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Homogeneous Coordinates

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homogeneouscoords.pptx

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Homogeneous Coordinates: Adding a 4th Value to an XYZ Triple

We usually think of a 3D point as being represented by a triple: (x,y,z).

Using homogeneous coordinates, we add a 4th number: (x,y,z,w)

A graphics system, by convention, performs transformations and clipping using (x,y,z,w) and then divides x, y, and z by w before it uses them.

$$X = \frac{x}{w}, Y = \frac{y}{w}, Z = \frac{z}{w}$$

Thus (1,2,3,1) , (2,4,6,2) , (-1,-2,-3,-1) all represent the same 3D point.

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This Seems Awkward – Why Do It?

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One reason is that it allows for perspective division within the matrix mechanism. The OpenGL call *glFrustum(left, right, bottom, top, near, far)* creates this matrix:

$$\begin{Bmatrix} x' \\ y' \\ z' \\ w' \end{Bmatrix} = \begin{bmatrix} \frac{2 \cdot near}{right - left} & 0 & \frac{right + left}{right - left} & 0 \\ 0 & \frac{2 \cdot near}{top - bottom} & \frac{top + bottom}{top - bottom} & 0 \\ 0 & 0 & \frac{-(far + near)}{far - near} & \frac{-2 \cdot far \cdot near}{far - near} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix}$$

This gives w' = -z, which is the necessary divisor for perspective.

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Another Reason is to be able to Represent Points at Infinity

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This is useful to be able specify a parallel light source by placing the light source location at infinity.

The point (1,2,3,1) represents the 3D point (1,2,3)

The point (1,2,3,.5) represents the 3D point (2,4,6)

The point (1,2,3,.01) represents the point (100,200,300)

So, (1,2,3,0) represents a point at infinity, but along the ray from the origin through (1,2,3)

Points-at-infinity are used for parallel light sources and some shadow algorithms

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However, When Using Homogeneous Coordinates, You Sometimes Just Need to be able to get a Vector Between Two Points

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To get a vector between two homogeneous points, we subtract them:

$$(x_b, y_b, z_b, w_b) - (x_a, y_a, z_a, w_a) = \frac{(x_b, y_b, z_b)}{w_b} - \frac{(x_a, y_a, z_a)}{w_a}$$

$$= \frac{(w_a x_b, w_a y_b, w_a z_b) - (w_b x_a, w_b y_a, w_b z_a)}{w_a w_b}$$

Fortunately, most of the time that we do this, we only want a **unit vector** in that direction, not the full vector. So, we can ignore the denominator, and just say:

$$\hat{v} = normalize(w_a x_b - w_b x_a, w_a y_b - w_b y_a, w_a z_b - w_b z_a);$$

```
vec3
VectorBetween( vec4 a, vec4 b )
{
   return normalize( vec3( a.w*b.x – b.w*a.x , a.w*b.y – b.w*a.y , a.w*b.z – b.w*a.z ) );
}
```

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