Inverse Kinematics

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**Inverse Kinematics** solves the problem “if I know the link transformation parameters, where are the links?”. 

**Inverse Kinematics (IK)** solves the problem “If I know where I want the links to be \((X^*, Y^*)\), what link transformation parameters will put them there?”
Inverse Kinematics (IK):
Things Need to Move – What Parameters Will Make Them Do That?
Cyclic Coordinate Descent (CCD) Method

The idea is to change $\Theta_1$ so that $(X,Y)$ are as close to $(X^*,Y^*)$ as possible. Then change $\Theta_2$. Then change $\Theta_3$. Then change $\Theta_1$. Then change $\Theta_2$. Then change $\Theta_3$. Then change $\Theta_1$.

$\cdots$
Changing $\Theta_1$

Holding $\Theta_2$ and $\Theta_3$ constant, rotate $\Theta_1$ towards $(X^*, Y^*)$ so that the dashed purple lines line up.
Changing $\Theta_1$
Changing $\Theta_2$

Holding $\Theta_1$ and $\Theta_3$ constant, rotate $\Theta_2$ towards $(X^*,Y^*)$ so that the dashed purple lines line up.
Changing $\Theta_2$
Changing $\Theta_3$

Holding $\Theta_1$ and $\Theta_2$ constant, rotate $\Theta_3$ towards $(X^*,Y^*)$ so that the dashed purple lines line up.
Changing $\Theta_3$
Now, do it again -- Changing $\Theta_1$
Now, do it again -- Changing $\Theta_1$
Now, do it again -- Changing $\Theta_2$
Now, do it again -- Changing $\Theta_2$
Now, do it again -- Changing $\Theta_3$
Now, do it again -- Changing $\Theta_3$
Computing how much to change a rotation by
(in this example, we are changing $\theta_2$)

Where we are now: $(X_3, Y_3)$

Where we want to be: $(X^*, Y^*)$

Use the C/C++ `atan2( )` function:

$$\theta^* = \text{atan2}( Y^* - Y_2, X^* - X_2 );$$

$$\theta = \text{atan2}( Y_3 - Y_2, X_3 - X_2 );$$

$$\Delta \theta_2 = \theta^* - \theta$$

**Do not use the C/C++ `atan( )` function:**

$$\theta^* = \text{atan}( (Y^* - Y_2) / (X^* - X_2) );$$

$$\theta = \text{atan}( (Y_3 - Y_2) / (X_3 - X_2) );$$

$$\Delta \theta_2 = \theta^* - \theta$$