Tessellation Shaders

Why do we need a Tessellation step right in the pipeline?

- You can perform adaptive subdivision based on a variety of criteria (size, curvature, etc.)
- You can provide coarser models, but have finer ones displayed (= geometric compression)
- You can apply detailed displacement maps without supplying equally detailed geometry
- You can adapt visual quality to the required level of detail
- You can create smoother silhouettes
- You can do all of this, and someone else will supply the patterns!

What built-in patterns can the Tessellation shaders produce?

- Lines
- Triangles
- Quads (subsequently broken into triangles)
The Tessellation Shaders: Where Do they Fit in the Pipeline?

**Tessellation Shader Organization**

- **Tessellation Control Shader**
  - Transformed xyz Patch Vertices from the Vertex Shader
  - New Patch Vertices in xyz, How much to tessellate, Per-vertex attributes

- **Tessellation Primitive Generator**
  - Tessellates the curve or surface into uvw coordinates

- **Tessellation Evaluation Shader**
  - uvw vertices for the tessellated primitives
  - Possibly apply a displacement map

- **Primitive Assembly**
  - xyz vertices

- **Geometry Shader**
  - Primitive Assembly

- **Rasterizer**
  - Fragment Shader
The **Tessellation Control Shader (TCS)** transforms the input coordinates to a regular surface representation. It also computes the required tessellation level based on distance to the eye, screen space spanning, hull curvature, or displacement roughness. There is one invocation per output vertex.

The **Fixed-Function Tessellation Primitive Generator (TPG)** generates semi-regular u-v-w coordinates in specific patterns. (In fact, if it had been up to me, this would have been called the **Tessellation Pattern Generator**.)

The **Tessellation Evaluation Shader (TES)** evaluates the surface in \(uvw\) coordinates. It interpolates attributes and applies displacements. There is one invocation per generated vertex.

There is a new “Patch” primitive – it is the face and its neighborhood:

```
gBegin( GL_PATCHES )
gVertex3f( ... );
gVertex3f( ... );
gEnd();
```

These have no implied topology – they will be given to you in an array. It’s up to your shader to interpret the order.

```
glPatchParameteri( GL_PATCH_VERTICES, num );   // #of vertices in each patch
```

In **GLSL**:

```
#version 400
#extension GL_ARB_tessellation_shader : enable
```

---

In the OpenGL Program

```
gBegin( GL_PATCHES );
gVertex3f( ... );
gVertex3f( ... );
gEnd();
```

GLuint tcs = glCreateShader( GL_TESS_CONTROL_SHADER );

GLuint tes = glCreateShader( GL_TESS_EVALUATION_SHADER );

**Check the OpenGL extension:**

“GL_ARB_tessellation_shader”

**In GLSL:**

```
#version 400
#extension GL_ARB_tessellation_shader : enable
```
**TCS Inputs**

`gl_in[]` is an array of structures:

```c
struct {
    vec4 gl_Position;
    float gl_PointSize;
    float gl_ClipDistance[6];
} gl_in[];
```

`gl_InvocationID` tells you which output vertex you are working on. This must be the index into the `gl_in[]` array.

`gl_PatchVerticesIn` is the number of vertices in each patch and the dimension of `gl_in[]`.

`gl_PrimitiveID` is the number of primitives since the last `glBegin()` (the first one is #0).

**TCS Outputs**

`gl_out[]` is an array of structures:

```c
struct {
    vec4 gl_Position;
    float gl_PointSize;
    float gl_ClipDistance[6];
} gl_out[];
```

All invocations of the TCS have read-only access to all the output information.

```
layout( vertices = n ) out;
```

Used to specify the number of vertices output to the TPG.

`gl_TessLevelOuter[4]` is an array containing up to 4 edges of tessellation levels.

`gl_TessLevelInner[2]` is an array containing up to 2 edges of tessellation levels.
In the TCS

User-defined variables defined per-vertex are qualified as “out”
User-defined variables defined per-patch are qualified as “patch out”

Defining how many vertices this patch will output:

```
layout( vertices = 16 ) out;
```

Tessellation Primitive Generator

Is “fixed-function”, i.e., you can’t change its operation except by setting parameters

 Consumes all vertices from the TCS and emits tessellated triangles, quads, or lines

 Outputs positions as coordinates in barycentric (u,v,w)

 All three coordinates (u,v,w) are used for triangles

 Just (u,v) are used for quads and isolines
**TES Inputs**

Reads one vertex of 0 \(\leq (u,v,w) \leq 1\) coordinates in variable `vec3 gl_TessCoord`

User-defined variables defined per-vertex are qualified as "out"

User-defined variables defined per-patch are qualified as "patch out"

\[
\text{gl_in}\[\]\ is \ an\ array\ \ of\ \ structures\ \ coming\ \ from\ \ the\ \ TCS:
\]

\[
\text{struct}
\{
\quad \text{vec4 gl\_Position;}
\quad \text{float gl\_PointSize;}
\quad \text{float gl\_ClipDistance[ 6 ];}
\}\ \text{gl\_in}\[\]:
\]

Triangle pattern
Quad pattern
Isoline pattern

**Tessellation Primitive Generator (TPG)**

- Is "fixed-function", i.e., you can’t change its operation except by setting parameters
- Consumes all vertices from the TCS and emits vertices for the **triangles**, **quads**, or **isolines** patterns
- TPG outputs a series of vertices as coordinates in barycentric \((u,v,w)\) parametric space
- All three coordinates \((u,v,w)\) are used for triangles
- Just \((u,v)\) are used for quads and isolines
TES Output Topologies: the Quad Pattern

\[
\begin{array}{c}
(u=0,v=0) \\
OL0 \\
(u=0,v=1) \\
OL1 \\
(u=1,v=0) \\
OL2 \\
(u=1,v=1) \\
OL3
\end{array}
\]

TES Output Topologies: the Isolines Pattern

Top line not drawn

OL0 == 1 implies that you just want to draw a single curve
**Examples**

In these examples:

1. We are using `glman` to run them. The only necessary input files are the `glman .glib` file and the shader files. If you aren’t using `glman`, you can do this from a full OpenGL program.

2. All of the surface examples use the Geometry Shader triangle-shrink shader. This isn’t necessary, but is educational to really see how much and where the surfaces have been tessellated.
Example: A Bézier Curve

\[ P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3 \]

1. You program the Tessellation Control Shader to decide how much to tessellate the curve based on screen area, curvature, etc.

   Can even tessellate non-uniformly if you want

   The OpenGL tessellation can also do 1D curves. Just set OL0 = 1.
2. The Tessellation Primitive Generator generates $u, v, w$ values for as many subdivisions as the TCS asked for.

$$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u) P_2 + u^3 P_3$$

3. The Tessellation Evaluation Shader computes the $x, y, z$ coordinates based on the TPG’s $u$ values

$$P(u) = u^3(-P_0 + 3P_1 - 3P_2 + P_3) + u^2(3P_0 - 6P_1 + 3P_2) + u(-3P_0 + 3P_1) + P_0$$
In an OpenGL Program

```cpp
glPatchParameteri( GL_PATCH_VERTICES, 4 );
gBegin( GL_PATCHES );
glVertex3f( x0, y0, z0 );
glVertex3f( x1, y1, z1 );
glVertex3f( x2, y2, z2 );
glVertex3f( x3, y3, z3 );
gEnd( );
```

In a .glib File

```cpp
##OpenGL GLIB
Perspective 70

Vertex  beziercurve.vert
Fragment  beziercurve.frag
TessControl  beziercurve.tcs
TessEvaluation  beziercurve.tes
Program BezierCurve  uOuter0 <0 1 5>  uOuter1 <3 5 50>

Color 1 .5  0.  1.

NumPatchVertices 4
gBegin gl_patches
gVertex 0. 0. 0.
gVertex 1. 1. 1.
gVertex 2. 1. 0.
gVertex 3. 0. 1.
gEnd
```
In the TCS Shader

```cpp
#version 400
#extension GL_ARB_tessellation_shader: enable

uniform int uOuter0, uOuter1;

layout( vertices = 4 ) out;

void main()
{
    gl_out[ gl_InvocationID ].gl_Position = gl_in[ gl_InvocationID ].gl_Position;

    gl_TessLevelOuter[0] = float(uOuter0);
    gl_TessLevelOuter[1] = float(uOuter1);
}
```

Assigning the intermediate pi’s is here to make the code more readable. We assume that the compiler will optimize this away.

In the TES Shader

```cpp
#version 400
#extension GL_ARB_tessellation_shader: enable

layout(isolines, equal_spacing) in;

void main()
{
    vec4 p0 = gl_in[0].gl_Position;
    vec4 p1 = gl_in[1].gl_Position;
    vec4 p2 = gl_in[2].gl_Position;
    vec4 p3 = gl_in[3].gl_Position;

    float u = gl_TessCoord.x;

    // the basis functions:
    float b0 = (1.-u) * (1.-u) * (1.-u);
    float b1 = 3. * u * (1.-u) * (1.-u);
    float b2 = 3. * u * u * (1.-u);
    float b3 = u * u * u;

    gl_Position = b0*p0 + b1*p1 + b2*p2 + b3*p3;
}
```
Example: A Bézier Curve

Outer1 = 5

Outer1 = 50

Example: A Bézier Surface

P00 P01 P02 P03 P10 P11 P12 P13 P20 P21 P22 P23 P30 P31 P32 P33 P11 P12 P13 P21 P22 P23 P31 P32 P33
Bézier Surface Parametric Equations

\[ P(u, v) = \begin{bmatrix} (1-u)^3 & 3u(1-u)^2 & 3u^2(1-u) & u^3 \end{bmatrix} \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} (1-v)^3 \\ 3v(1-v)^2 \\ 3v^2(1-v) \\ v^3 \end{bmatrix} \]

In an OpenGL Program

gPatchParameteri( GL_PATCH_VERTICES, 16 );

gBegin( GL_PATCHES );
gVertex3f( x_{00}, y_{00}, z_{00} );
gVertex3f( x_{10}, y_{10}, z_{10} );
gVertex3f( x_{20}, y_{20}, z_{20} );
gVertex3f( x_{30}, y_{30}, z_{30} );
gVertex3f( x_{01}, y_{01}, z_{01} );
gVertex3f( x_{11}, y_{11}, z_{11} );
gVertex3f( x_{21}, y_{21}, z_{21} );
gVertex3f( x_{31}, y_{31}, z_{31} );
gVertex3f( x_{02}, y_{02}, z_{02} );
gVertex3f( x_{12}, y_{12}, z_{12} );
gVertex3f( x_{22}, y_{22}, z_{22} );
gVertex3f( x_{32}, y_{32}, z_{32} );
gVertex3f( x_{03}, y_{03}, z_{03} );
gVertex3f( x_{13}, y_{13}, z_{13} );
gVertex3f( x_{23}, y_{23}, z_{23} );
gVertex3f( x_{33}, y_{33}, z_{33} );
gEnd( );

This order is not set by OpenGL. It is set by you. Pick a convention yourself and stick to it! GLSL doesn’t care as long as you are consistent.
In the .glib File

In the TCS Shader

#version 400
#extension GL_ARB_tessellation_shader : enable
uniform float uOuter02, uOuter13, uInner0, uInner1;
layout( vertices = 16 ) out;
void main() {
  gl_out[gl_invocationID].gl_Position = gl_in[gl_invocationID].gl_Position;
  gl_TessLevelOuter[0] = gl_TessLevelOuter[2] = uOuter02;
  gl_TessLevelInner[0] = uInner0;
  gl_TessLevelInner[1] = uInner1;
}
In the TES Shader

```glsl
//version 400 compatibility
#extension GL_ARB_tessellation_shader : enable
layout( quads, equal_spacing, ccw ) in;
out vec3 teNormal;

void main() {
    vec4 p00 = gl_in[0].gl_Position;
    vec4 p10 = gl_in[1].gl_Position;
    vec4 p20 = gl_in[2].gl_Position;
    vec4 p30 = gl_in[3].gl_Position;
    vec4 p01 = gl_in[4].gl_Position;
    vec4 p11 = gl_in[5].gl_Position;
    vec4 p21 = gl_in[6].gl_Position;
    vec4 p31 = gl_in[7].gl_Position;
    vec4 p02 = gl_in[8].gl_Position;
    vec4 p12 = gl_in[9].gl_Position;
    vec4 p22 = gl_in[10].gl_Position;
    vec4 p03 = gl_in[12].gl_Position;
    vec4 p13 = gl_in[13].gl_Position;
    vec4 p23 = gl_in[14].gl_Position;
    vec4 p33 = gl_in[15].gl_Position;

    float u = gl_TessCoord.x;
    float v = gl_TessCoord.y;

    float bu0 = (1.-u) * (1.-u) * (1.-u);
    float bu1 = 3. * u * (1.-u) * (1.-u);
    float bu2 = 3. * u * u * (1.-u);
    float bu3 = u * u * u;
    float dbu0 = -3. * (1.-u) * (1.-u);
    float dbu1 = 3. * (1.-u) * (1.-3.*u);
    float dbu2 = 3. * u * (2.-3.*u);
    float dbu3 = 3. * u * u;

    float bv0 = (1.-v) * (1.-v) * (1.-v);
    float bv1 = 3. * v * (1.-v) * (1.-v);
    float bv2 = 3. * v * v * (1.-v);
    float bv3 = v * v * v;
    float dbv0 = -3. * (1.-v) * (1.-v);
    float dbv1 = 3. * (1.-v) * (1.-3.*v);
    float dbv2 = 3. * v * (2.-3.*v);
    float dbv3 = 3. * v * v;

    gl_Position =
        bu0 * ( bv0*p00 + bv1*p01 + bv2*p02 + bv3*p03 )
        + bu1 * ( bv0*p10 + bv1*p11 + bv2*p12 + bv3*p13 )
        + bu2 * ( bv0*p20 + bv1*p21 + bv2*p22 + bv3*p23 )
        + bu3 * ( bv0*p30 + bv1*p31 + bv2*p32 + bv3*p33 );
}
```

Assigning the intermediate pij’s is here to make the code more readable. We assume that the compiler will optimize this away.
In the TES Shader –
Computing the Normal, given a u and v

Tangent Vectors

```cpp
vec4 dpdu = dbu0 * ( bv0*p00 + bv1*p01 + bv2*p02 + bv3*p03 ) + dbu1 * ( bv0*p10 + bv1*p11 + bv2*p12 + bv3*p13 ) + dbu2 * ( bv0*p20 + bv1*p21 + bv2*p22 + bv3*p23 ) + dbu3 * ( bv0*p30 + bv1*p31 + bv2*p32 + bv3*p33 );

vec4 dpdv = bu0 * ( dbv0*p00 + dbv1*p01 + dbv2*p02 + dbv3*p03 ) + bu1 * ( dbv0*p10 + dbv1*p11 + dbv2*p12 + dbv3*p13 ) + bu2 * ( dbv0*p20 + dbv1*p21 + dbv2*p22 + dbv3*p23 ) + bu3 * ( dbv0*p30 + dbv1*p31 + dbv2*p32 + dbv3*p33 );

teNormal = normalize( cross( dpdu.xyz, dpdv.xyz ) );
```

Example: A Bézier Surface

```
uOuter02 = uOuter13 = 10
uInner0  = uInner1   = 5
```

```
uOuter02 = uOuter13 = 5
uInner0  = uInner1   = 5
```
Tessellation Levels and Smooth Shading

Smoothing edge boundaries is one of the reasons that you can set Outer and Inner tessellation levels separately.

Example: Whole-Sphere Subdivision

```
spheresubd.glib

##OpenGL GLIB
Vertex spheresubd.vert
Fragment spheresubd.frag
TessControl spheresubd.tcs
TessEvaluation spheresubd.tes
Geometry spheresubd.geom
Program SphereSubd
  uDetail <1 30 200>
  uScale <0.1 1. 10.>
  uShrink <0. 1. 1.>
  uFlat <false>
  uColor {1. 1. 0.}
  uLightX <-10. 5. 10.>
  uLightY <-10. 10. 10.>
  uLightZ <-10. 10. 10.>
Color 1. 1. 0.
NumPatchVertices 1

glBegin gl_patches
  glVertex 0. 0. 0. .2
  glVertex 0. 1. 0. .3
  glVertex 0. 0. 1. .4
  glEnd
```

Using the x, y, z, and w to specify the center and radius of the sphere.
Example: Whole-Sphere Subdivision

spheresubd.vert

```
#version 400 compatibility

out vec3 vCenter;
out float vRadius;

void main()
{
    vCenter = gl_Vertex.xyz;
    vRadius = gl_Vertex.w;

    gl_Position = vec4(0., 0., 0., 1.); // doesn't matter now – we will in the cords later
}
```

Using the x, y, z, and w to specify the center and radius of the sphere

Example: Whole-Sphere Subdivision

spheresubd.tcs

```
#version 400 compatibility
#extension GL_ARB_tessellation_shader : enable

in float vRadius[ ];
in vec3 vCenter[ ];
patch out float tcRadius;
patch out vec3 tcCenter;
uniform float uDetail;
uniform float uScale;
layout( vertices = 1 ) out;

void main()
{
    gl_out[ gl_InvocationID ].gl_Position = gl_in[ 0 ].gl_Position; // (0,0,0,1)
    tcCenter = vCenter[ 0 ];
    tcRadius = vRadius[ 0 ];

    gl_TessLevelOuter[ 0 ] = 2.0;
    gl_TessLevelOuter[ 1 ] = uScale * tcRadius * uDetail;
    gl_TessLevelOuter[ 2 ] = 2.0;
    gl_TessLevelOuter[ 3 ] = uScale * tcRadius * uDetail;
    gl_TessLevelInner[ 0 ] = uScale * tcRadius * uDetail;
    gl_TessLevelInner[ 1 ] = uScale * tcRadius * uDetail;
```

Using the scale and the radius to help set the tessellation detail.

Outer[0] and Outer[2] are the number of divisions at the poles. Outer[1] and Outer[3] are the number of divisions at the vertical seams. Inner[0] and Inner[1] are the inside sphere detail.
Example: Whole-Sphere Subdivision

```glsl
// Version 400 compatibility
#extension GL_ARB_tessellation_shader : enable
uniform float uScale;
layout( quads, equal_spacing, ccw ) in;
patch in float tcRadius;
patch in vec3 tcCenter;
out vec3 teNormal;
const float PI = 3.14159265;
void main( )
{
  vec3 p = gl_in[0].gl_Position.xyz;
  float u = gl_TessCoord.x;
  float v = gl_TessCoord.y;
  float w = gl_TessCoord.z;
  float phi = PI * ( u - .5 );
  float theta = 2. * PI * ( v - .5 );
  float cosphi = cos(phi);
  vec3 xyz = vec3( cosphi*cos(theta), sin(phi), cosphi*sin(theta) );
  teNormal = xyz;
  xyz *= ( uScale * tcRadius );
  xyz += tcCenter;
  gl_Position = gl_ModelViewMatrix * vec4( xyz,1. );
}
```

Turning u and v into spherical coordinates

---

**Oregon State University**

**Computer Graphics**
Making the Whole-Sphere Subdivision Adapt to Screen Coverage

sphereadapt.tcs, I

```glsl
#version 400 compatibility
#extension GL_ARB_tessellation_shader : enable

in float vRadius[];
in vec3 vCenter[];

patch out float tcRadius;
patch out vec3 tcCenter;

uniform float uDetail;
layout( vertices = 1 ) out;

void main() {
    gl_out[ gl_InvocationID ].gl_Position = gl_in[ 0 ].gl_Position; // (0,0,0,1)
    tcCenter = vCenter[ 0 ];
    tcRadius = vRadius[ 0 ];

    vec4 mx = vec4( vCenter[0] - vec3( vRadius[0], 0., 0. ), 1. );
    vec4 px = vec4( vCenter[0] + vec3( vRadius[0], 0., 0. ), 1. );
    vec4 my = vec4( vCenter[0] - vec3( 0., vRadius[0], 0. ), 1. );
    vec4 py = vec4( vCenter[0] + vec3( 0., vRadius[0], 0. ), 1. );
    vec4 mz = vec4( vCenter[0] - vec3( 0., 0., vRadius[0] ), 1. );
    vec4 pz = vec4( vCenter[0] + vec3( 0., 0., vRadius[0] ), 1. );
}
```

We no longer use uScale or tcRadius. But, we do use uDetail to provide a way to convert from NDC to Screen Space or to indicate the quality you'd like (I.e., uDetail depends on how good you want the spheres to look and on how large the window is in pixels.)
Making the Whole-Sphere Subdivision Adapt to Screen Coverage

sphereadapt.tes

```glsl
#version 400 compatibility
#extension GL_ARB_tessellation_shader: enable
layout(quads, equal_spacing, ccw) in;
patch in float tcRadius;
patch in vec3 tcCenter;
out vec3 teNormal;
const float PI = 3.14159265;
void main()
{
  vec3 p = gl_in[0].gl_Position.xyz;
  float u = gl_TessCoord.x;
  float v = gl_TessCoord.y;
  float w = gl_TessCoord.z;
  float phi = PI * (u - .5);
  float theta = 2. * PI * (v - .5);
  float cosphi = cos(phi);
  vec3 xyz = vec3(cosphi*cos(theta), sin(phi), cosphi*sin(theta));
  teNormal = xyz;
  xyz *= tcRadius;
  xyz += tcCenter;
  gl_Position = gl_ModelViewMatrix * vec4(xyz,1.);
}
```

Original

Triangles Shrunk

Zoomed In

Zoomed Out

Rotated

Notice that the number of triangles adapts to the screen coverage of each sphere, and that the size of the tessellated triangles stays about the same, regardless of radius or transformation.
Example: PN Triangles

General idea: turn each triangle into a triangular Bézier patch. Create the Bézier control points by using the surface normals at the corner vertices. The Bézier patch equation can then be interpolated to any level of tessellation.

Example: PN Triangles

#version 400 compatibility
uniform float uScale;
out vec3 vNormal;
void main() {
    vec3 xyz = gl_Vertex.xyz;
    xyz *= uScale;
    gl_Position = gl_ModelViewMatrix * vec4( xyz, 1.);
    vNormal = normalize( gl_NormalMatrix * gl_Normal );
}

pntriangles.vert

Example: PN Triangles

#version 400 compatibility
#extension GL_ARB_tessellation_shader : enable
uniform int uOuter, uInner;
layout( vertices = 3 ) out;
in vec3 vNormal[];
out vec3 tcNormals[];
void main() {
    tcNormals[ gl_InvocationID ] = vNormal[ gl_InvocationID ];
    gl_out[ gl_InvocationID ].gl_Position = gl_in[ gl_InvocationID ].gl_Position;
    gl_TessLevelOuter[0] = uScale * float(uOuter);
    gl_TessLevelOuter[1] = uScale * float(uOuter);
    gl_TessLevelOuter[2] = uScale * float(uOuter);
    gl_TessLevelInner[0] = uScale * float(uInner);
}

pntriangles.tcs
Example: PN Triangles

```glsl
#version 400
#extension GL_ARB_tessellation_shader : enable
in vec3 tcNormals[ ];
out vec3 teNormal;
layout( triangles, equal_spacing, ccw ) in;
void main() {
    vec3 p1 = gl_in[0].gl_Position.xyz;
    vec3 p2 = gl_in[1].gl_Position.xyz;
    vec3 p3 = gl_in[2].gl_Position.xyz;
    vec3 n1 = tcNormals[0];
    vec3 n2 = tcNormals[1];
    vec3 n3 = tcNormals[2];
    float u = gl_TessCoord.x;
    float v = gl_TessCoord.y;
    float w = gl_TessCoord.z;
    vec3 b300 = p1;
    vec3 b030 = p2;
    vec3 b003 = p3;
    float w12 = dot( p2 - p1, n1);
    float w21 = dot( p1 - p2, n2);
    float w13 = dot( p3 - p1, n1);
    float w31 = dot( p1 - p3, n3);
    float w23 = dot( p3 - p2, n2);
    float w32 = dot( p2 - p3, n3);
    vec3 b210 = ( 2.*p1 + p2 - w12*n1 ) / 3.;
    vec3 b120 = ( 2.*p2 + p1 - w21*n2 ) / 3.;
    vec3 b021 = ( 2.*p2 + p3 - w23*n2 ) / 3.;
    vec3 b012 = ( 2.*p3 + p2 - w32*n3 ) / 3.;
    vec3 b102 = ( 2.*p3 + p1 - w31*n3 ) / 3.;
    vec3 b201 = ( 2.*p1 + p3 - w13*n1 ) / 3.;
    vec3 ee = ( b210 + b120 + b021 + b012 + b102 + b201 ) / 6.;
    vec3 vv = ( p1 + p2 + p3 ) / 3.;
    vec3 b111 = ee + ( ee - vv ) / 2.;
    vec3 xyz = 1.*b300*w*w*w + 1.*b030*u*u*u + 1.*b003*v*v*v +
                3.*b210*u*w*w + 3.*b120*u*u*w + 3.*b201*v*w*w +
                3.*b021*u*u*v + 3.*b102*v*v*w + 3.*b012*u*v*v +
                6.*b111*u*v*w;
    float v12 = 2. * dot( p2-p1, n1+n2 ) / dot( p2-p1, p2-p1 );
    float v23 = 2. * dot( p3-p2, n2+n3 ) / dot( p3-p2, p3-p2 );
    float v31 = 2. * dot( p1-p3, n3+n1 ) / dot( p1-p3, p1-p3 );
    vec3 n200 = n1;
    vec3 n020 = n2;
    vec3 n002 = n3;
    vec3 n110 = normalize( n1 + n2 - v12*(p2-p1) );
    vec3 n011 = normalize( n2 + n3 - v23*(p3-p2) );
    vec3 n101 = normalize( n3 + n1 - v31*(p1-p3) );
    Normal = n200*w*w + n020*u*u + n002*v*v +
             n110*w*u + n011*u*v + n101*w*v;
    gl_Position = vec4( xyz, 1. );
}
```
Example: PN Triangles

### pntriangles.geom

```glsl
#version 400

precision GL_gpu_shader4: enable
precision GL_geometry_shader4: enable

uniform float uShrink;
in vec3 teNormal[];
out float gLightIntensity;
const vec3 LIGHTPOS = vec3(5., 10., 10.);
vec3 V[3];
vec3 CG;

void ProduceVertex(int v)
{
    gLightIntensity = abs(dot(normalize(LIGHTPOS - V[v]), normalize(teNormal[v])));
    gl_Position = gl_ProjectionMatrix * vec4(CG + uShrink * (V[v] - CG), 1.);
    EmitVertex();
}

void main()
{
    V[0] = gl_PositionIn[0].xyz;
    V[1] = gl_PositionIn[1].xyz;
    CG = (V[0] + V[1] + V[2]) / 3.;
    ProduceVertex(0);
    ProduceVertex(1);
    ProduceVertex(2);
}
```

### pntriangles.frag

```glsl
#version 400

precision GL_gpu_shader4: enable
precision GL_geometry_shader4: enable

in float gLightIntensity;
const vec3 COLOR = vec3(1., 1., 0.);

void main()
{
    gl_FragColor = vec4(gLightIntensity*COLOR, 1.);
}
```
The Cow's Tail is a Good Example of using PN Triangles

Notice how much improvement there is just by increasing the outer tessellation. This is because smooth shading already helps the inner parts of triangles, but does nothing for the silhouettes.

The Difference Between Tessellation Shaders and Geometry Shaders

By now, you are probably confused about when to use a Geometry Shader and when to use a Tessellation Shader. Both are capable of creating new geometry from existing geometry. See if this helps.

**Use a Geometry Shader when:**

1. You need to convert an input topology into a different output topology, such as in the silhouette and hedgehog shaders (triangles→lines) or the explosion shader (triangles→points)

2. You need some sort of geometry processing to come after the Tessellation Shader (such as how the shrink shader was used).

**Use a Tessellation Shader when:**

1. One of the built-in tessellation patterns will suit your needs.

2. You need more than 6 input vertices to define the surface being tessellated.

3. You need more output vertices than a Geometry Shader can provide.
Demonstrating the Limits of Tessellation Shaders

This tessellation is using 64x64 (the maximum allowed).

This is pretty good-looking, but doesn’t come close to using the full 4096x2276 resolution available for the bump-map.