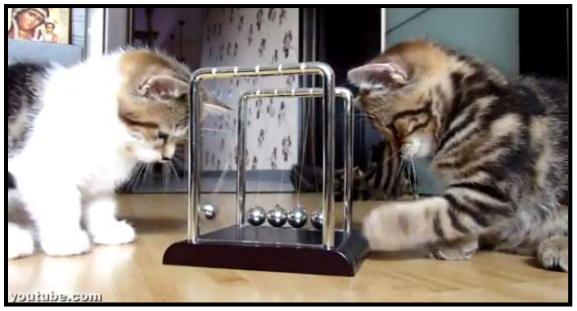
Collision Physics



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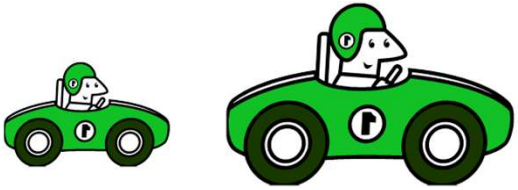
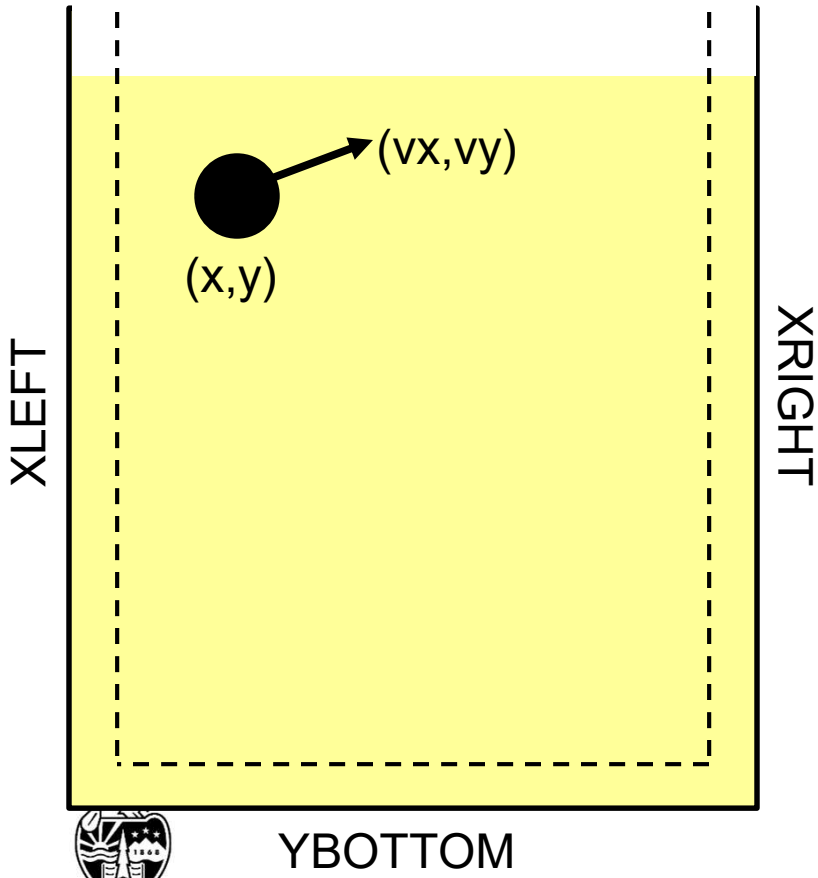


There are some cases when you can compute the collision in closed-form



We need to answer the question:

What will cause the ball to stop flying freely through the air?

The answer is – it will hit a wall or the floor.
From our rules of physics, the next bounce will happen at one of the solutions to these equations:

$$XRIGHT - RADIUS = x + v_x t$$

$$XLEFT + RADIUS = x + v_x t$$

$$YBOTTOM + RADIUS = y + v_y t + \frac{1}{2} g t^2$$

Note: g < 0.

There are some cases when you can compute the collision in closed-form

The next bounce will happen at one of the solutions to these equations:

$$XRIGHT - RADIUS = x + v_x t \rightarrow t_1 = \frac{XRIGHT - RADIUS - x}{v_x}$$

$$XLEFT + RADIUS = x + v_x t \rightarrow t_2 = \frac{XLEFT + RADIUS - x}{v_x}$$

$$YBOTTOM + RADIUS = y + v_y t + \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 + v_y t + (y - YBOTTOM - RADIUS) = 0 \rightarrow t_3, t_4 = \frac{-v_y \pm \sqrt{v_y^{\,2} - 2g(y - YBOTTOM - RADIUS)}}{g}$$

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How do you know which of the 4 ***t*** values is the one that you should use?

There are some cases when you can compute the collision in closed-form

The next bounce will happen at one of the solutions to these equations:

$$t_1 = \frac{XRIGHT - RADIUS - x}{v_x}$$

$$t_2 = \frac{XLEFT + RADIUS - x}{v_x}$$

$$t_3, t_4 = \frac{-v_y \pm \sqrt{v_y^{\ 2} - 2g(y - YBOTTOM - RADIUS)}}{g}$$

How do you know which of the 4 ***t*** values is the one that you should use?

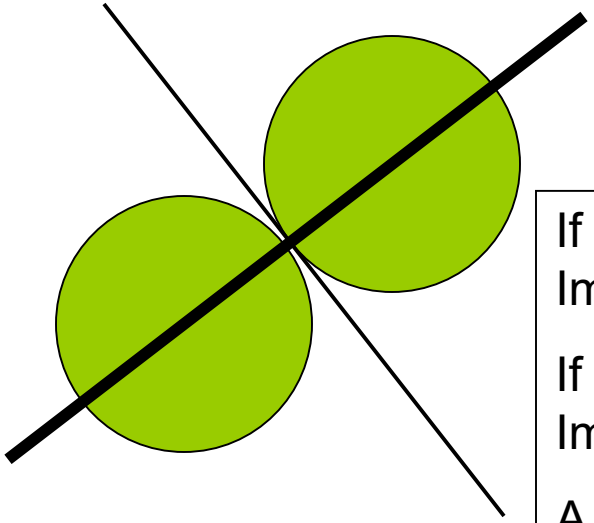
Use the time that is the smallest positive number.

We want positive, because negative time values happened ***ago***.

We want smallest because we are interested in the collision that happens ***first***.

The Physics of Collisions -- Definitions

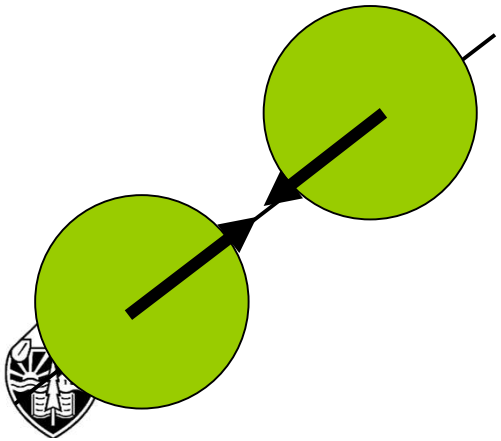
Line of Impact



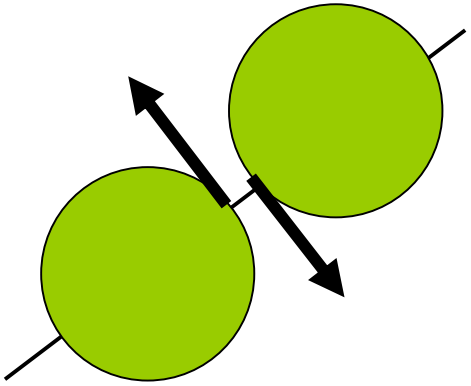
If the objects' velocities are parallel to the Line of Impact, this is a ***Direct Impact***

If the objects' velocities are perpendicular to the Line of Impact, this is a ***Tangential Impact***

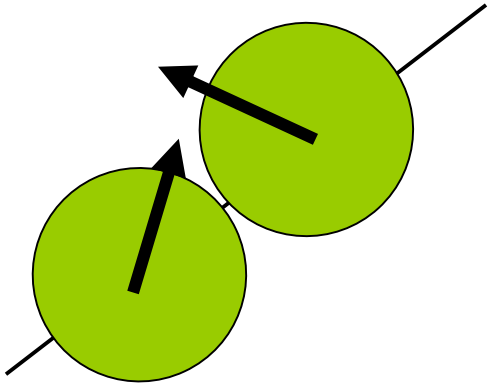
A combination of the two is called an ***Oblique Impact***



Direct Impact



Tangential Impact



Oblique Impact
(both direct and tangential)

The Physics of Collisions – Fundamental Quantities

The ***momentum*** of an object is defined as its mass multiplied by its velocity:

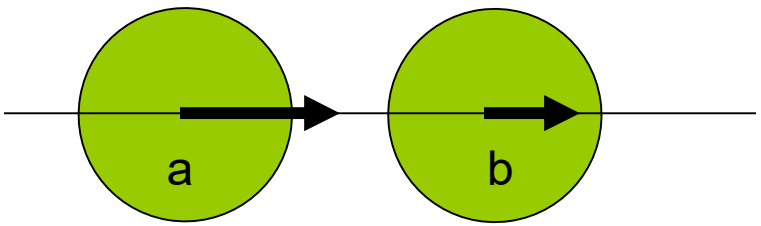
$$Momentum = mv$$

The ***energy*** of an object is defined as one half of its mass multiplied by its velocity squared:

$$Energy = \frac{1}{2}mv^2$$

The Physics of Collisions – Conservation of Momentum

In a collision, the total momentum after the impact is equal to the total momentum before the impact. Always.



$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

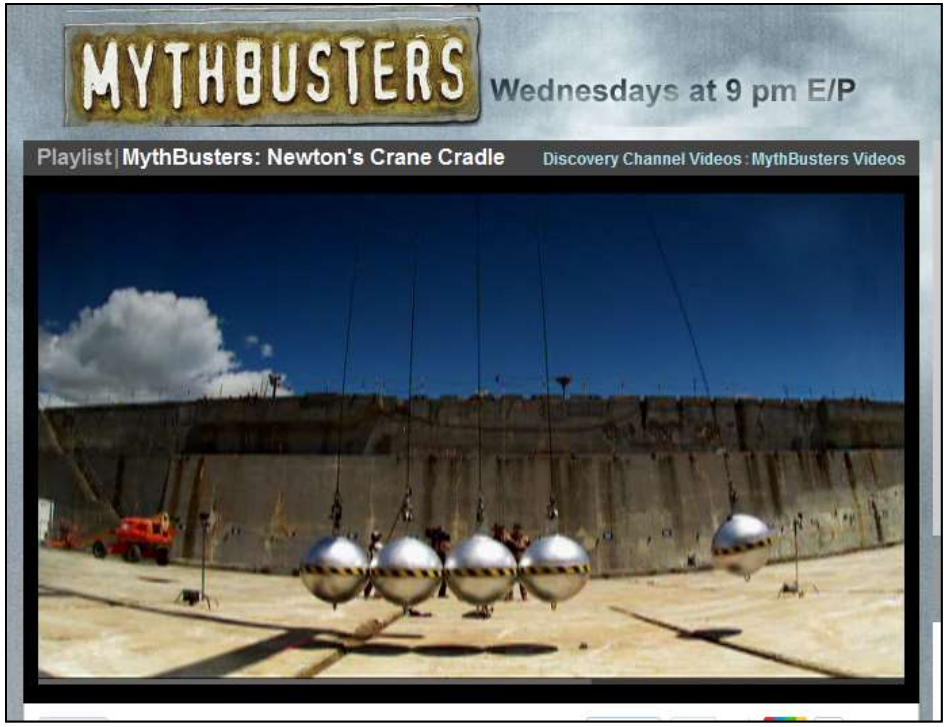
where the primes ′ refer to velocities after the impact

This is referred to as the ***Conservation of Momentum Law***

Momentum is always conserved through **any** collision

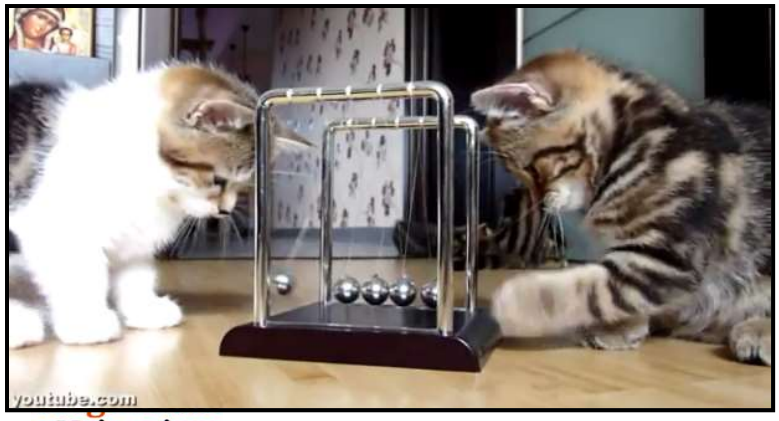
Conservation of Momentum as Explained by Newton's Cradle

http://www.grand-illusions.com/acatalog/Newtons_Cradle.html



http://dsc.discovery.com/videos/mythbusters-newtons-crane-cradle/

And, of course, where would *any* topic be without kittens? ☺

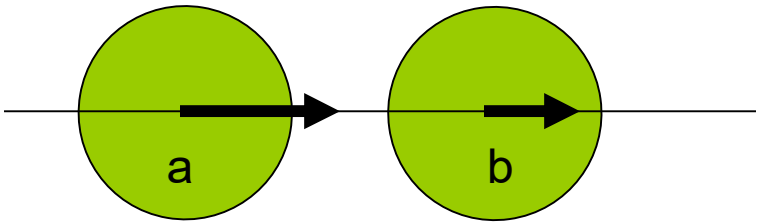


http://www.buzzfeed.com/koolnewsblog/cats-for-the-love-of-physics-1dgl

The Physics of Collisions – Coefficient of Restitution

In a collision, energy is conserved in the entire system, but not necessarily in the form of velocities. (It can become permanent deformation, heat, light, etc..)

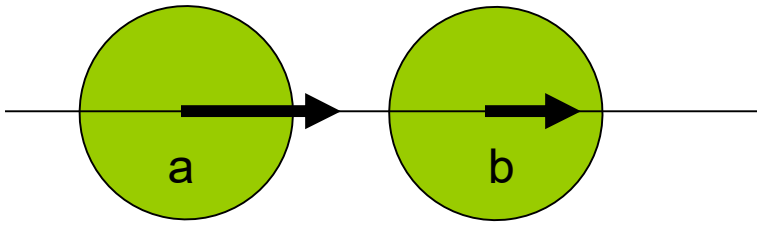
This loss of velocity is expressed as the ***Coefficient of Restitution*** (COR). The COR, e, is how much less the relative velocities of the objects are after impact than they were before impact:



$$v_b' - v_a' = -e(v_b - v_a)$$

(the negative sign is there to indicate the “bounce”)

The Physics of Collisions – Combining Momentum and Restitution Laws



Starting with these two equations:

$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

$$v'_b - v'_a = -e(v_b - v_a)$$

Treat the two initial velocities as inputs and solve for the two resulting velocities. This gives:

$$v'_a = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

$$v'_b = \frac{m_a v_a + m_b v_b - e m_a (v_b - v_a)}{m_a + m_b}$$

The Physics of Collisions with Immoveable Objects

To treat the case of mass b being an immoveable object, such as the ground or a solid wall, solve for the resulting velocities taking the limit:

$$\lim_{m_b \to \infty} v'_a = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

$$= \lim_{m_b \to \infty} \left[\frac{m_a v_a}{m_a + m_b} + \frac{m_b v_b}{m_a + m_b} + \frac{e m_b (v_b - v_a)}{m_a + m_b} \right]$$

$$= \left[0 + v_b + e(v_b - v_a) \right]$$

Since mass b is immoveable, its velocity is zero, so that a's post-collision velocity is:

$$\boxed{v'_a = -e v_a}$$

Collisions – Experimentally Determining the Coefficient of Restitution

Velocities are hard to measure live, but distances are not.

So, drop the object from a height ***h***, and measure its bounce to a height **h'** :

Before the bounce:

$$v_2^{\,2} = 0^2 + 2gh$$

$$v = \sqrt{2gh}$$

After the bounce:

$$0^2 = v'^2 - 2gh'$$

$$v' = \sqrt{2gh'}$$

$$|v'| = e|v|$$

$$\boxed{e = \frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}}}$$

Collisions – Some Coefficients of Restitution of Balls Bounced on a Concrete Surface

Ball Material	CoR
range golf ball	0.858
tennis ball	0.712
billiard ball	0.804
hand ball	0.752
wooden ball	0.603
steel ball bearing	0.597
glass marble	0.658
ball of rubber bands	0.828
hollow, hard plastic ball	0.688

http://hypertextbook.com/facts/2006/restitution.shtml

The Physics of Collisions – Totally Plastic Collisions

$$v_a' = \frac{m_a v_a + m_b v_b + e m_b (v_b - v_a)}{m_a + m_b}$$

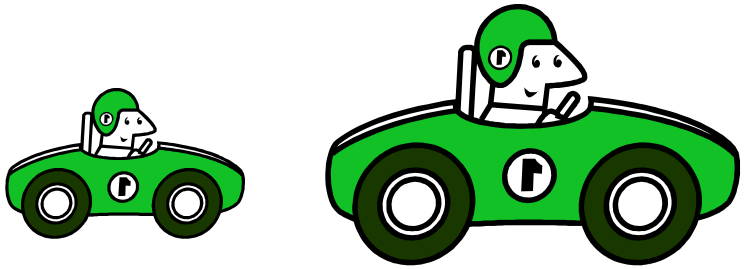
$$v_b' = \frac{m_a v_a + m_b v_b - e m_a (v_b - v_a)}{m_a + m_b}$$

If e=0, then the two objects stick together and end up with the same resulting velocity:

$$\boxed{v_a' = v_b' = \frac{m_a v_a + m_b v_b}{m_a + m_b}}$$

One of my Jury Duties: Two vehicles collide. One is very damaged, the other hardly at all. What happened? Who's right?

Very damaged **Hardly damaged**



B **A**

25 mph → **0 mph**

v_a = 0. mph
v_b = 25 mph = 11.2 m/sec
m_a = 2.0
m_b = 1.0
e = .30

$$v'_a = \frac{v_b + ev_b}{m_a + m_b} = \frac{11.2(1.3)}{3} = 4.9 m/\sec$$

$$v'_b = \frac{m_b v_b - em_a v_b}{m_a + m_b} = \frac{11.2(1-.3*2)}{3} = 1.5 m/\sec$$

How much velocity change did each car undergo?
(**Δv** is an approximate measure of damage.)

Δv_a = 4.9 – 0. = 4.9 m/sec = 11.0 mph

Δv_b = 1.5 – 11.2 = -9.7 m/sec = -21.7 mph

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The Physics of Collisions – Totally Elastic Collisions

What happens when *e*=1? The two fundamental equations are

$$m_b v_b' + m_a v_a' = m_b v_b + m_a v_a$$
$$v_b' - v_a' = -e(v_b - v_a) = (v_a - v_b)$$

Rearranging:

$$m_a \left(v_a' - v_a \right) = m_b \left(v_b - v_b' \right)$$
$$v_a' + v_a = v_b + v_b'$$

The Physics of Collisions – Elastic Collisions

$$m_a\left(v'_a - v_a\right) = m_b\left(v_b - v'_b\right)$$
$$v'_a + v_a = v_b + v'_b$$

Then, multiplying the two together gives:

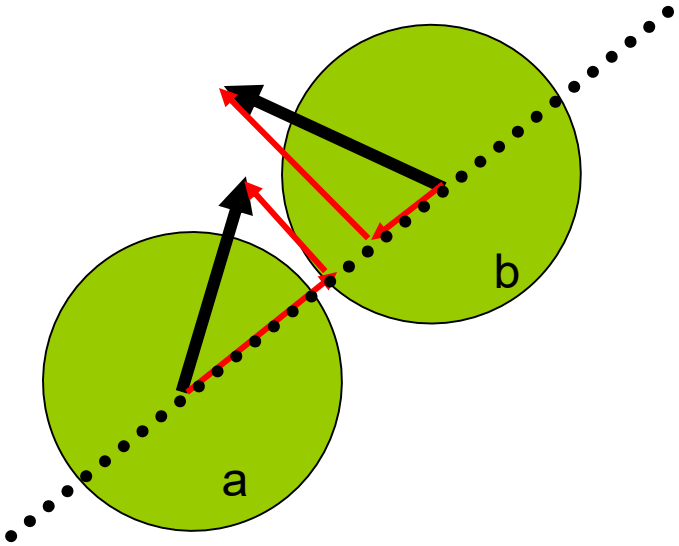
$$m_a v'^2_a - m_a v_a{}^2 = m_b v_b{}^2 - m_b v'^2_b$$

Or:

$$\boxed{\frac{1}{2} m_b v'^2_b + \frac{1}{2} m_a v'^2_a = \frac{1}{2} m_b v_b{}^2 + \frac{1}{2} m_a v_a{}^2}$$

This shows that energy is conserved when the Coefficient of Restitution is 1.0

The Physics of Collisions – Oblique Impacts



Oblique Impacts are then handled by using vector math to determine the direct and tangential velocity components with respect to the **Line of Impact**.

The direct components are changed using the equations we just derived.

The tangential components are left unchanged. (This assumes no friction.)

The new components are then combined to produce the resulting velocity vectors.