Bump Mapping:
Surface Local Coordinate System

- $N$ is the surface normal
- $T$ is the tangent, which must be consistently oriented from vertex to vertex (glman does this automatically in the Sphere primitive)
- $B$ is the Binormal
Bump Mapping:
A Problem

The problem is that lighting information is in Eye Coordinates, but the bump information is in Surface Local Coordinates!

We need to:
1. Figure out how to convert from one to the other, and,
2. Decide which of light information or bump information gets converted to the other’s coordinate system

While we are at it, let's also rename the Surface Local coordinates to \((s, t, h)\) for \((\text{texture}_s, \text{texture}_t, \text{bump}_h)\). This is the same as \((B, T, N)\), but uses terminology that is more bump-specific.

Bump Mapping:
Converting Between Coordinate Systems

Converting from Eye Coordinates to Surface Local Coordinates:

\[
\begin{bmatrix}
  s \\
  t \\
  h
\end{bmatrix}
=
\begin{bmatrix}
  B_x & B_y & B_z \\
  T_x & T_y & T_z \\
  N_x & N_y & N_z
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
\]

(The "Orange Book" uses this to convert the light vector to Surface Local Coordinates.)

Converting from Surface Local Coordinates to Eye Coordinates:

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
=
\begin{bmatrix}
  B_x & T_x & N_x \\
  B_y & T_y & N_y \\
  B_z & T_z & N_z
\end{bmatrix}
\begin{bmatrix}
  s \\
  t \\
  h
\end{bmatrix}
\]

(I prefer to use this one to convert the bump normal to Eye Coordinates.)
Bump Mapping:
Two Ways to Establish the Surface Local Coordinate System

There are 2 good ways to get the tangent and binormal vectors:

1. Have the Tangent already defined (gman’s Sphere does this)
2. Pick a general rule, e.g., "Tangent ≈ up"
   2a. Use Gram-Schmidt to correctly orthogonalize it wrt the Normal
   2b. Use two cross-products to correctly orthogonalize it wrt the Normal

Note: 2a and 2b give the same result; but some people find 2b easier to understand

If the vectors B-T-N form an X-Y-Z-looking right handed coordinate system:

vec3 N = normalize( gl_NormalMatrix * gl_Normal );
vec3 T;
vec3 B;
#define GRAM_SCHMIDT_METHOD
#if defined HAVE_TANGENT_METHOD
T = normalize( vec3( gl_ModelViewMatrix*vec4(Tangent,0.) ) );
B = normalize( cross(T,N) );
#endif
#define GRAM_SCHMIDT_METHOD
#if defined CROSS_PRODUCT_METHOD
T = vec3( 0.,1.,0.);
float d = dot( T, N );
T = normalize( T - d*N );
B = normalize( cross(T,N) );
#endif

Gram-Schmidt Orthogonalization

T = vec3( 0.,1.,0.);
float d = dot( T, N );
T = normalize( T - d*N );
B = normalize( cross(T,N) );

(1) Given that N is correct, how do we change T to be exactly perpendicular to N?
(2) How much of T is in the same direction as N?
(3) How much of T to get rid of so that none of it is in the same direction as N?
(4) The resulting T' is exactly perpendicular to N
Bump Mapping:
Establishing the Surface Local Coordinate System

// Produce the transformation from Surface coords to Eye coords:
BTNx = vec3(B.x, T.x, N.x);
BTNy = vec3(B.y, T.y, N.y);
BTNz = vec3(B.z, T.z, N.z);

// where the light is coming from:
vec3 LightPosition = vec3(LightX, LightY, LightZ);
vec3 ECposition = (gl_ModelViewMatrix * gl_Vertex).xyz;
DirToLight = normalize(LightPosition - ECposition);
gl_Position = gl_ModelViewProjectionMatrix * gl_Vertex;

Bump Mapping:
Using the Surface-Local-to-Eye-Coordinate Transform

vec3
ToXyz( vec3 sth )
{
    sth = normalize( sth );
    vec3 xyz;
    xyz.x = dot( BTNx, sth );
    xyz.y = dot( BTNy, sth );
    xyz.z = dot( BTNz, sth );
    return normalize( xyz );
}
Bump Mapping:
Using the Surface Local Transform

void main()
{
    const float PI = 3.14159265;
    vec2 st = gl_TexCoord[0].st;  // locate the bumps based on (s,t)
    float Swidth  = 1. / BumpDensity;
    float Theight = 1. / BumpDensity;
    float numInS = floor( st.s / Swidth );
    float numInT = floor( st.t / Theight );

    vec2 center;
    center.s = numInS * Swidth + Swidth/2.;
    center.t = numInT * Theight + Theight/2.;
    vec2 stp = st - center;  // st' is now wrt the center of the bump

    float theta = atan( stp.t, stp.s );

    vec3 normal = ToXyz( vec3( 0., 0., 1. ) );  // un-bumped normal
    if( abs(stp.s) > Swidth/4.  ||  abs(stp.t) > Theight/4. )
    {
        normal = ToXyz( vec3( 0., 0., 1. ) );
    }
    else
    {
        if( PI/4. <= theta  &&  theta <= 3.*PI/4. )
        {
            normal = ToXyz( vec3( 0., Theight/4., 0. ) );
        }
        else if( -PI/4. <= theta  &&  theta <= PI/4. )
        {
            normal = ToXyz( vec3( Height, 0., Swidth/4. ) );
        }
        else if( -3.*PI/4. <= theta  &&  theta <= -PI/4. )
        {
            normal = ToXyz( vec3( 0., -Height, Theight/4. ) );
        }
        else if( theta >= 3.*PI/4.  ||  theta <= -3.*PI/4. )
        {
            normal = ToXyz( vec3( -Height, 0., Swidth/4. ) );
        }
    }

    float intensity = Ambient + (1.-Ambient)*dot(normal, DirToLight);
    vec3 litColor = SurfaceColor.rgb * intensity;
    gl_FragColor = vec4( litColor, SurfaceColor.a );
}
Changing the Bump Height

Changing the Bump Density
It's handy to not need a Program-supplied Tangent Vector

Combining Bump and Cube Mapping:
A Good Reason to Work in Eye Coordinates instead of Surface Local Coordinates
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A Good Reason to Work in Eye Coordinates instead of Surface Local Coordinates
It's Even Easier When You Know the Bump Equation in World Coordinates:
Bump-mapping to Create Ripples

In 2D, a slope $m = \frac{dy}{dx}$. It can be expressed as the vector $[1, m]$.

The normal to the shape is the vector perpendicular to the vector slope:

$$[-m, 1]$$

Note that $[1, m] \cdot [-m, 1] = 0$, as it must be.

So, if $z = -Amp \cdot \cos(2\pi x/Pd - 2\pi Time)$, then the slope $dz/dx$ is:

$$\frac{dz}{dx} = Amp \cdot 2\pi/Pd \cdot \sin(2\pi x/Pd - 2\pi Time)$$

and the vector slope is:

$$\text{Slope} = [1, 0, Amp \cdot 2\pi/Pd \cdot \sin(2\pi x/Pd - 2\pi Time)]$$
Following the pattern from before, the normal vector is:

\[
[ \text{Normal} ] = [-\text{Amp} \times 2\pi/Pd \times \sin(2\pi x/Pd - 2\pi \text{Time}), 0., 1.]
\]

This is true along just the X axis. The trick now is to rotate the normal vector into where we really are. Because we are just talking about a rotation, the transformation is the same as if we were rotating a vertex.

\[
\begin{align*}
N_x' &= N_x \times \cos \Theta - N_y \times \sin \Theta = N_x \times \cos \Theta \\
N_y' &= N_x \times \sin \Theta + N_y \times \cos \Theta = N_x \times \sin \Theta \\
N_z' &= N_z = 1.
\end{align*}
\]

(Note that in the final version, you will substitute R for x in the slope equation)

Because each linear ripple has an angle \( \Theta \), we can think of its direction and perpendicular normal like this:

The linear ripple goes through the point C in the direction \([\cos \Theta, \sin \Theta]\)

The normal is then \([-\sin \Theta, \cos \Theta]\)

(Note that slope \( \times \) normal = 0, as it must be.)

The distance, s, of a Model Coordinate position perpendicular to the linear ripple is:

\[s = (\text{MCposition-C}) \cdot (\sin \Theta, \cos \Theta)\]
The amplitude of the wave, \( z \), is:
\[
z = -Amp \cdot \cos\left(\frac{2\pi s}{P} - 2\pi \text{Time}\right)
\]
(where \( P \) is the wave period)

And the slope \( dz/ds \) is:
\[
dz/ds = Amp \cdot \frac{2\pi}{P} \cdot \sin\left(\frac{2\pi s}{P} - 2\pi \text{Time}\right)
\]

If we start by assuming that the ripple angle is 0° (i.e., the wave is propagating in \( y \)), then the vector slope of the wave is:
\[
slope = [0, 1, \frac{dz}{dy}]
\]
\[
= [0, 1, Amp \cdot \frac{2\pi}{P} \cdot \sin\left(\frac{2\pi s}{P} - 2\pi \text{Time}\right)]
\]

So the wave’s vector normal while propagating in \( y \) is:
\[
\text{normal} = [0, -Amp \cdot \frac{2\pi}{P} \cdot \sin\left(\frac{2\pi s}{P} - 2\pi \text{Time}\right), 1]
\]

This is true if the wave is propagating in \( y \), i.e., the ripple angle is 0°. The trick now is to rotate the normal vector into where we really are. Because we are just talking about a rotation, the transformation is the same as if we were rotating a vertex.

\[
\begin{align*}
N_x' &= N_x \cdot \cos \Theta - N_y \cdot \sin \Theta \\
N_y' &= N_x \cdot \sin \Theta + N_y \cdot \cos \Theta \\
N_z' &= N_z
\end{align*}
\]

\[
\text{vec3 normal} = \text{normalize}(\text{vec3}(N_x', N_y', N_z'));
\]

So, for any MCposition of a fragment, we compute the normal vector to the simulated rippled surface. We then make this interact with the light source location to make variations in intensity give the rippled appearance.
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Visualization: Terrain Height Bump-Mapping

Visualization by Nick Gebbie