Animating Wave Motion using Gerstner Waves

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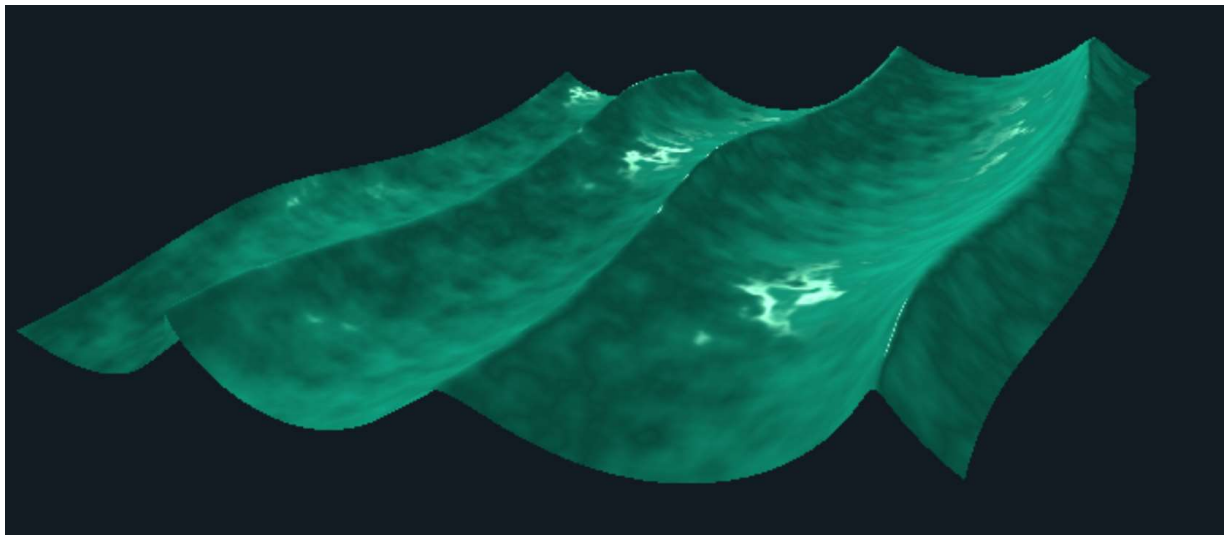

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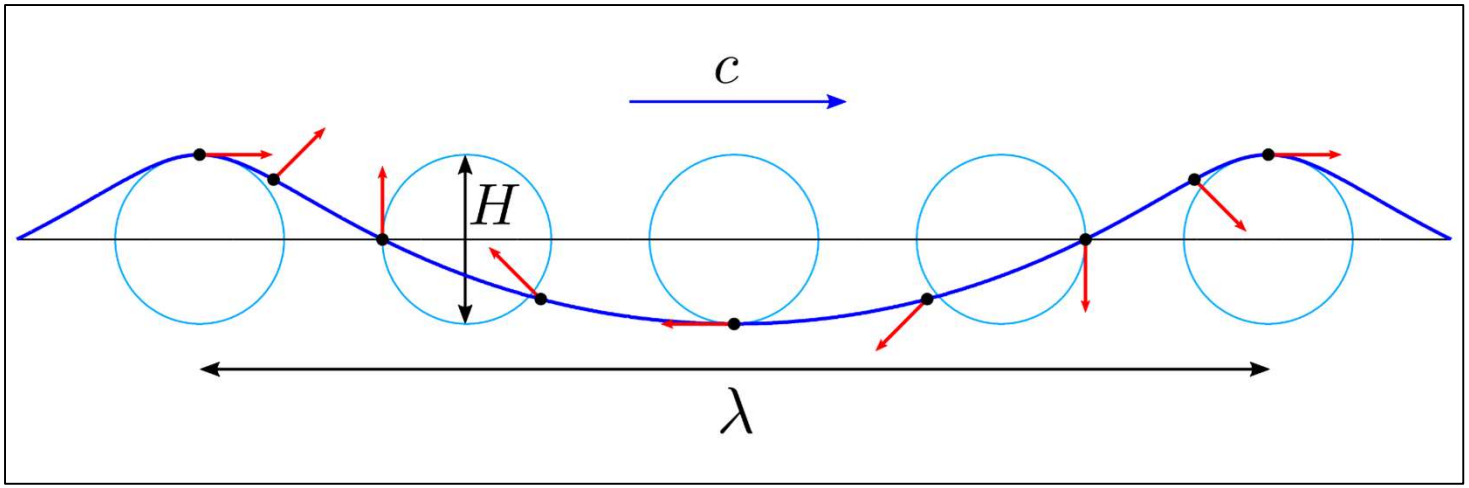
Computer Graphics

How Do Waves Work?

First of all, the water in waves doesn't "flow". It moves in a circular pattern. The equation for this is called a Trochoidal wave, or a Gerstner wave, named after mathematician Franz Josef Gerstner who discovered this in 1802.

Click on the Wikipedia link below for more information. It's an interesting read.



https://en.wikipedia.org/wiki/Trochoidal_wave

If you scroll down in the Wikipedia article, you will see a section called **In Computer Graphics**. I adapted the following equations and code from that section. I am assuming deep water so that the hyperbolic tangent term drops out. Feel free to put it back. I also changed the wave density components to an angular direction (γ) instead.

Gerstner Wave Equations

Horizontal Motion → $x' = x - \sum_{m=0}^{M-1} A_m cos\gamma_m sin\theta_m$

Vertical Motion → $y' = \sum_{m=0}^{M-1} A_m cos\theta_m$

Horizontal Motion → $z' = z - \sum_{m=0}^{M-1} A_m sin\gamma_m sin\theta_m$

$(x, y, z) = original\ vertex\ coordinates$

$(x', y', z') = displaced\ vertex\ coordinates$

$\theta_m = k_m cos\gamma_m x + k_m sin\gamma_m y - \omega_m t - \emptyset_m$

$A_m = Amplitude$

$\omega_m = \sqrt{g k_m}$

$\gamma_m = Wave\ propagation\ angle$

$t = time$

$k_m = Wave\ density$

$\emptyset_m = Wave\ phase\ shift$

gerstner.glib


```
##OpenGL GLIB
Perspective 70
LookAt 0 0 7  0 0 0  0 1 0
Timer 60

Vertex   gerstner.vert
Fragment gerstner.frag
Program  Gerstner                                   \
                uTimeScale <1. 2. 100.>             \
\
                uAm0 <0. .2 1.>                     \
                uKm0 <0.1 1. 5.>                    \
                uGamma0 <-1.57080 0. 1.57080>       \
\
                uAm1 <0. 0. 1.>                     \
                uKm1 <0.1 2. 5.>                    \
                uPhiM1 <0. 0. 6.28>                 \
                uGamma1 <-1.57080 0. 1.57080>       \
\
                uLightX <-20. 0. 20.>               \
                uLightY <1. 10. 20.>                \
                uLightZ <-20. -20. 20.>             \
                uKa <0. .1 1.>                      \
                uKd <0. .6 1.>                      \
                uKs <0. .3 1.>                      \
                uShininess < 1. 2. 200.>            \
                uColor {.1 1. .8 1.}                \
                uNoiseAmp <0. 0. 1.>                \
                uNoiseFreq <.1 .1 2.>               \
\

QuadXZ  -0.2  3.  300  300
```

gerstner.vert, I


```
#version 330 compatibility

uniform float	uTimeScale;
//uniform float	uG;
//uniform float	uH;

uniform float	uAm0;
uniform float	uKm0;
uniform float	uGamma0;

uniform float	uAm1;
uniform float	uKm1;
uniform float	uPhiM1;
uniform float	uGamma1;

uniform float	Timer;

uniform float	uLightX, uLightY, uLightZ;
vec3 eyeLightPosition = vec3(  uLightX,  uLightY,  uLightZ );

out vec3 			vMC;
out vec3 			vEs;
out vec3 			vLs;
out vec3 			vNs;

const float PI = 3.14159265;
const float G  = 1.;

void
main( )
{
	float newx = gl_Vertex.x;
	float newy = 0.;
	float newz = gl_Vertex.z;

	float dxda = 1.;
	float dyda = 0.;
	float dzda = 0.;

	float dxdb = 0.;
	float dydb = 0.;
	float dzdb = 1.;
```

gerstner.vert, II


```
// m = 0
{
		float phiM0 = 0.;  // m=0 is the phase baseline
		float wm0 = sqrt( G*uKm0 );
		float thetam = gl_Vertex.x*uKm0*cos(uGamma0)+ gl_Vertex.z*uKm0*sin(uGamma0) - wm0*Timer*uTimeScale - phiM0;
		newx -= uAm0*cos(uGamma0)*sin(thetam);
		newy += uAm0 * cos(thetam);
		newz -= uAm0*sin(uGamma0)*sin(thetam);

		float dthetamda = uKm0*cos(uGamma0);
		float dthetamdb = uKm0*sin(uGamma0);
		dxda -= uAm0*cos(uGamma0)*cos(thetam)*dthetamda;
		dyda -= uAm0*sin(thetam)*dthetamda;
		dzda -= uAm0*sin(uGamma0)*cos(thetam)*dthetamda;
		dxdb -= uAm0*cos(uGamma0)*cos(thetam)*dthetamdb;
		dydb -= uAm0*sin(thetam)*dthetamdb;
		dzdb -= uAm0*sin(uGamma0)*cos(thetam)*dthetamdb;
}

// m = 1
{
		float wm1 = sqrt( G*uKm1 );
		float thetam = gl_Vertex.x*uKm1*cos(uGamma1)+ gl_Vertex.z*uKm1*sin(uGamma1) - wm1*Timer*uTimeScale - uPhiM1;
		newx -= uAm1*cos(uGamma1)*sin(thetam);
		newy += uAm1 * cos(thetam);
		newz -= uAm1*sin(uGamma1)*sin(thetam);

		float dthetamda = uKm1*cos(uGamma1);
		float dthetamdb = uKm1*sin(uGamma1);
		dxda -= uAm1*cos(uGamma1)*cos(thetam)*dthetamda;
		dyda -= uAm1*sin(thetam)*dthetamda;
		dzda -= uAm1*sin(uGamma1)*cos(thetam)*dthetamda;
		dxdb -= uAm1*cos(uGamma1)*cos(thetam)*dthetamdb;
		dydb -= uAm1*sin(thetam)*dthetamdb;
		dzdb -= uAm1*sin(uGamma1)*cos(thetam)*dthetamdb;
}
```

gerstner.vert, III


```
        vec3 newVertex = vec3( newx, newy, newz );
        vMC = newVertex;

        vec3 ta = vec3( dxda, dyda, dzda );
        vec3 tb = vec3( dxdb, dydb, dzdb );
        vNs = normalize( gl_NormalMatrix*cross( tb, ta ) );
                                                        // surface normal vector

        vec4 ECposition = gl_ModelViewMatrix * vec4( newVertex, 1. );
        vLs = normalize( eyeLightPosition - ECposition.xyz );   // vector from the point
                                                                // to the light position
        vEs = normalize( vec3( 0., 0., 0. ) - ECposition.xyz );  // vector from the point
                                                                 // to the eye position

        gl_Position = gl_ModelViewProjectionMatrix * vec4( newVertex, 1.);
}
```

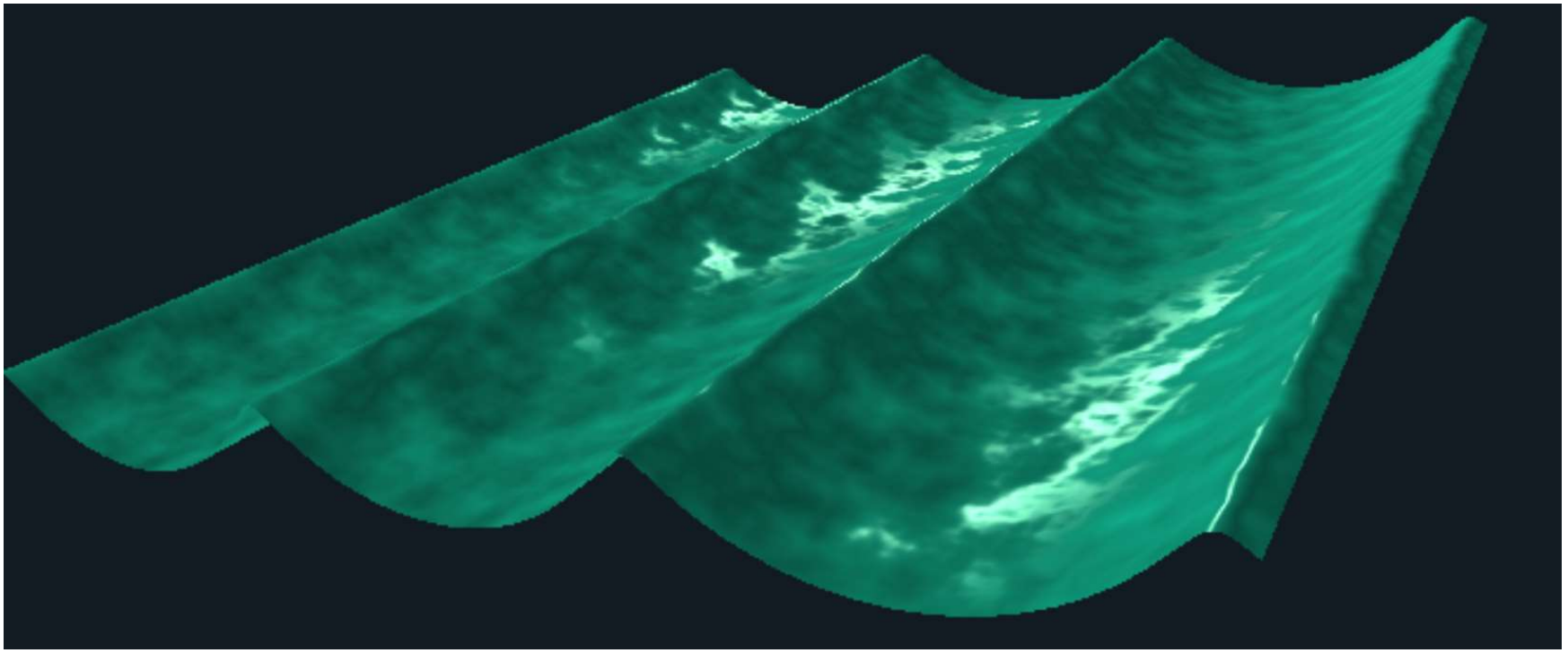
gerstner.frag, I


```
#version 330 compatibility

in vec3                         vMC;
in vec3                         vNs;
in vec3                         vLs;
in vec3                         vEs;

uniform float                   uKa, uKd, uKs;
uniform vec4                    uColor;
uniform float                   uShininess;
uniform sampler3D               Noise3;
uniform float                   uNoiseAmp;
uniform float                   uNoiseFreq;

const vec4 WHITE = { 1., 1., .8, 1. };

vec3
RotateNormal( float angx, float angy, vec3 n )
{
        float cx = cos( angx );
        float sx = sin( angx );
        float cy = cos( angy );
        float sy = sin( angy );

        // rotate about x:
        float yp =  n.y*cx - n.z*sx;            // y'
        n.z      =  n.y*sx + n.z*cx;            // z'
        n.y      =  yp;
        // n.x     =  n.x;

        // rotate about y:
        float xp =  n.x*cy + n.z*sy;            // x'
        n.z      = -n.x*sy + n.z*cy;            // z'
        n.x      =  xp;
        // n.y     =  n.y;

        return normalize( n );
}
```

gerstner.frag, II

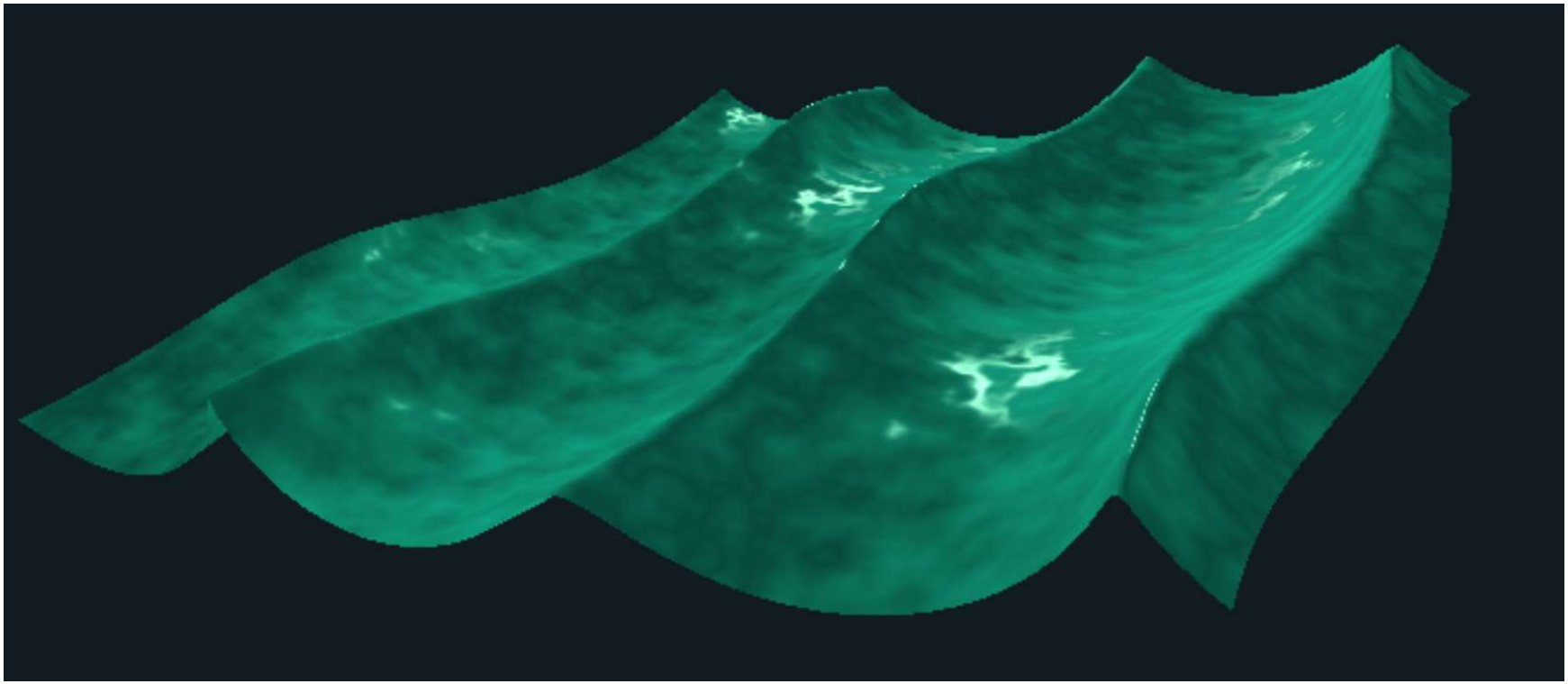

```
void
main( )
{
	vec4 nvx = texture3D( Noise3, uNoiseFreq*vMC );
	vec4 nvy = texture3D( Noise3, uNoiseFreq*vec3(vMC.xy,vMC.z+0.5) );

	float angx = nvx.r + nvx.g + nvx.b + nvx.a;		//  1. -> 3.
	angx = angx - 2.;
	// -1. -> 1.
	angx *= uNoiseAmp;

	float angy = nvy.r + nvy.g + nvy.b + nvy.a;		//  1. -> 3.
	angy = angy - 2.;
	// -1. -> 1.
	angy *= uNoiseAmp;

	vec3 normal = normalize( vNs );
	vec3 light  = normalize( vLs );
	vec3 eye    = normalize( vEs );

	normal = RotateNormal( angx, angy, normal );

	vec4 ambient = uKa * uColor;

	float d = max( dot(normal,light), 0. );
	d = abs( dot(normal,light));
	vec4 diffuse = uKd * d * uColor;

	float s = 0.;
	if( dot(normal,light) > 0. )				// only do specular if the light can see the point
	{
		vec3 ref = normalize( 2. * normal * dot(normal,light) - light );
		s = pow( max( dot(eye,ref),0. ), uShininess );
	}
	vec4 specular = uKs * s * WHITE;
	gl_FragColor = vec4( ambient.rgb + diffuse.rgb + specular.rgb, 1. );
}
```

Example



m = 0

Example



m = 0, 1