Fractals
IN
Computer Graphics
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The Definition of Fractal

- A fractal is a never-ending pattern.
- Fractals are infinitely complex patterns that are self-similar across different scales.

Koch Snowflake

Sierpinski Carpet

The Definition of Fractal

- Exact self-similarity
  The strongest kind of self-similarity, and fractals look the same at all scales.
  Fractals defined by iterative function systems often exhibit the exact self-similarity.

Mandelbrot Set
The Definition of Fractal

- Quasi self-similarity
  Fractals appear roughly the same at different scales. Quasi self-similarity fractals contain small copies of the entire fractal in distorted and degenerate forms. Fractals defined by recurrence relations are usually quasi self-similar, but they are not exactly self-similar.

Mandelbrot Set Fractal

- Based on a recurrence relation
  \[ z_n = z_{n-1}^2 + c \]
  \[ c \in \mathbb{C} \text{ and } z_0 = 0 \]

- Mandelbrot Set
  \[ M = \{ c \mid \limsup_{n \to \infty} |z_n| < \infty \} \]

- Show the Mandelbrot Set on the Complex plane
  \[ c = x + y \cdot i \]

Mandelbrot Set Fractal

- Simplify the condition
  Do limited time of iteration
  Limitation of Supremum of Z
  \[ M = \{ c \mid \limsup_{n \to \infty} |z_n| \leq 2 \} \]

- Represent z and c by using 2D vector
  \[ \text{vec2} z = \text{vec2}(\text{Real Part, Imaginary Part}); \]
  \[ z \cdot z^* = \text{vec2}(z, x \cdot x^* - z y \cdot z^* y x^* + z^* y + z y^* + z^* y z^*); \]
Julia Set
Fractal

- Based on a recurrence relation
  \[ z_{n+1} = z_n^2 + c \]
  \( c \in \mathbb{C} \)

- Differences from Mandelbrot Set
  \( c \) is given.
  \[ J = \{ z : \lim \sup |z_n| = \infty \} \]

Koch Snowflake
Fractal

- Based on a Iterative Relation
  1. Divide the segment into three equal parts (AC, CD, DB)
  2. Take the CD as the bottom and draw an equilateral triangle LMC outward
  3. Remove the segment CD
  4. Repeat 1 to 3 for AC, CM, MD, and DB, respectively.

Koch Snowflake
Fractal

- L-System
  An L-system or Lindenmayer system is a parallel rewriting system and a type of formal grammar.
  Simulate the morphology of various organisms.
  Very similar to Chomsky normal form.

- Koch Snowflake Iterative Relation
  \[ V = \{ F \} \]
  \( S = \{ +, - \} \)
  \( \omega = F \)
  \( P = \{ (F \rightarrow +F-F+F), (F \rightarrow -F+F+F) \} \)
  - Draw a line (The length of line is 1/3ⁿ)
  - CCW rotate 60°
  - Clockwise rotation 60°
  - \( n = 0 \)
  \( n = 1 \) F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F
  \( n = 2 \) F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F
  \( n = 3 \) F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F
  \( n = 4 \) F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F

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Koch Snowflake
Fractal

References


Any Questions?

Thank you for listening!