

Parametric Line Equation

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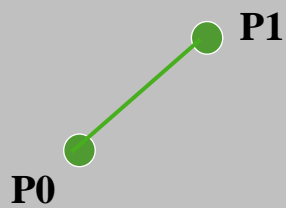
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The Line Equation:

Why " $y=mx+b$ " Isn't Good Enough for Graphics Applications Software



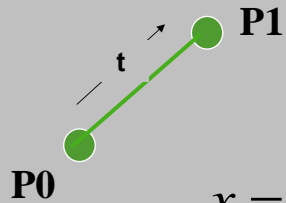
1. Cannot represent vertical lines ($m = \infty$)
2. Can only represent infinite lines, not finite line segments
3. Can only represent 2D lines, not 3D



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Parametric Line Equation



$$x = X_0 + t(X_1 - X_0)$$

$$y = Y_0 + t(Y_1 - Y_0)$$

$$z = Z_0 + t(Z_1 - Z_0)$$

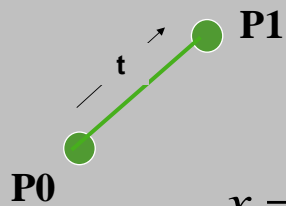


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$$0. \leq t \leq 1.$$

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Can Also Be Thought of As a Blending Function



$$x = (1-t)X_0 + tX_1$$

$$y = (1-t)Y_0 + tY_1$$

$$z = (1-t)Z_0 + tZ_1$$

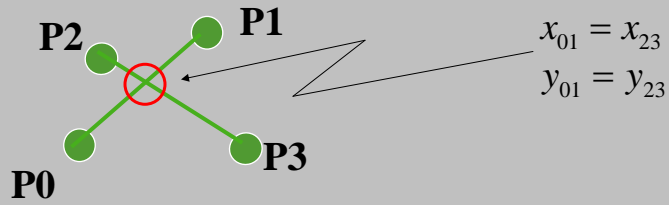


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$$0. \leq t \leq 1.$$

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Example Use: Intersection of two 2D Line Segments



$$\begin{aligned} x_{01} &= X_0 + t(X_1 - X_0) & x_{23} &= X_2 + u(X_3 - X_2) \\ y_{01} &= Y_0 + t(Y_1 - Y_0) & y_{23} &= Y_2 + u(Y_3 - Y_2) \end{aligned}$$

Since, at the point of intersection, $x_{01} = x_{23}$ and $y_{01} = y_{23}$

we have 2 equations in 2 unknowns (t and u)

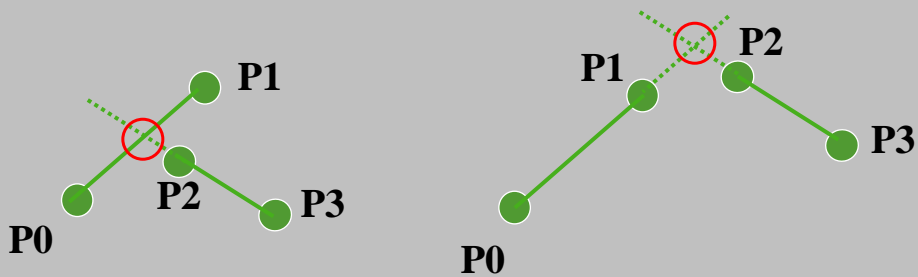
Solve for t^* and u^* . If they are not each between 0. and 1., then the infinite lines intersect, but not the finite line segments. Plug t^* and u^* back into the equations above to find (x^*, y^*) , the point of intersection.



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What if t^* or u^* is $< 0.$ or $> 1.$?



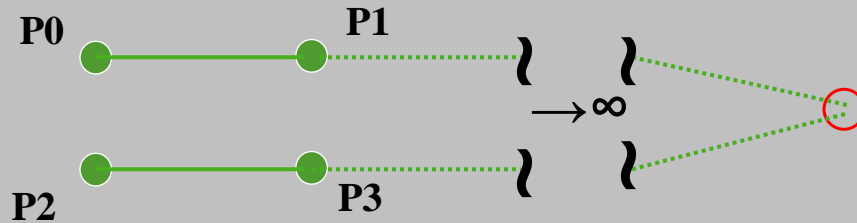
It means you have a situation like one of these. The infinite lines intersect, but not the finite line segments.



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What if the lines are Parallel ?



Well, they *do* intersect, but at infinity. This shows up in the math by the expression for t^* and u^* becoming infinitely large, that is, there would be a divide by zero.

So, you need to check the denominator before doing the division.

Since t^* and u^* would both be infinity in this case, which is most certainly > 1 , that means that the line segments do not intersect.



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Linear Blending Shows Up in a Lot of Computer Graphics Applications

You can linearly blend *any* two quantities with:

$$Q = (1 - t)Q_0 + tQ_1$$

color, shape, location, angle, scale factors, ...



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