Physics

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Physics

Quantity	Units
Linear position	Meters
Linear velocity	Meters/second
Linear acceleration	Meters/second^2
Force	Newtons
Mass	Kilograms
Weight	Newtons
Density	Kilograms/meter^3
Time	Seconds
Pressure	Newtons/meter^s
Momentum	Kilograms*meters/second
Angular position	Radians
Angular velocity	Radians/second
Angular acceleration	Radians/second^2
Moment (=torque)	Newtons*meters
Moment of Inertia	Kilograms*meters^3
Temperature	º Celsius

Some Useful Conversions

1 meter = 39.37 inches = 3.28 feet

1 mile = 1,610 meters = 1.610 kilometers

1 mile per hour = 1.467 feet per second

1 mile per hour = 0.447 meters per second

1 gallon = 3.79 liters

1 cubic foot = 7.48 gallons = 28.35 liters

1 kilogram = 2.2 pounds (mass, at Earth's surface)

1 Newton = 0.224 pounds (force)

1 pound = 4.45 Newtons (force)

1 radian = 57.3º

Some Useful Conversions

- A gram is about the mass of a paper clip
- A nickel has a mass of about 5 grams
- A liter is half of a soda 2-liter bottle, or about a fourth of a gallon of milk
- A kilogram is twice as much as a pound
- A Newton is about ¼ of a pound
- A meter is a little more than a yard
- A kilometer is about $^3/_5$ of a mile
- Water freezes at 0º Celsius
- A hot day is 30º Celsius

Acceleration Due to Gravity

Newton's Gravitational Law says that the attraction between two objects is the product of their masses times the gravitational constant G, divided by the square of the distance between them:

$$F = \frac{G m_1 m_2}{d_{12}^2}$$

where: $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

and d_{12} is the distance between body 1 and body 2

For an object at or near the surface of the Earth, this simplifies to:

$$F = mg$$

where: $g = 9.8 \frac{m}{\sec^2}$

g is known as the ***Acceleration Due to Gravity***

Constant-Acceleration Formulas

$$v_1 = v_0 + at$$

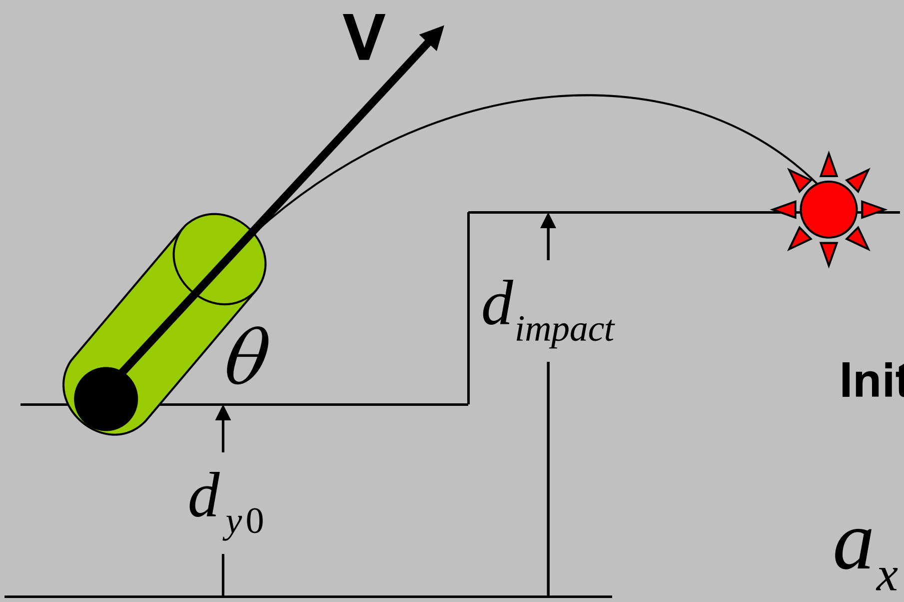
$$v_1^2 = v_0^2 + 2a(d_1 - d_0)$$

$$d_1 = d_0 + v_0 t + \frac{1}{2}at^2$$

Note: as long as you are near the surface of the Earth, *g* is considered to be "constant acceleration":

$$d_y = d_{y0} + v_{y0} t - \frac{1}{2}gt^2$$

Projectile Motion



Initial Quantities:

$$a_x = 0 \qquad a_y = -g$$

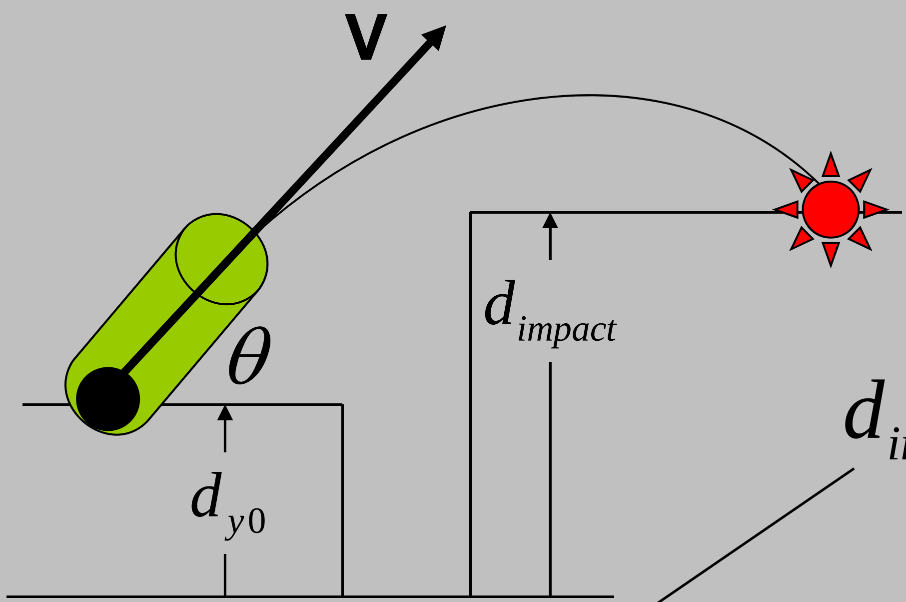
$$v_{x0} = V\cos\theta \qquad v_{y0} = V\sin\theta$$

Quantities in Flight:

$$d_x = d_{x0} + v_{x0}t$$

$$d_y = d_{y0} + v_{y0}t - \frac{1}{2}gt^2$$

Projectile Motion:
How Long will the Projectile Stay in Flight?



Y Distance Equation:

$$d_{impact} = d_{y0} + v_{y0}t - \frac{1}{2}gt^2$$

$$At^2 + Bt + C = 0$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Solve for the time:

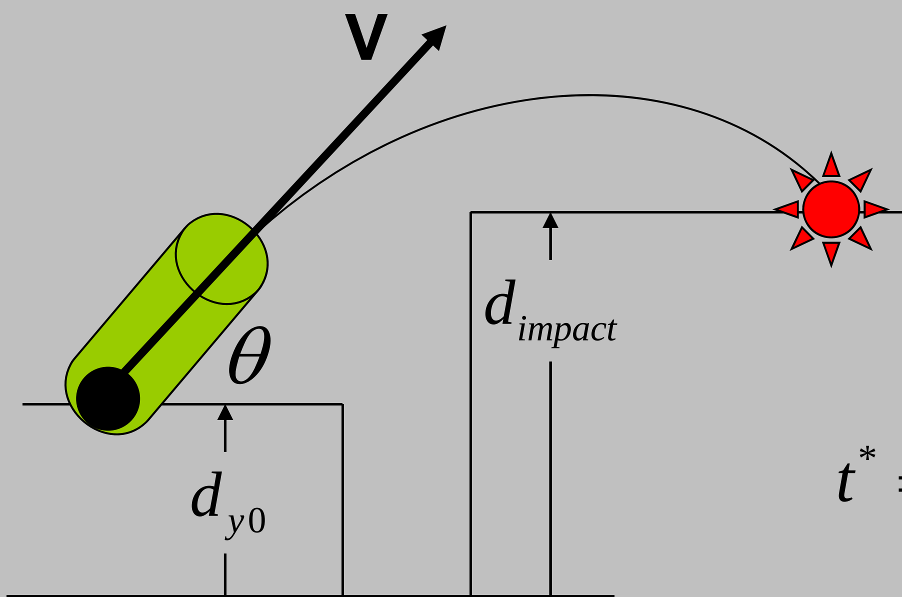
$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 + 2g(d_{y0} - d_{impact})}}{-g}$$

How Far will it Go?

Solve for the X Distance:

$$d_x = d_{x0} + v_{x0} \cdot t^*$$

Projectile Motion:
How Long will the Projectile Stay in Flight?

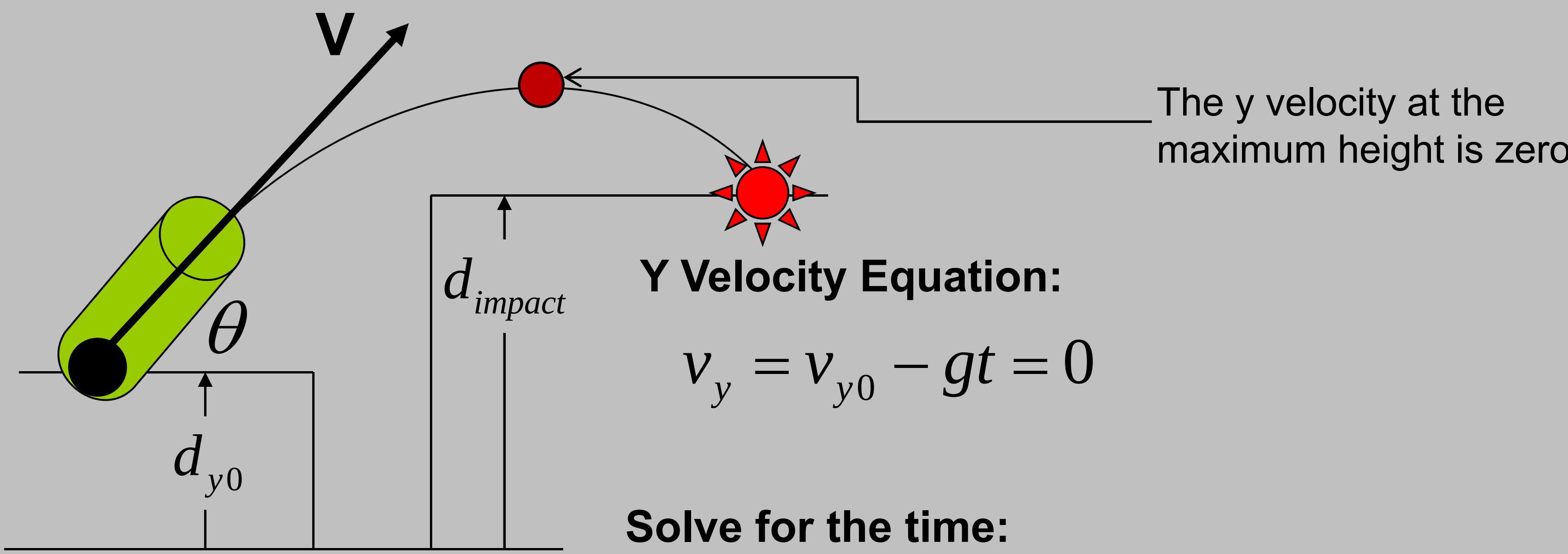


$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 + 2g(d_{y0} - d_{impact})}}{-g}$$

Why are there 2 solutions for t*?

How do you know which one is the correct one?

Projectile Motion:
When and Where will the Projectile Reach its Maximum Height?



Y Velocity Equation:

$$v_y = v_{y0} - gt = 0$$

Solve for the time:

$$t^* = \frac{v_{y0}}{g} = \frac{V \sin\theta}{g}$$

How High will it Go?

Solve for the Y Distance:

$$d^* = d_{y0} + v_{y0} t^* - \frac{1}{2} g \left(t^*\right)^2$$

Dynamics

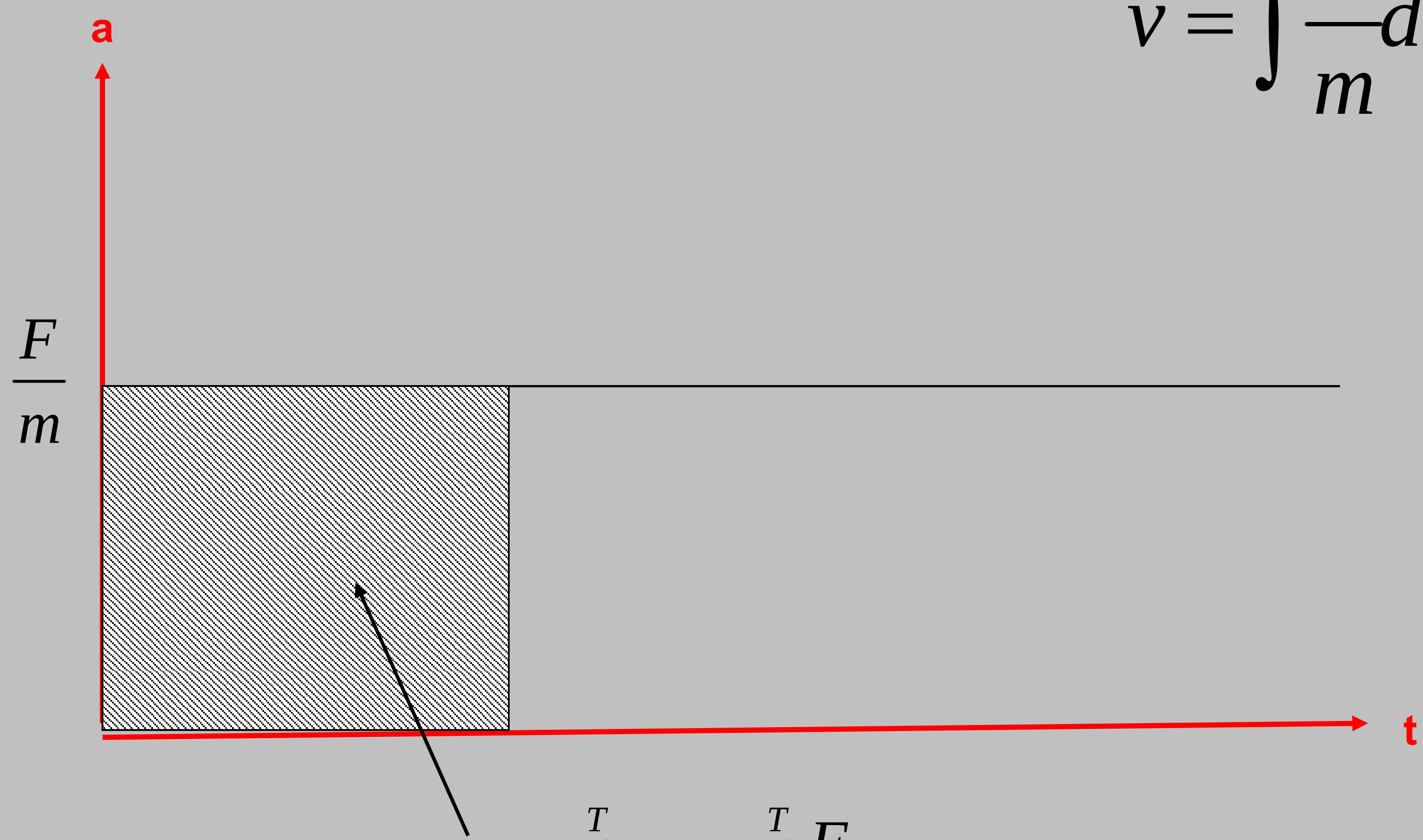
$$F = ma = m\frac{d^2x}{dt^2} \equiv \ddot{x}$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{F}{m}$$

$$v = \int a\,dt = \int \frac{F}{m}dt \qquad x = \int v\,dt$$

Integrating the Physics Equations

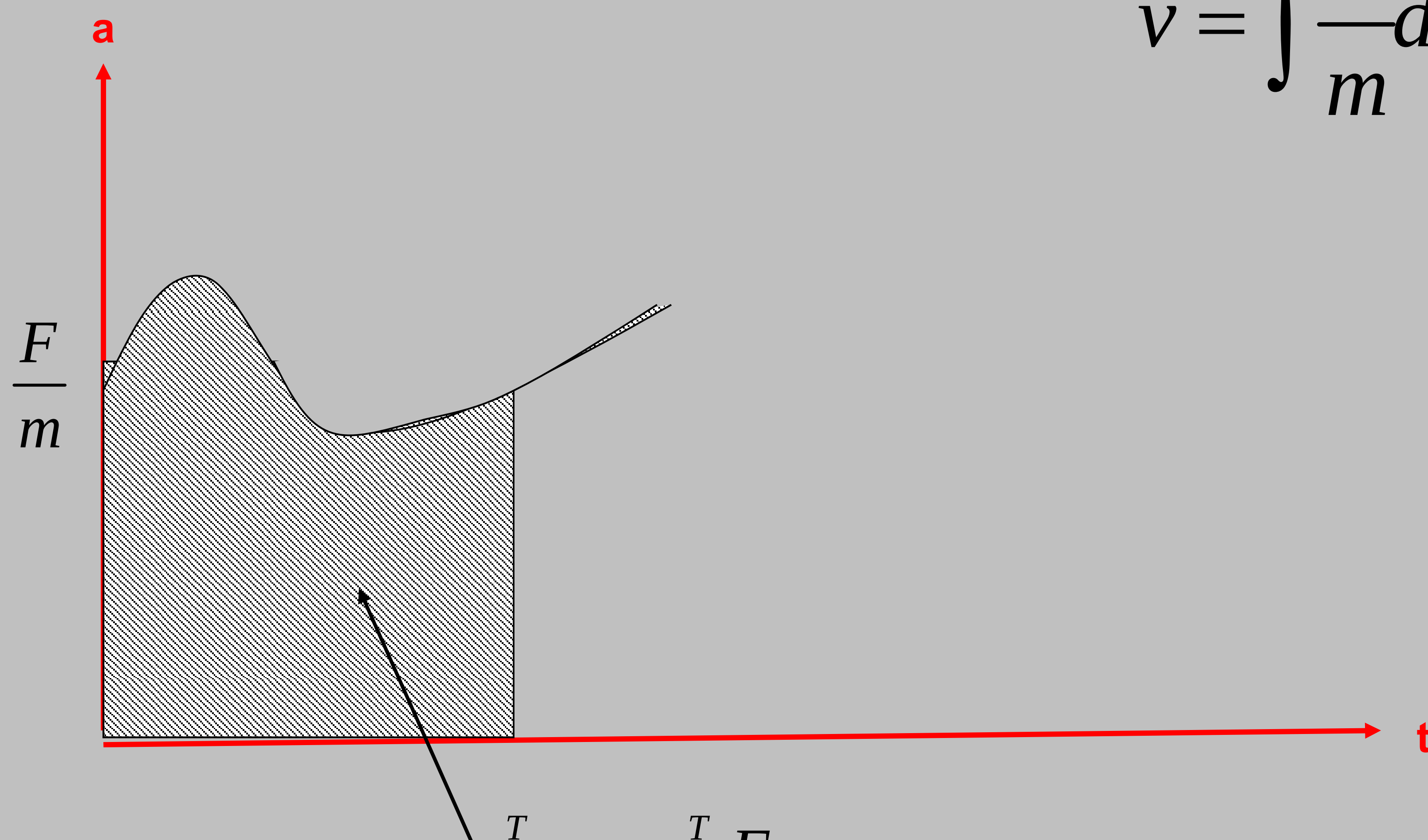
$$v = \int \frac{F}{m} dt$$



$$area = \int_0^T a\,dt = \int_0^T \frac{F}{m} dt = velocity\ after\ time\ T$$

What if Force and mass are not constant?

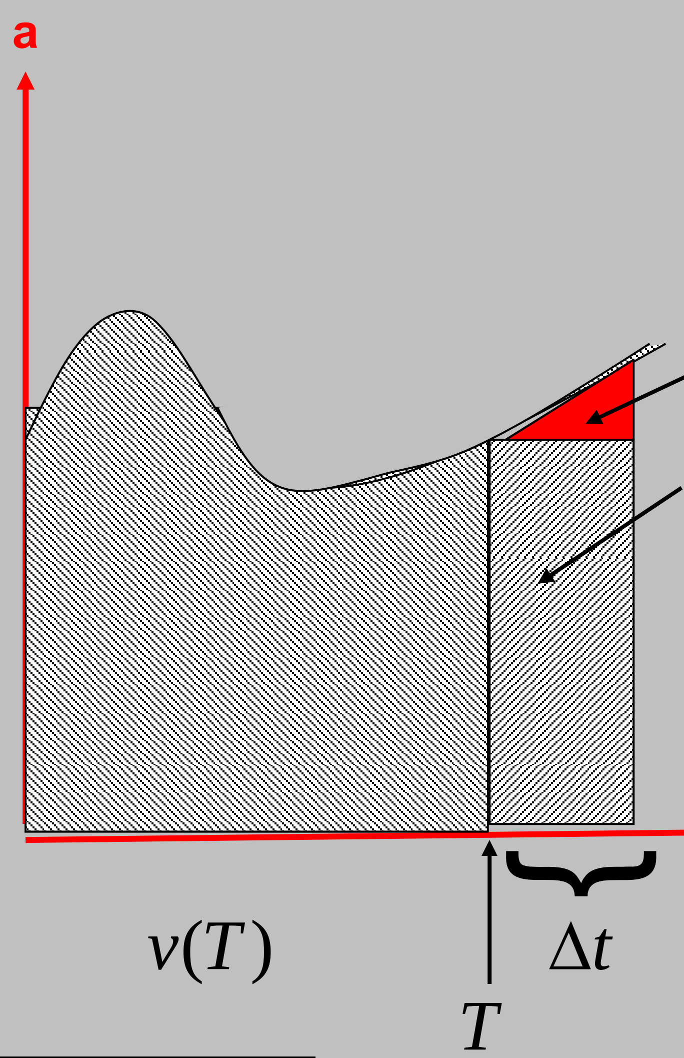
$$v = \int \frac{F}{m} dt$$



$$area = \int_0^T a\,dt = \int_0^T \frac{F}{m} dt = v(T)$$

What is v(T+ΔT) ?

$$v = \int \frac{F}{m} dt$$



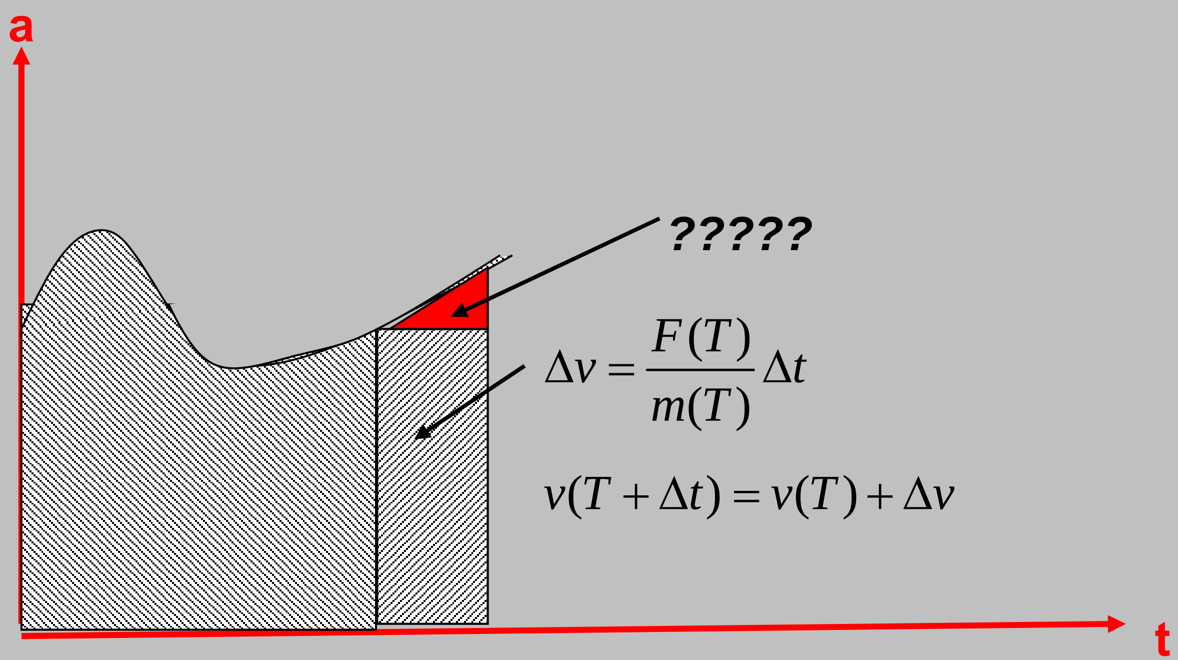
The velocity is off by this much !

$$\Delta v = a(T) \bullet \Delta t = \frac{F(T)}{m(T)} \Delta t$$

$$v(T + \Delta t) = v(T) + \Delta v$$

This is close, but is clearly not right!

What is v(T+ΔT) ?



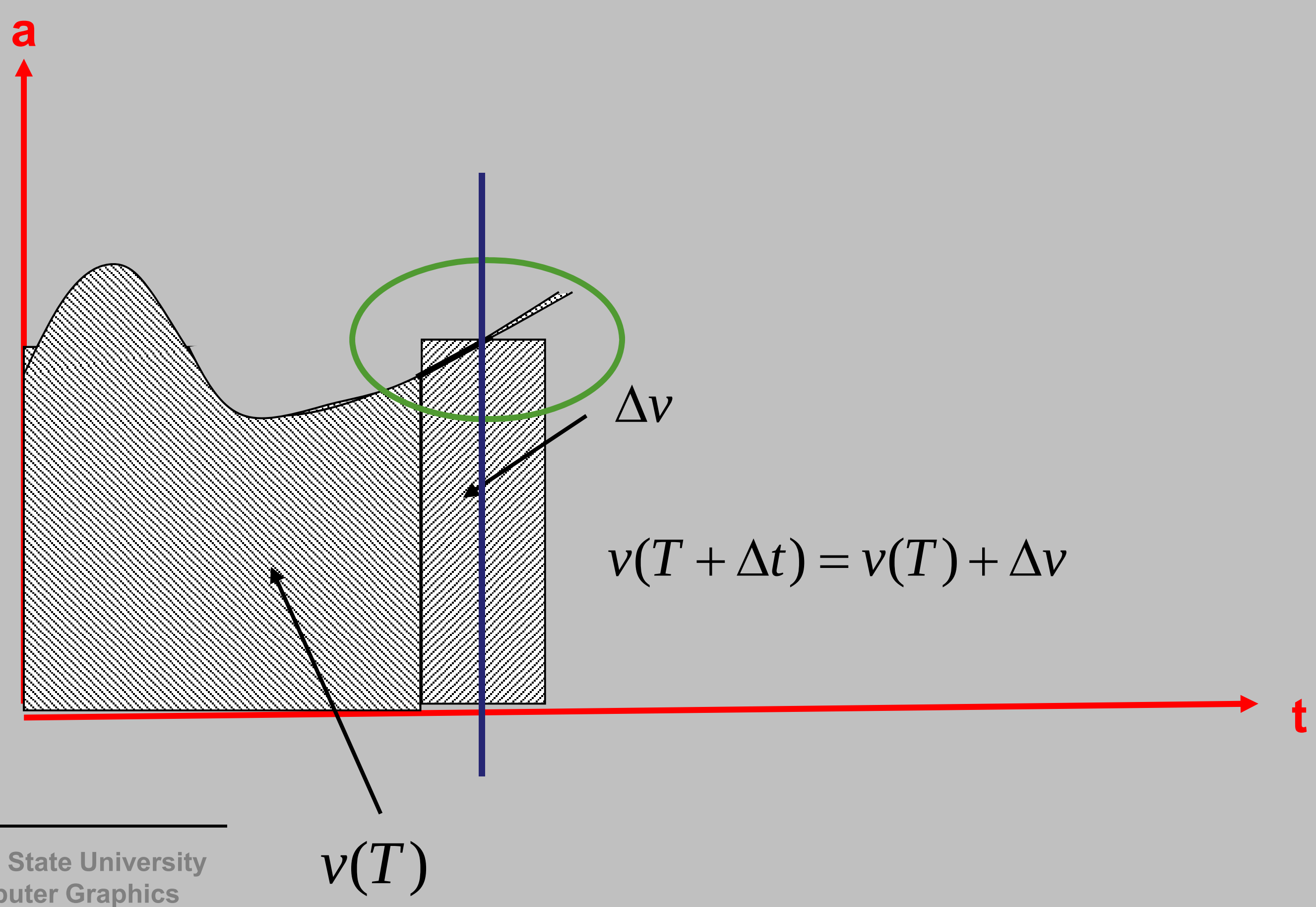
$$\Delta v = \frac{F(T)}{m(T)} \Delta t$$

$$v(T + \Delta t) = v(T) + \Delta v$$

The problem is that we are treating all of the quantities as if they were constant throughout the interval, even though they are not. This is known as a *First Order solution*.

What is v(T+ΔT) ?

A *Second Order solution* can be obtained by doing the First Order solution, determining all quantities at $t = T + \Delta t$ then averaging them with the properties at $t = T$ and then treating them as constant throughout the interval.



What does a Second Order solution look like in a Program?

```
void
GetVelAcc( float t, float x, float vx, float *v, float *a )
{ . . . }

. . .

GetVelAcc( tnow, xnow, vnow, &vnew1, &anew1 );
Δv = anew1 * Δt;
Δx = vnew1 * Δt;

GetVelAcc( tnow+ Δt, xnow+Δx, vnow+Δv, &vnew2, &anew2 );

aavg = ( anew1 + anew2 ) / 2.;
vavg = ( vnew1 + vnew2 ) / 2.;

xnow = xnow + vavg * Δt;
vnow = vnow + aavg * Δt;
tnow = tnow + Δt ;
```

This function returns a velocity and acceleration

This is called the "state vector" and consists of all values necessary to produce the acceleration

The Runge-Kutta Fourth Order Solution

$$\begin{Bmatrix} v_1 \\ a_1 \end{Bmatrix} = GetVelAcc(t, x, v)$$

$$\begin{Bmatrix} v_2 \\ a_2 \end{Bmatrix} = GetVelAcc(t + \frac{\Delta t}{2}, x + v_1 \frac{\Delta t}{2}, v + a_1 \frac{\Delta t}{2})$$

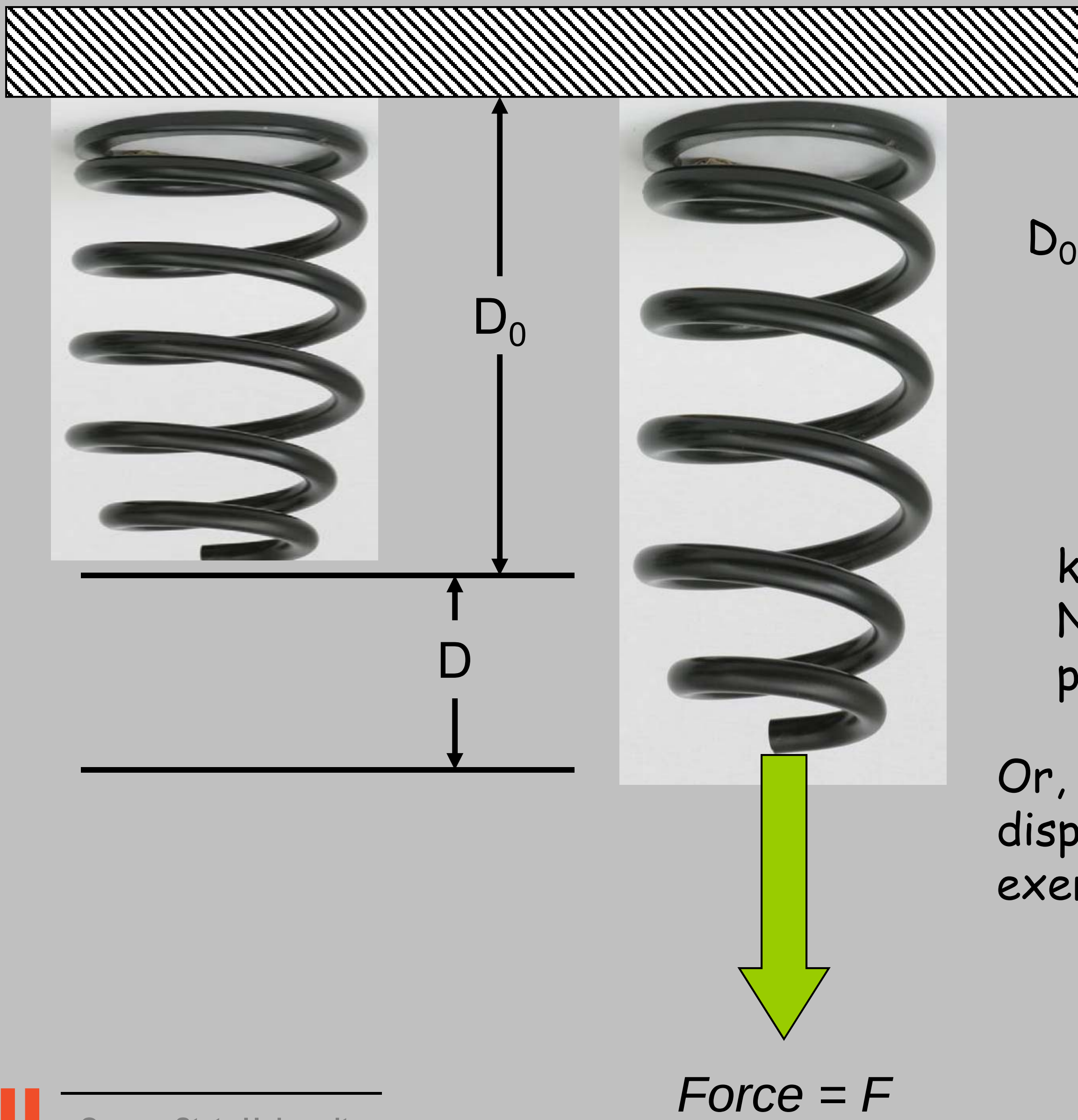
$$\begin{Bmatrix} v_3 \\ a_3 \end{Bmatrix} = GetVelAcc(t + \frac{\Delta t}{2}, x + v_2 \frac{\Delta t}{2}, v + a_2 \frac{\Delta t}{2})$$

$$\begin{Bmatrix} v_4 \\ a_4 \end{Bmatrix} = GetVelAcc(t + \Delta t, x + v_3 \Delta t, v + a_3 \Delta t)$$

$$\boxed{\begin{Bmatrix} x(t+\Delta t) \\ v(t+\Delta t) \end{Bmatrix} = \begin{Bmatrix} x \\ v \end{Bmatrix} + \frac{\Delta t}{6}(\begin{Bmatrix} v_1 \\ a_1 \end{Bmatrix} + 2\begin{Bmatrix} v_2 \\ a_2 \end{Bmatrix} + 2\begin{Bmatrix} v_3 \\ a_3 \end{Bmatrix} + \begin{Bmatrix} v_4 \\ a_4 \end{Bmatrix})}$$

Adapted from: http://en.wikipedia.org/wiki/Runge-Kutta

How do Springs Work?



D_0 = unloaded spring length

$$D = \frac{F}{k}$$

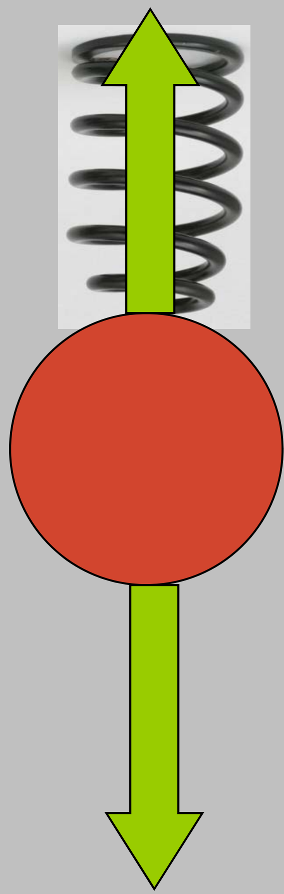
k = spring stiffness in Newtons/meter or pounds/inch

Or, if you know the displacement, the force exerted by the spring must be:

$$F = kD$$

Solving Motion where there is a Spring

$$F_{spring} = ky$$



$$v(y,t) = \int \frac{\sum F}{m} dt$$

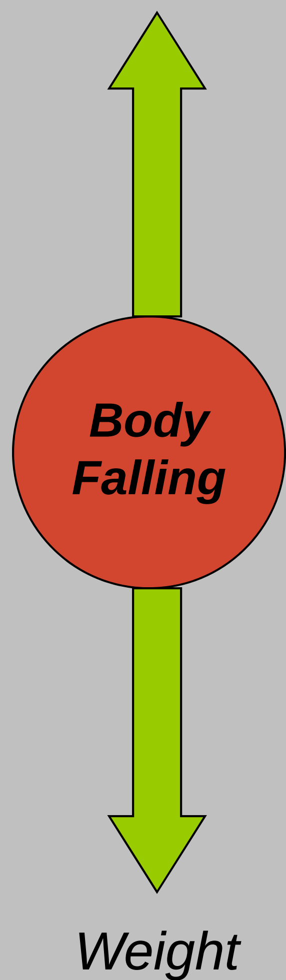
$$v(y,t) = \int \frac{W - ky}{m} dt$$

```
void
GetVelAcc( float t, float y, float vy, float *v, float *a )
{
        *v = vy;
        *a = ( W – K*y  ) /  MASS;
}
```

Air Resistance Force

$$F_{drag} = \frac{1}{2}\rho v_y^2 C_d A$$

- Fluid density (ρ)
- Y Velocity (v_y)
- Drag Coefficient (C_d)
- Cross-sectional area (A)



Some Drag Coefficients

C_d	Item
2.1	a smooth brick
0.9	a typical bicycle plus cyclist
0.4	rough sphere
0.1	smooth sphere
0.001	laminar flat plate
0.005	turbulent flat plate
0.295	bullet
1.0-1.3	person (upright position)
1.28	flat plate perpendicular to flow
1.0-1.1	skier
1.0-1.3	wires and cables
1.3-1.5	Empire State Building
1.8-2.0	Eiffel Tower

http://en.wikipedia.org/wiki/Drag_coefficient

Solving Motion where there is Air Resistance

$$v(y,t) = \int \frac{\sum F}{m} dt$$

$$v(y,t) = \int \frac{W - \frac{1}{2}\rho v_y^2 C_d A}{m} dt$$

$$\rho_{air} = 1.293 \ \frac{kg}{m^3}$$

```
void
GetVelAcc( float t, float y, float vy , float *v, float *a )
{
        *v = vy;
        *a = ( W – .5*DENSITY*vy*vy*DRAG*AREA  ) /  MASS;
}
```

Terminal Velocity

When a body is in free fall, it is being accelerated by the force of gravity. However, as it accelerates, it is encountering more and more air resistance force. At some velocity, these two forces balance each other out and the velocity is constant. This is known as the **terminal velocity**.

$$v(y,t) = \int \frac{W - \frac{1}{2}\rho v_y^2 C_d A}{m} dt$$

The velocity becomes constant when the sum of the forces equals 0

$$W - \frac{1}{2}\rho v_y^2 C_d A = 0$$

$$v_t = \sqrt{\frac{2W}{\rho C_d A}}$$

Human Terminal Velocity

Assume:

Weight = 200 pounds = 890 Newtons

C_d = 1.28

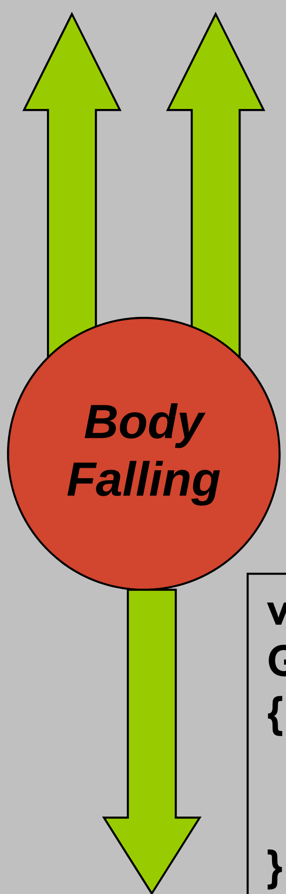
A = 6 ft^2 = 0.558 m^2

$$\rho_{air} = 1.293 \frac{kg}{m^3}$$

$$v_t = \sqrt{\frac{2W}{\rho C_d A}} = 43.90 \frac{m}{\sec} \approx 98 mph$$

How about a Cliff Jumper on a Bungee Cord?

$$F_{spring} = ky \qquad F_{drag} = \frac{1}{2}\rho v_y^2 C_d A$$



Weight

$$v(y) = \int \frac{\sum F}{m} dt$$

$$v(y) = \int \frac{W - ky - \frac{1}{2}\rho v_y^2 C_d A}{m} dt$$

```
void
GetVelAcc( float t, float y, float vy , float *v, float *a )
{
        *v = vy;
        *a = ( W – K*y - .5*DENSITY*vy*vy*DRAG*AREA  ) /  MASS;
}
```

Buoyancy – Another Good Force to Know About

Archimedes' Principle says that the buoyancy force on an object in a fluid is the weight of the fluid that is being displaced by the object.

$$\left.\begin{aligned}\rho_{air} &= 4.66x10^{-5}\ pounds / in^3 \\ \rho_{helium} &= 0.65x10^{-5}\ pounds / in^3\end{aligned}\right\}\ \text{Densities}$$

So, for a helium balloon that is one foot in diameter (i.e., radius=6 inches), it has its weight pulling it down and a buoyancy force pushing it up. The net force pushing it up because of the gas inside the balloon is:

$$F_{buoyancy} = V_{balloon}\rho_{helium} - V_{balloon}\rho_{air} = V_{balloon}(\rho_{helium} - \rho_{air})$$

$$V_{balloon} = \frac{4}{3}\pi r^3 = 904.78 in^3$$

$$F_{buoyancy} = 904.78 in^3 (4.01x10^{-5}\ pounds / in^3) = 0.036\ pounds$$

Note that this must still counterbalance the weight of the balloon material, or the balloon will not fly.