


## Physics

Mike Bailey

Oregon State University



OSU Oregon State University  
Computer Graphics

## Physics

Quantity	Units
Linear position	Meters
Linear velocity	Meters/second
Linear acceleration	Meters/second <sup>2</sup>
Force	Newtons
Mass	Kilograms
Weight	Newtons
Density	Kilograms/meter <sup>3</sup>
Time	Seconds
Pressure	Newtons/meter <sup>2</sup>
Momentum	Kilograms*meters/second
Angular position	Radians
Angular velocity	Radians/second
Angular acceleration	Radians/second <sup>2</sup>
Moment (+torque)	Newtons*meters
Moment of Inertia	Kilograms*meters <sup>2</sup>
Temperature	° Celsius

OSU Oregon State University  
Computer Graphics

## Some Useful Conversions

- 1 meter = 39.37 inches = 3.28 feet
- 1 mile = 1,610 meters = 1.610 kilometers
- 1 mile per hour = 1.467 feet per second
- 1 mile per hour = 0.447 meters per second
- 1 gallon = 3.79 liters
- 1 cubic foot = 7.48 gallons = 28.35 liters
- 1 kilogram = 2.2 pounds (mass, at Earth's surface)
- 1 Newton = 0.224 pounds (force)
- 1 pound = 4.45 Newtons (force)
- 1 radian = 57.3°

OSU Oregon State University  
Computer Graphics

## Some Useful Conversions

- A gram is about the mass of a paper clip
- A nickel has a mass of about 5 grams
- A liter is half of a soda 2-liter bottle, or about a fourth of a gallon of milk
- A kilogram is twice as much as a pound
- A Newton is about ¼ of a pound
- A meter is a little more than a yard
- A kilometer is about ¾ of a mile
- Water freezes at 0° Celsius
- A hot day is 30° Celsius

OSU Oregon State University  
Computer Graphics

## Acceleration Due to Gravity

Newton's Gravitational Law says that the attraction between two objects is the product of their masses times the gravitational constant G, divided by the square of the distance between them:

$$F = \frac{Gm_1m_2}{d_{12}^2} \quad \text{where: } G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

and  $d_{12}$  is the distance between body 1 and body 2

For an object at or near the surface of the Earth, this simplifies to:

$$F = mg \quad \text{where: } g = 9.8 \frac{m}{sec^2}$$

$g$  is known as the **Acceleration Due to Gravity**

OSU Oregon State University  
Computer Graphics

## Constant-Acceleration Formulas

$$v_1 = v_0 + at$$

$$v_1^2 = v_0^2 + 2a(d_1 - d_0)$$

$$d_1 = d_0 + v_0t + \frac{1}{2}at^2$$

Note: as long as you are near the surface of the Earth,  $g$  is considered to be "constant acceleration":

$$d_y = d_{y0} + v_{y0}t - \frac{1}{2}gt^2$$

OSU Oregon State University  
Computer Graphics

**Projectile Motion**

**Initial Quantities:**

$$a_x = 0 \quad a_y = -g$$

$$v_{x0} = V \cos \theta \quad v_{y0} = V \sin \theta$$

**Quantities in Flight:**

$$d_x = d_{x0} + v_{x0}t$$

$$d_y = d_{y0} + v_{y0}t - \frac{1}{2}gt^2$$

DSU Oregon State University Computer Graphics mjb - September 23, 2009

**Projectile Motion: How Long will the Projectile Stay in Flight?**

**Y Distance Equation:**

$$d_{\text{impact}} = d_{y0} + v_{y0}t - \frac{1}{2}gt^2$$

**Solve for the time:**

$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 + 2g(d_{y0} - d_{\text{impact}})}}{-g}$$

**How Far will it Go?**

**Solve for the X Distance:**

$$d_x = d_{x0} + v_{x0} \cdot t^*$$

**Quadratic Equation:**

$$At^2 + Bt + C = 0$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

DSU Oregon State University Computer Graphics mjb - September 23, 2009

**Projectile Motion: How Long will the Projectile Stay in Flight?**

$$t^* = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 + 2g(d_{y0} - d_{\text{impact}})}}{-g}$$

Why are there 2 solutions for t\*?  
How do you know which one is the correct one?

DSU Oregon State University Computer Graphics mjb - September 23, 2009

**Projectile Motion: When and Where will the Projectile Reach its Maximum Height?**

The y velocity at the maximum height is zero

**Y Velocity Equation:**

$$v_y = v_{y0} - gt = 0$$

**Solve for the time:**

$$t^* = \frac{v_{y0}}{g} = \frac{V \sin \theta}{g}$$

**How High will it Go?**

**Solve for the Y Distance:**

$$d^* = d_{y0} + v_{y0}t^* - \frac{1}{2}g(t^*)^2$$

DSU Oregon State University Computer Graphics mjb - September 23, 2009

**Dynamics**

$$F = ma = m \frac{d^2x}{dt^2} \equiv \ddot{x}$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{F}{m}$$

$$v = \int a dt = \int \frac{F}{m} dt \quad x = \int v dt$$

DSU Oregon State University Computer Graphics mjb - September 23, 2009

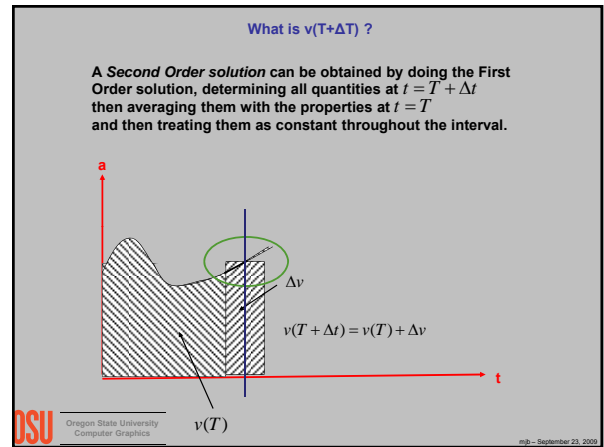
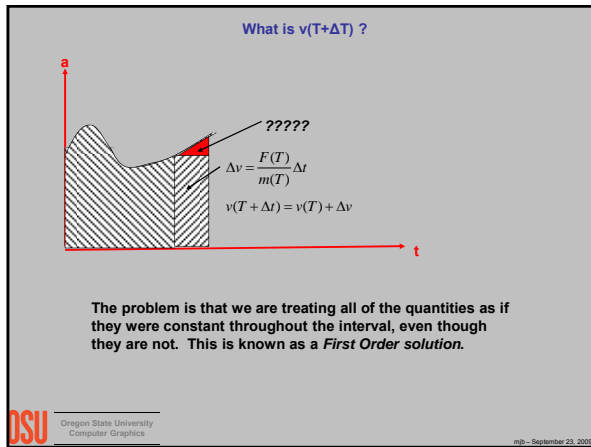
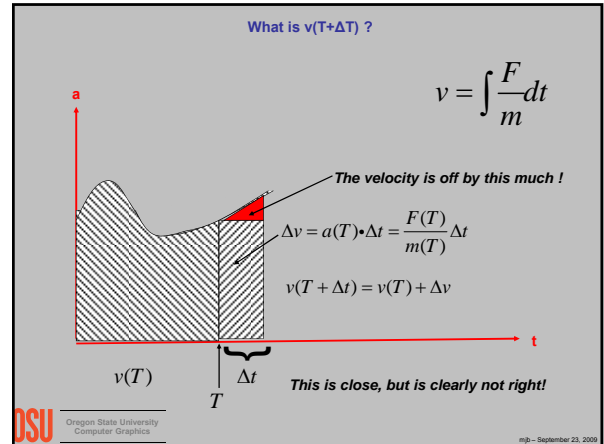
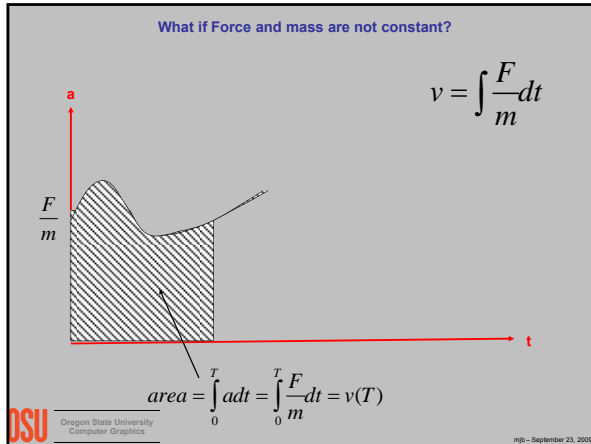
**Integrating the Physics Equations**

$$v = \int \frac{F}{m} dt$$

**area =**

$$\int_0^T a dt = \int_0^T \frac{F}{m} dt = \text{velocity after time } T$$

DSU Oregon State University Computer Graphics mjb - September 23, 2009



What does a Second Order solution look like in a Program?

```

void
GetVelAcc( float t, float x, float vx, float *v, float *a )
{ ... }
    
```

This function returns a velocity and acceleration

... This is called the "state vector" and consists of all values necessary to produce the acceleration

```

GetVelAcc( tnow, xnow, vnow, &vnew1, &anew1 );
Δv = anew1 * Δt;
Δx = vnew1 * Δt;

GetVelAcc( tnow + Δt, xnow + Δx, vnow + Δv, &vnew2, &anew2 );
aavg = ( anew1 + anew2 ) / 2.;
vavg = ( vnew1 + vnew2 ) / 2.;
xnow = xnow + vavg * Δt;
vnow = vnow + aavg * Δt;
tnow = tnow + Δt;
    
```

DSU Oregon State University Computer Graphics mjb - September 23, 2009

The Runge-Kutta Fourth Order Solution

$$\begin{cases} v_1 \\ a_1 \end{cases} = \text{GetVelAcc}(t, x, v)$$

$$\begin{cases} v_2 \\ a_2 \end{cases} = \text{GetVelAcc}\left(t + \frac{\Delta t}{2}, x + v_1 \frac{\Delta t}{2}, v + a_1 \frac{\Delta t}{2}\right)$$

$$\begin{cases} v_3 \\ a_3 \end{cases} = \text{GetVelAcc}\left(t + \frac{\Delta t}{2}, x + v_2 \frac{\Delta t}{2}, v + a_2 \frac{\Delta t}{2}\right)$$

$$\begin{cases} v_4 \\ a_4 \end{cases} = \text{GetVelAcc}(t + \Delta t, x + v_3 \Delta t, v + a_3 \Delta t)$$

$$\begin{cases} x(t + \Delta t) \\ v(t + \Delta t) \end{cases} = \begin{cases} x \\ v \end{cases} + \frac{\Delta t}{6} \left( \begin{cases} v_1 \\ a_1 \end{cases} + 2 \begin{cases} v_2 \\ a_2 \end{cases} + 2 \begin{cases} v_3 \\ a_3 \end{cases} + \begin{cases} v_4 \\ a_4 \end{cases} \right)$$

Adapted from: <http://en.wikipedia.org/wiki/Runge-Kutta>

DSU Oregon State University Computer Graphics mjb - September 23, 2009

### How do Springs Work?

$D_0$  = unloaded spring length

$$D = \frac{F}{k}$$

$k$  = spring stiffness in Newtons/meter or pounds/inch

Or, if you know the displacement, the force exerted by the spring must be:

$$F = kD$$

Force = F

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Solving Motion where there is a Spring

$$F_{spring} = ky$$

$$v(y,t) = \int \frac{\sum F}{m} dt$$

$$v(y,t) = \int \frac{W - ky}{m} dt$$

```
void
GetVelAcc( float t, float y, float vy, float *v, float *a )
{
    *v = vy;
    *a = ( W - K*y ) / MASS;
}
```

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Air Resistance Force

$$F_{drag} = \frac{1}{2} \rho v_y^2 C_d A$$

← Cross-sectional area

← Fluid density

← Drag Coefficient

← Y Velocity

Body Falling

Weight

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Some Drag Coefficients

$C_d$	Item
2.1	a smooth brick
0.9	a typical bicycle plus cyclist
0.4	rough sphere
0.1	smooth sphere
0.001	laminar flat plate
0.005	turbulent flat plate
0.295	bullet
1.0-1.3	person (upright position)
1.28	flat plate perpendicular to flow
1.0-1.1	skier
1.0-1.3	wires and cables
1.3-1.5	Empire State Building
1.8-2.0	Eiffel Tower

[http://en.wikipedia.org/wiki/Drag\\_coefficient](http://en.wikipedia.org/wiki/Drag_coefficient)

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Solving Motion where there is Air Resistance

$$v(y,t) = \int \frac{\sum F}{m} dt$$

$$v(y,t) = \int \frac{W - \frac{1}{2} \rho v_y^2 C_d A}{m} dt$$

$\rho_{air} = 1.293 \frac{kg}{m^3}$

```
void
GetVelAcc( float t, float y, float vy, float *v, float *a )
{
    *v = vy;
    *a = ( W - .5*DENSTY*vy*vy*DRAG*AREA ) / MASS;
}
```

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Terminal Velocity

When a body is in free fall, it is being accelerated by the force of gravity. However, as it accelerates, it is encountering more and more air resistance force. At some velocity, these two forces balance each other out and the velocity is constant. This is known as the **terminal velocity**.

$$v(y,t) = \int \frac{W - \frac{1}{2} \rho v_y^2 C_d A}{m} dt$$

The velocity becomes constant when the sum of the forces equals 0

$$W - \frac{1}{2} \rho v_y^2 C_d A = 0$$

$$v_t = \sqrt{\frac{2W}{\rho C_d A}}$$

DSU Oregon State University Computer Graphics mp - September 23, 2009

### Human Terminal Velocity

**Assume:**

Weight = 200 pounds = 890 Newtons  
 $C_d = 1.28$   
 $A = 6 \text{ ft}^2 = 0.558 \text{ m}^2$   
 $\rho_{air} = 1.293 \frac{\text{kg}}{\text{m}^3}$

$$v_t = \sqrt{\frac{2W}{\rho C_d A}} = 43.90 \frac{\text{m}}{\text{sec}} \approx 98 \text{mph}$$

Oregon State University Computer Graphics      mp - September 23, 2009

### How about a Cliff Jumper on a Bungee Cord?

$F_{spring} = ky$ 
 $F_{drag} = \frac{1}{2} \rho v_y^2 C_d A$ 

Weight

$$v(y) = \int \frac{\sum F}{m} dt$$

$$v(y) = \int \frac{W - ky - \frac{1}{2} \rho v_y^2 C_d A}{m} dt$$

```
void
GetVelAcc( float t, float y, float vy , float *v, float *a )
{
    *v = vy;
    *a = ( W - K*y - .5*DENSIY*vy*vy*DRAG*AREA ) / MASS;
}
```

Oregon State University Computer Graphics      mp - September 23, 2009

### Buoyancy – Another Good Force to Know About

*Archimedes' Principle* says that the buoyancy force on an object in a fluid is the weight of the fluid that is being displaced by the object.

$\rho_{air} = 4.66 \times 10^{-5} \text{ pounds / in}^3$   
 $\rho_{helium} = 0.65 \times 10^{-5} \text{ pounds / in}^3$

}

Densities

So, for a helium balloon that is one foot in diameter (i.e., radius=6 inches), it has its weight pulling it down and a buoyancy force pushing it up. The net force pushing it up because of the gas inside the balloon is:

$$F_{buoyancy} = V_{balloon} \rho_{helium} - V_{balloon} \rho_{air} = V_{balloon} (\rho_{helium} - \rho_{air})$$

$$V_{balloon} = \frac{4}{3} \pi r^3 = 904.78 \text{in}^3$$

$$F_{buoyancy} = 904.78 \text{in}^3 (4.01 \times 10^{-5} \text{ pounds / in}^3) = 0.036 \text{ pounds}$$

Note that this must still counterbalance the weight of the balloon material, or the balloon will not fly.

Oregon State University Computer Graphics      mp - September 23, 2009