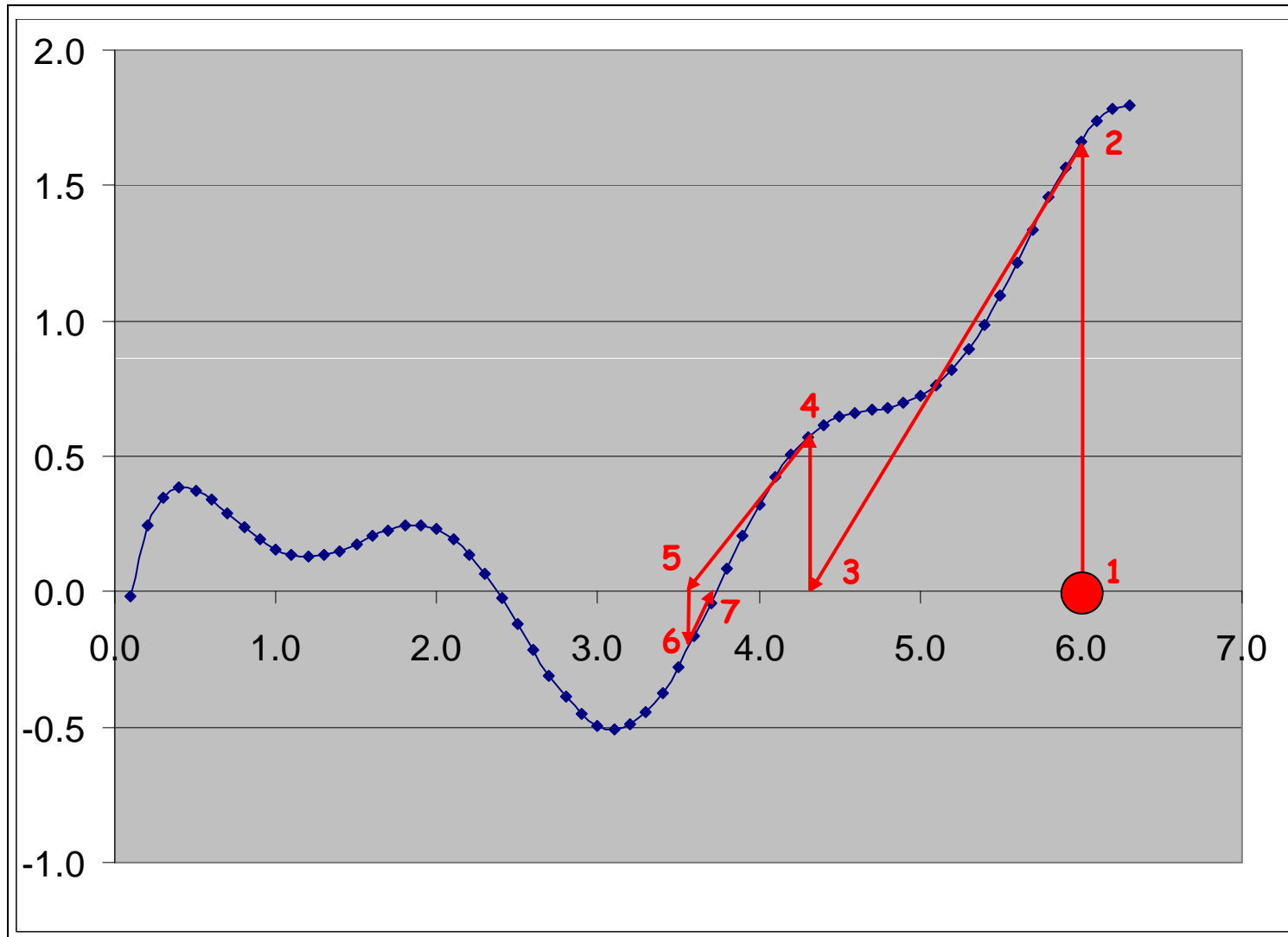
$$y = \cos^3 x + \log_e x = 0$$

$$\frac{dy}{dx} = -3 * \sin x * \cos^2 x + \frac{1}{x}$$

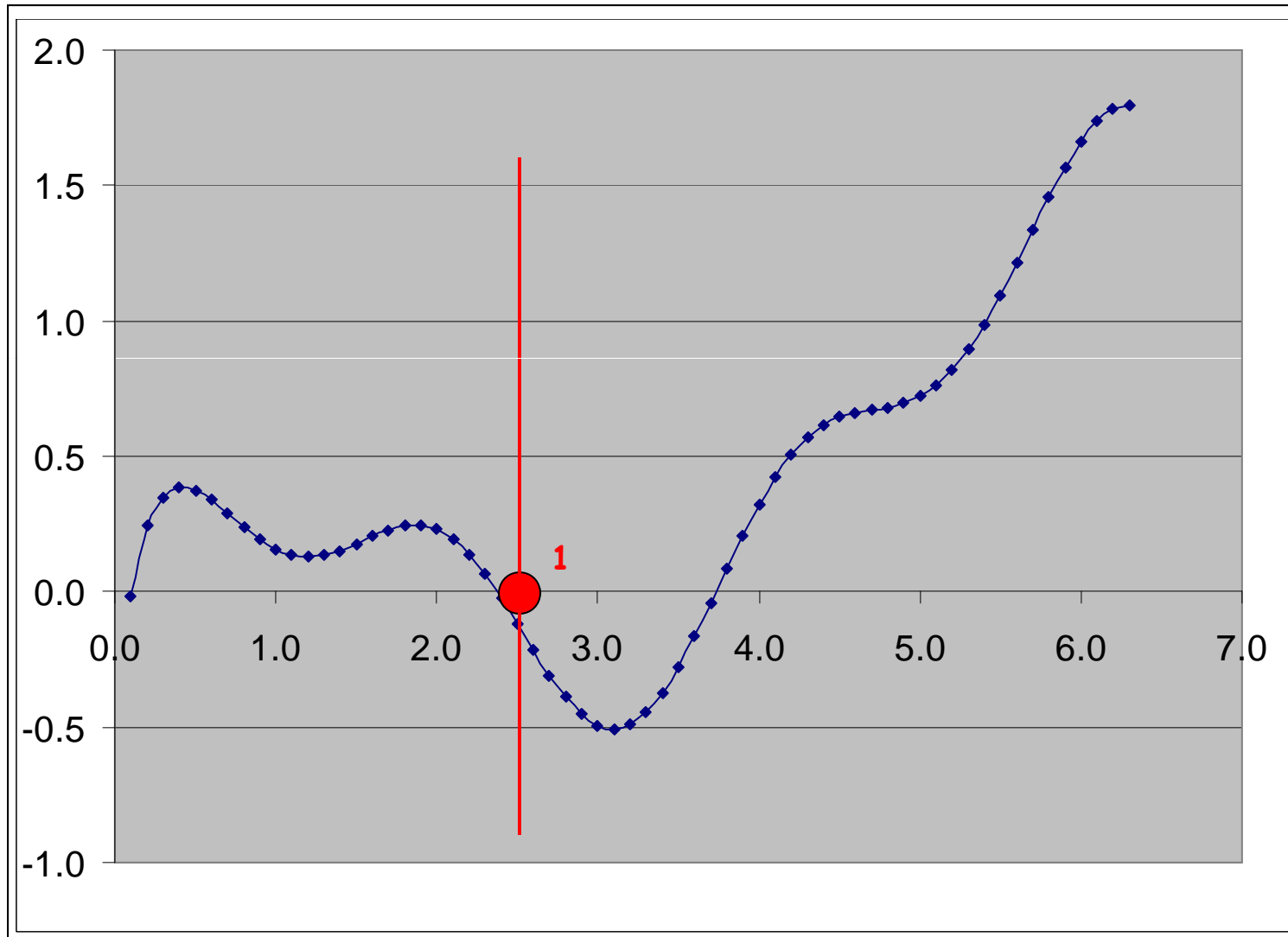


xhave	yhave	dydx	xnext
6.00000	1.66336	0.93947	4.22947
4.22947	0.52615	0.80937	3.57939
3.57939	-0.18910	1.32263	3.72236
3.72236	-0.01354	1.41914	3.73191
3.73191	-0.00144	1.42045	3.73292
3.73292	-0.00015	1.42056	3.73303
3.73303	-0.00002	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304

## Here's what is really going on



What would have happened if we had started with  $x=2.5$ ?



What would have happened if we had started with  $x=1.5$ ?

