



## **Transforming a Surface Normal**



## Before transformation: T = (P0 - P)

 $N \cdot T = 0$ , or expressed in matrix notation:

$$\begin{cases} \{N_i\}^T & \{T_i\} = 0 \\ 1x3 & 3x1 \end{cases}$$

## After transformation:

$$T' = (P_0' - P') = ([M]\{P_0\} - [M]\{P\}) = [M](\{P_0\} - \{P\}) = [M]\{T\}$$

N  $^{\prime}\cdot T$   $^{\prime}$  = ~0~ , or, expressed in matrix notation:

$$\begin{bmatrix} \{N'\}^T & \{T'\} = 0 \\ 1x3 & 3x1 \end{bmatrix}$$

## If [Q] is the matrix which needs to transform the normal, then:

 $(\begin{array}{cccc} [Q]\{N\} \end{array})^T & \{T\ '\} = 0 & \text{, then, substituting for } \{T'\} \text{:} \\ (3x3\ 3x1\ )^T & 3x1 & \end{array}$ 

( [Q]{N} )  $^T$  [M] {T} = 0 , then, distributing the transpose:

 $\{N\}^{{ \mathrm{\scriptscriptstyle T} }}[Q]^{{ \mathrm{\scriptscriptstyle T} }}[M]\;\{T\}=0\;$  , then, associating the 2 middle terms:

 $\{N\}^T$  (  $[Q]^T[M]$  )  $\{T\}$  = 0 , then, remembering that  $\{N\ \}^T$   $\{T\ \}$  =  $\,0$ :

 $[Q]^T[M] = [I]$  , so that Q must equal:

 $Q = (\ [M]^{-1}\ )^T$