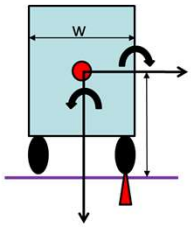
Kinematic Physics for Simulation and Game Programming



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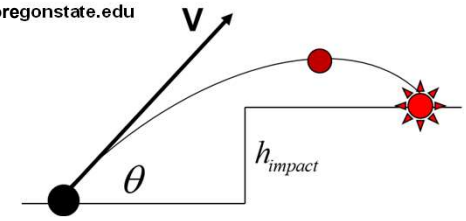

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1

SI Physics Units (International System of Units)

Quantity	Units
Linear position	Meters
Linear velocity	Meters/second
Linear acceleration	Meters/second^2
Force	Newtons (kg·m/s^2)
Energy	Joules (N·m)
Power	Watts (J/s)
Mass	Kilograms
Weight	Newtons
Density	Kilograms/meter^3
Time	Seconds
Pressure	Pascals (N/m^2)
Momentum	Kilograms-meters/second
Angular position	Radians
Angular velocity	Radians/second
Angular acceleration	Radians/second^2
Moment (=torque)	Newton-meters
Moment of Inertia	Kilogram-meters^3
Temperature	° Kelvin

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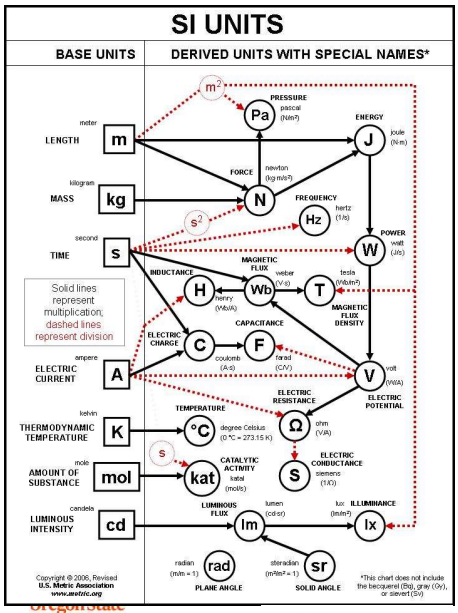
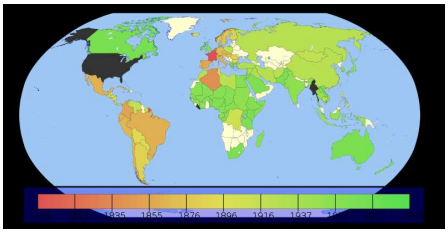
2

SI Physics Units (International System of Units)



US Metric Association

Metrification of the World:



http://en.wikipedia.org/wiki/International_System_of_Units

Some Useful Conversions

1 meter = 39.37 inches = 3.28 feet

1 mile = 1,610 meters = 1.610 kilometers

1 mile per hour = 1.467 feet per second

1 mile per hour = 0.447 meters per second

1 gallon = 3.79 liters

1 cubic foot = 7.48 gallons = 28.35 liters

1 kilogram = 2.2 pounds (mass, at Earth's surface)

1 Newton = 0.224 pounds (force)

1 pound = 4.45 Newtons (force)

1 radian = 57.3º

5

What's the Difference Between Mass and Weight?

Mass is the resistance to acceleration and deceleration. You can also think of it as *inertia* -- how difficult it is to accelerate a wagon with something in it.

Weight is the force pulling you towards the center of whatever planetary body you happen to be standing on.

On the moon, your mass would be the same as it is on Earth. It would still require the same amount of force to push you in a (frictionless) wagon.

On the moon, however, your weight would be about $^1/_6$ of what it is on Earth.

Because most of us are stuck on Earth, within a mile or two of sea level, in common practice, "mass" and "weight" designate about the same thing.

5

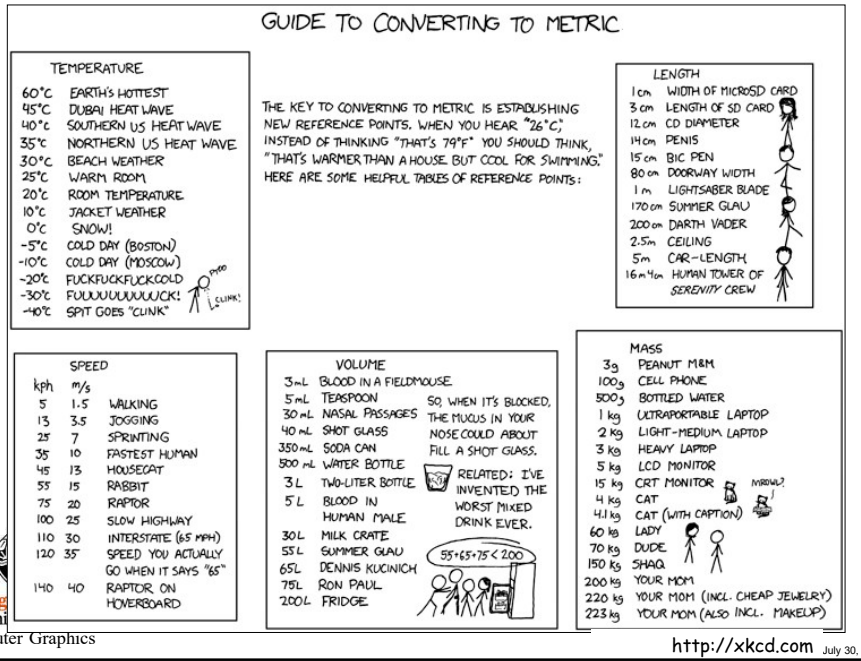
6

Some Useful Conversions

- A gram is about the mass of a paper clip
- A nickel has a mass of about 5 grams
- A liter is half of a 2-liter soda bottle, or about a fourth of a gallon of milk
- A kilogram is a little more than twice as much as a pound (on Earth)
- A Newton is about ¼ of a pound
- A meter is a little more than a yard
- A kilometer is about $^5/_8$ of a mile
- Water freezes at 0º Celsius
- A comfortable day is around 24º Celsius
- A really hot day is around 35º Celsius
- Your body temperature is about 37º Celsius

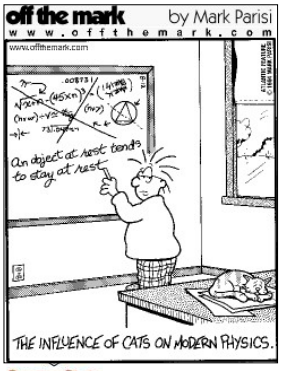
6

Some Different Useful Conversions



Newton's Three Laws of Motion

1. Every object in motion keeps that same motion (i.e., same speed and direction) unless an external force acts on it.
2. Force equals mass times acceleration (F=ma)
3. For every action, there is an equal and opposite reaction.



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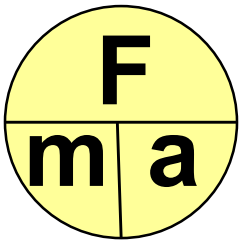

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Acceleration Due to Gravity

Newton's Gravitational Law says that the attraction force between two objects is the product of their masses times the gravitational constant G, divided by the square of the distance between them:

$$F_{12} = \frac{G m_1 m_2}{d_{12}^2} \quad \text{where: } G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

and d_{12} is the distance between body 1 and body 2

For an object, m, at or near the surface of the Earth (i.e., d_{12} is the radius of the Earth) this simplifies to:

$$F = mg \quad \text{where: } g = -9.8 \frac{meters}{\sec^2} = -32.2 \frac{feet}{\sec^2}$$

g is known as the ***Acceleration Due to (Earth's) Gravity***

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Fun facts -- gravitational acceleration on other bodies

Body	Gravity Acceleration (m/sec^2)	g's
Mercury	3.70	0.38
Venus	8.87	0.90
Earth	9.81	1.00
Moon	1.62	0.17
Mars	3.71	0.38
Jupiter	24.79	2.53
Saturn	10.44	1.06
Uranus	8.69	0.89
Neptune	11.15	1.14

https://www.universetoday.com/35565/gravity-on-other-planets/

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Constant-Acceleration Formulas
(these need to be memorized!)

$$v_1 = v_0 + at$$
$$d_1 = d_0 + v_0 t + \frac{1}{2} a t^2$$

If you are moving vertically, then the acceleration, a, will be the acceleration due to gravity, g:

$$d_y = d_{y0} + v_{y0} t + \frac{1}{2} g t^2$$

g = -9.8 meters/sec^2 = -32.2 feet/sec^2

If you are moving horizontally, there is no acceleration unless some outside horizontal force creates it.

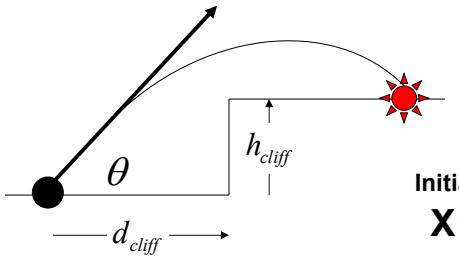
$$d_x = d_{x0} + v_{x0} t$$

The following formula is handy because it relates all the usual quantities, but doesn't require you to know the elapsed time:

$$v_1^2 = v_0^2 + 2a(d_1 - d_0)$$

Projectile Motion

$$\vec{V} = V\cos\theta, V\sin\theta$$



Initial Quantities:

X:

$$a_x = 0$$
$$v_{x0} = V\cos\theta$$

Y:

$$a_y = g$$
$$v_{y0} = V\sin\theta$$

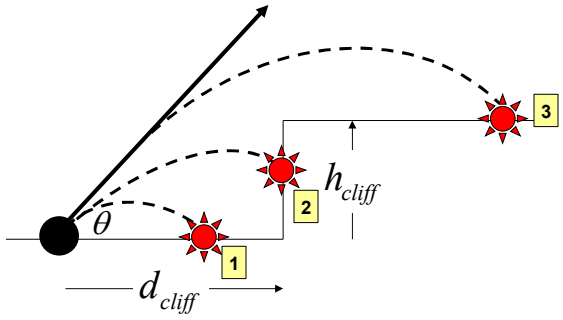
Quantities in Flight:

X: $$d_x = d_{x0} + v_{x0} t$$

Y: $$h_y = h_{y0} + v_{y0} t + \frac{1}{2} g t^2$$

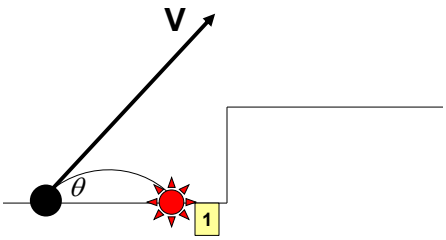
A Projectile Launches – Where Does it End Up?

$$\vec{V}_0 = V_0 \cos\theta, V_0 \sin\theta$$



Strategy: Treat each case separately. Figure out what limitation makes the projectile stop moving, calculate the time to get to that, and then see where the projectile would have ended up first.

Case 1. What if it Never Reaches the Cliff?



Y Distance Equation:

$$0. = 0. + v_{y0}t + \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 + v_{y0}t = 0$$

Note: g < 0.

Solve for the time:

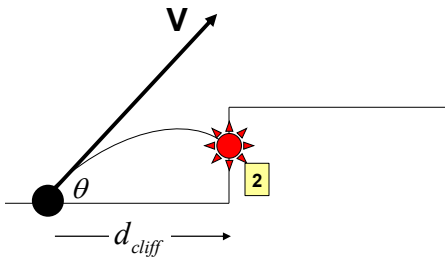
$$t(\frac{1}{2}gt + v_{y0}) = 0 \qquad t^* = 0, \frac{2v_{y0}}{-g}$$

How Far will it Go?

Solve for the X Distance:

$$d_x = d_{x0} + v_{x0} \cdot t^*$$

Case 2. What if it Hits the Side of the Cliff?



X Distance Equation:

$$d_{cliff} = 0. + v_{x0}t$$

$$\downarrow$$

$$d_{cliff} = v_{x0}t$$

Solve for the time:

$$t^* = \frac{d_{cliff}}{v_{x0}}$$

How High will it Go?

Solve for the Y Distance:

$$h_y = 0. + v_{y0}t + \frac{1}{2}gt^2$$

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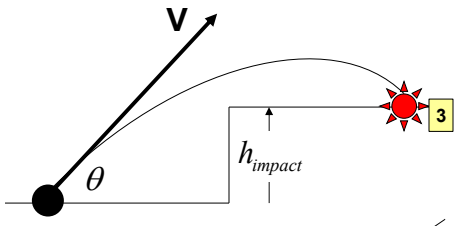
Case 3. What if it Lands on Top of the Cliff?



Y Distance Equation:

$$h_{impact} = 0. + v_{y0}t + \frac{1}{2}gt^2$$

$$\downarrow$$

$$\frac{1}{2}gt^2 + v_{y0}t - h_{impact} = 0$$

Note: g < 0.

Solve for the time:

$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 + 2gh_{impact}}}{g}$$

$$At^2 + Bt + C = 0$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Note: this will also work if $h_{impact} < 0.$

How Far will it Go?

Solve for the X Distance:

$$d_x = d_{x0} + v_{x0} \cdot t^*$$

Note: if $h_{impact} == 0.$, this becomes Case #1

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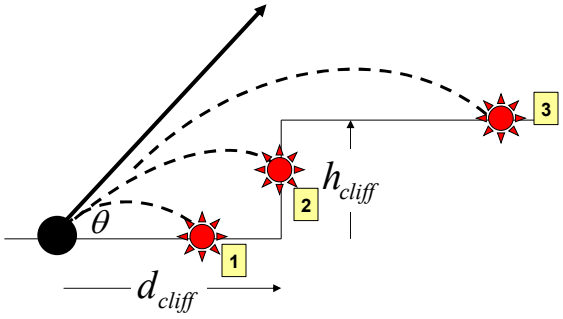
How do you decide if Case 1, 2, or 3 is the Correct Solution?



Consider Case #1: If the horizontal distance that it travels is ≤ d_{cliff}, then this is what happened.

Consider Case #2: If the height of the projectile is ≤ h_{cliff} as its horizontal distance passes d_{cliff}, then this is what happened.

Otherwise, Case #3 is what happened.

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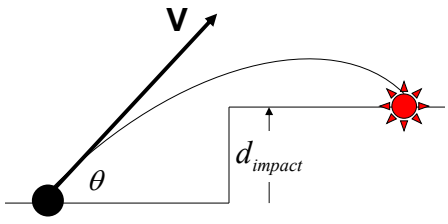
Case 3 Projectile Motion: How Long will the Projectile Stay in Flight?



$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 2g(d_{y0} - d_{impact})}}{g}$$

Why are there 2 solutions for t*?

How do you know which one is the correct one?

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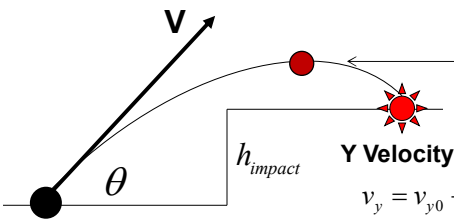
Case 1, 2, 3 Projectile Motion:
When and Where will the Projectile Reach its Maximum Height?



The y velocity at the maximum height is zero

Y Velocity Equation:

$$v_y = v_{y0} + gt = 0$$

Note: g < 0.

Solve for the time:

$$t^* = -\frac{v_{y0}}{g} = -\frac{V \sin\theta}{g}$$

How High will it Go?

Solve for the Y Distance:

$$h^* = 0. + v_{y0}t^* + \frac{1}{2}g\left(t^*\right)^2$$

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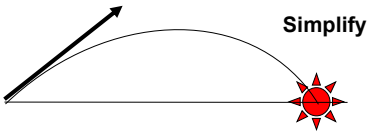
Let's Try it with Some Numbers

Find how much time it takes for the projectile to hit the ground.
Find how far the projectile travels horizontally before hitting the ground.
Find the maximum height the projectile reaches before starting back down.

V=(10.,10.) meters/sec

Simplify *g* to be -10 meters/sec^2



1. Deal with just the Y component first. What is the equation that relates distance travelled to initial velocity and gravitational acceleration?

$$d_{impact} = d_{y0} + v_{y0}t + \frac{1}{2}gt^2$$

2. Solve for t when $d_{impact}=d_{y0}$. Why are there 2 answers? What are they? Which one do you want?

$$d_{y0} = d_{y0} + 10t - \frac{1}{2}10t^2$$

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3. Now deal with the X component. What equation relates distance traveled to initial velocity and (zero) acceleration?

$$d_x = d_{x0} + v_{x0}t$$

4. Plug in the *t* you got in step #2. How far did the projectile travel?

$$d_x = 0 + 10t$$

5. Now deal with the maximum height. What is the Y velocity when the projectile reaches the maximum height?

0.0

6. What equation relates velocity achieved to initial velocity and distance travelled? (Hint: there is one that doesn't need *t*.)

$$v_1^2 = v_0^2 + 2a(d_1 - d_0)$$

7. Solve it for $(d_1\text{-}d_0)$.

$$0^2 = 10^2 - 2\square 10(d_1 - d_0)$$

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The Physics of Bouncing Against a Floor or Wall

To treat the case of an object bouncing against an immoveable object, such as a floor or a solid wall, the resulting velocity is:

$$v' = -ev$$

Where *e* is the Coefficient of Restitution (CoR), and is a measure of how much energy is retained during the bounce.

The amount of energy actually retained during the collision is:

$$E_{retained} = e^2$$

Which means that:

$$E_{lost} = \left(1 - e^2\right)$$

Balls Bounced on a Concrete Surface:

Ball Material	CoR
range golf ball	0.858
tennis ball	0.712
billiard ball	0.804
hand ball	0.752
wooden ball	0.603
steel ball bearing	0.597
glass marble	0.658
ball of rubber bands	0.828
hollow, hard plastic ball	0.688

http://hypertextbook.com/facts/2006/restitution.shtml

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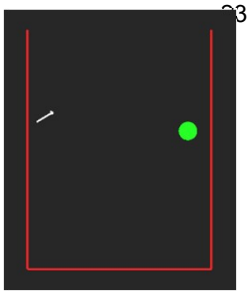
A Ball Bouncing in a Box



Current Position = (x,y)
Current Velocity = (v_x,v_y)
How long until the *next* bounce?

$$x' = x + v_x t = x_{left} + radius$$

$$x' = x + v_x t = x_{right} - radius$$

$$y' = y + v_y t + \frac{1}{2} g t^2 = y_{floor} + radius \rightarrow t^* = \frac{-v_y \pm \sqrt{v_y^2 - 2g\left(y - y_{floor} - radius\right)}}{g}$$

1. Figure out which equation produces the ***minimum positive*** t value
2. Advance the ball that much
3. Perform the bounce in the proper direction
4. Handle the rest of the time step

$$v_x' = -e v_x$$
$$v_y' = v_y$$

or:

$$v_x' = v_x$$
$$v_y' = -e v_y$$

If no bounce:

$$x' = x + v_x t$$
$$y' = y + v_y t + \frac{1}{2} g t^2$$
$$v_x' = v_x$$
$$v_y' = v_y + gt$$

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The Physics of Bouncing Against a Floor or Wall


```
void
Bounce( float dt )
{
	while( dt > EPSILON )
	{
		float tmin = dt;			// minimum time to do something
		int which = NOTHING_HIT;		// which reason was it for doing the something:

		float  tleft = ????;  // time to hit the left wall
		if( tleft > 0.  &&  tleft < tmin )
		{
			tmin = tleft;
			which = HIT_LEFT;
		}

		float tright = ????;    // time to hit the right wall
		  . . .
		float tfloor1 = ????;   // time to hit the floor
		  . . .
		float tfloor2 = ????;   // time to hit the floor (note there are 2 answers)
		  . . .

		// tmin is now set to the smallest positive of: dt, tleft, tright, tfloor1, tfloor2
		// which is set to:
		//	NOTHING_HIT, HIT_LEFT, HIT_RIGHT, HIT_FLOOR1, or HIT_FLOOR2
		// to show what was the first thing hit
```

$$x + v_x t = x_{left} + radius$$

$$t_{left} = \frac{x_{left} + radius - x}{v_x}$$

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The Physics of Bouncing Against a Floor or Wall


```
// take a time step of time length tmin, using the projectile motion equations:
// if a bounce is going to occur, tmin takes the ball right up to the surface:

Xnow  = ????;
Ynow  = ????;
Vxnow = ????;
Vynow = ????;
   // change the proper velocity component:
   // if nothing was hit in this time step, just return:

switch( which )
{
      case NOTHING_HIT:
            return;
      case HIT_LEFT:
            ?????;
            break;
      case HIT_RIGHT:
            ?????
            break;
      case HIT_FLOOR1:
            ?????
            break;
      case HIT_FLOOR2:
            ?????
            break;
}

dt -= tmin;    // after the bounce, we might still have some time step left
```

$$x' = x + v_x t$$
$$y' = y + v_y t + \frac{1}{2} g t^2$$
$$v_x' = v_x$$
$$v_y' = v_y + gt$$

$$v_x' = -ev_x$$
$$v_y' = v_y$$

or:

$$v_x' = v_x$$
$$v_y' = -ev_y$$