

**Vulkan.**

**Vulkan Ray Tracing – 5 New Shader Types!**




**Oregon State University**  
Mike Bailey  
mjb@cs.oregonstate.edu




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


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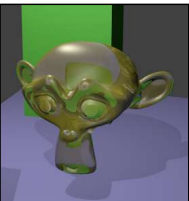
**Analog Ray Tracing Example**

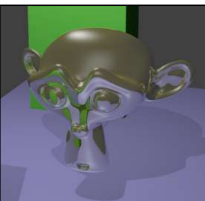


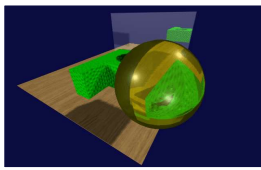
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**Digital Ray Tracing Examples**







Blender

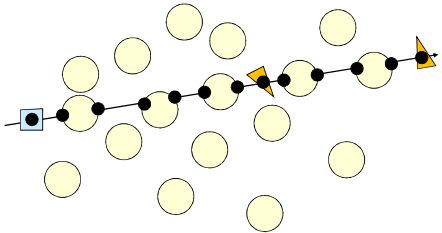




IronCad

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**In a Raytracing, each ray typically hits a lot of Things**



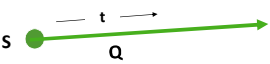



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**Parametrizing a Ray**



**Given:**  
**S** is the (x,y,z) starting point  
**Q** is the (x,y,z) direction of travel

Then, the (x,y,z) position of a point **p** at some position along its direction of travel is:



$$p = S + tQ$$

$$t \geq 0.$$

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**Example: The Ray Intersection Process for a Sphere**

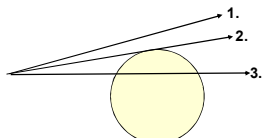
Sphere equation:  $(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = R^2$   
 Ray equation:  $(x,y,z) = (x_0,y_0,z_0) + t(dx,dy,dz)$



Plugging (x,y,z) from the second equation into the first equation and multiplying-through and simplifying gives:

$$At^2 + Bt + C = 0 \quad \rightarrow \quad t_1, t_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Solve for  $t_1, t_2$  and analyze the solution like this:

1. If both  $t_1$  and  $t_2$  are complex (i.e., have an imaginary component), then the ray missed the sphere completely.
2. If both  $t_1$  and  $t_2$  are real and identical, then the ray brushed the sphere at a tangent point.
3. If both  $t_1$  and  $t_2$  are real and different, then the ray entered and exited the sphere.



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### Parameterizing a Triangle

It's often useful to be able to parameterize a triangle into (u,v), like this:

Note! There is *no* place in this triangle where  $u = 1$  and  $v = 1$ .

$$p = P0 + u*(P1-P0) + v*(P2-P0)$$

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### The Setup

We want to find out where the ray intersects the triangle.  
That is, where is the point  $p$  that is common to both the ray and the triangle?

Such that:

$$\begin{matrix} t \geq 0. \\ 0 \leq u \leq 1. \\ 0 \leq v \leq 1-u \end{matrix}$$

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### Equation Setup

Triangle:  $p = P0 + u*(P1-P0) + v*(P2-P0)$   
Ray:  $p = S + tQ$

Re-arranging:  
 $P0 + u*(P1-P0) + v*(P2-P0) = S + tQ$

Re-arranging some more:  
 $-tQ + u*(P1-P0) + v*(P2-P0) = S - P0$

Then collecting terms, we get:  
 $At + Bu + Cv = D$

where:

$$\begin{matrix} A = -Q \\ B = P1-P0 \\ C = P2-P0 \\ D = S - P0 \end{matrix}$$

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### Three Equations, Three Unknowns

Remembering that this equation is really 3 equations in (x,y,z):

$$At + Bu + Cv = D$$

we have 3 equations with 3 unknowns, which can be cast into a matrix form

$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{Bmatrix} t \\ u \\ v \end{Bmatrix} = \begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix}$$

Our goal is to solve this for  $t^*$ ,  $u^*$ , and  $v^*$

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### Solve for (t\*,u\*,v\*) using Cramer's Rule

$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{Bmatrix} t \\ u \\ v \end{Bmatrix} = \begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix}$$

$$D_0 = \det \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix}$$

$$D_t = \det \begin{bmatrix} D_x & B_x & C_x \\ D_y & B_y & C_y \\ D_z & B_z & C_z \end{bmatrix} \quad t^* = \frac{D_t}{D_0}$$

$$D_u = \det \begin{bmatrix} A_x & D_x & C_x \\ A_y & D_y & C_y \\ A_z & D_z & C_z \end{bmatrix} \quad u^* = \frac{D_u}{D_0}$$

$$D_v = \det \begin{bmatrix} A_x & B_x & D_x \\ A_y & B_y & D_y \\ A_z & B_z & D_z \end{bmatrix} \quad v^* = \frac{D_v}{D_0}$$

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### The Steps

1. Compute  $D_0$
2. If  $D_0 = 0$ ., then the ray is *parallel* to the plane of the triangle **STOP**
3. Compute  $D_t$
4. Compute  $t^*$
5. If  $t^* < 0$ ., the ray goes away from the triangle **STOP**
6. Compute  $D_u$
7. Compute  $u^*$
8. If  $u^* < 0$ ., or  $u^* > 1$ ., then the ray hits outside the triangle **STOP**
9. Compute  $D_v$
10. Compute  $v^*$
11. If  $v^* < 0$ ., or  $v^* > 1-u^*$ ., then the ray hits outside the triangle **STOP**
12. The intersection is at the point  $p = S + Qt^*$

This is known as the **Möller-Trumbore Triangle Intersection Algorithm**

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