Solving a Nonlinear Equation: Newton's Method

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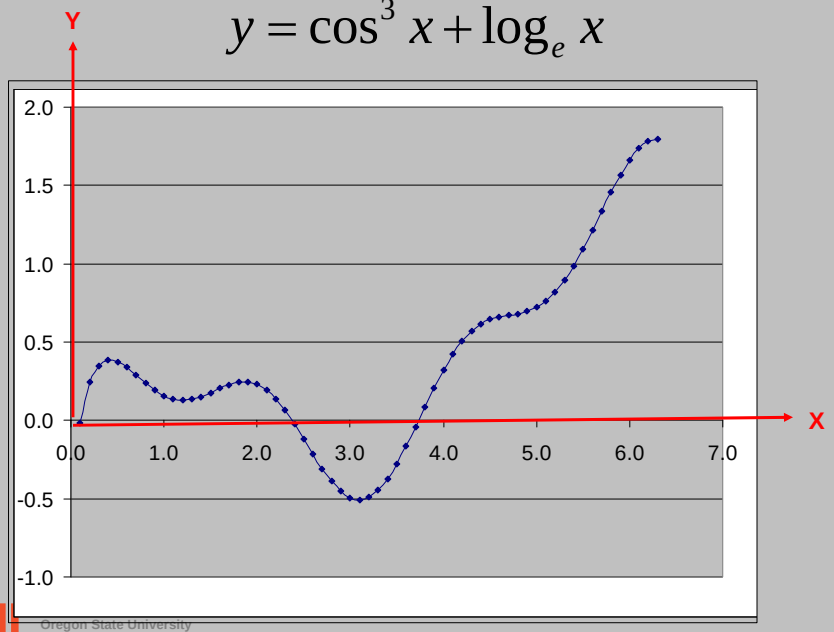
Newton's Method for Solving a Nonlinear Equation

Scenario: Have an equation y(x) = 0, but it is too messy to solve directly. You do have an initial guess at the correct value of x. It is close, but wrong. For example, solve:

$$y(x) = \cos^3 x + \log_e x = 0$$

Starting with x=6

$$y = \cos^3 x + \log_e x$$



Newton's Method for Solving a Nonlinear Equation

You can take the x you have and plug it into the equation to produce y_{have}. Now what?

From calculus, we know that:

$$\Delta y = y_{want} - y_{have} = 0. - y_{have} = \frac{dy}{dx}\Delta x$$

$$\Delta x = \frac{-y_{have}}{\frac{dy}{dx}}$$

We will use that to find the next value of x to try, and then repeat the process:

$$x_{have} = x_{have} + \Delta x$$

$$y_{have} = y(x_{have})$$

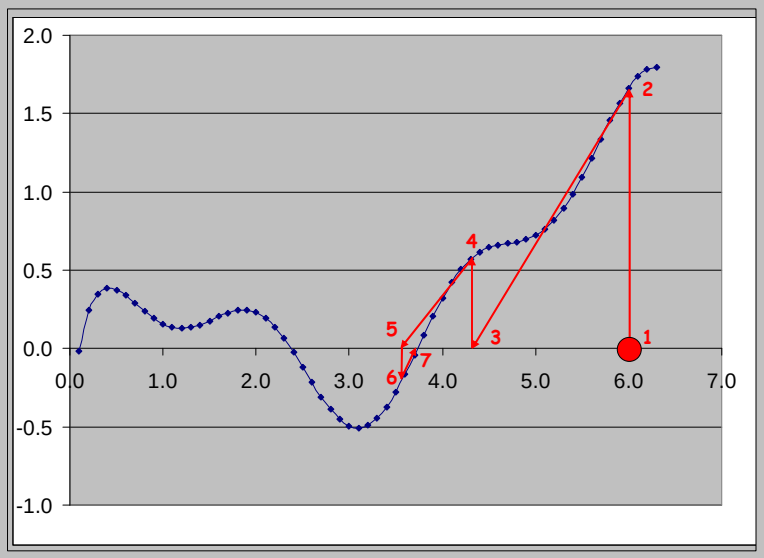
Watching Newton's Method Work

$$y = \cos^3 x + \log_e x = 0$$

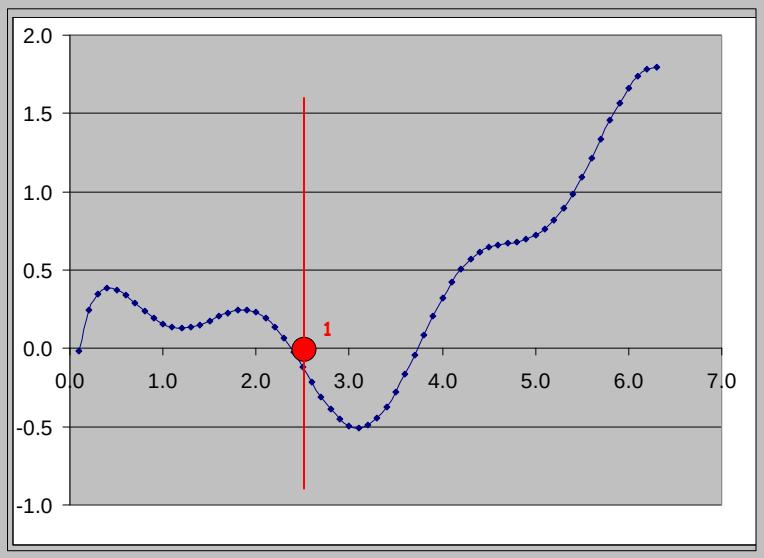
$$\frac{dy}{dx} = -3 * \sin x * \cos^2 x + \frac{1}{x}$$

xhave	yhave	dydx	xnext
6.00000	1.66336	0.93947	4.22947
4.22947	0.52615	0.80937	3.57939
3.57939	-0.18910	1.32263	3.72236
3.72236	-0.01354	1.41914	3.73191
3.73191	-0.00144	1.42045	3.73292
3.73292	-0.00015	1.42056	3.73303
3.73303	-0.00002	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304
3.73304	0.00000	1.42057	3.73304

Here's what is really going on



What would have happened if we had started with x=2.5?



What would have happened if we had started with x=1.5?