Procedural Geometry Generation: Making Fun Shapes with Math

- OR -

Applications of Function Mapping, Subdivision, Brownian Surfaces, Calculus, and other Geometric Tools over Manifolds

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Demo Videos

https://media.oregonstate.edu/media/t/0_glo5by26

https://media.oregonstate.edu/media/t/0_psqgd7p1
Manifolds

- Essentially, any topology that approximates euclidean space at any given point, assuming a small enough scope.
- In geometry, any section of a space that looks like a lower-dimension euclidean space is called a Flat.
- Flats include points, lines, planes, volumes, etc.
- All discrete representations of geometry, including triangles, are sections of flats.
The Point -

- Technically, any contiguous section of space, surfaces included, can be modeled using grids, as long as you connect the edges of the grid appropriately.
- This is impractical in some cases, but useful in many
Cartesian Grids

- A cube of space which is euclidean and tiled by cubes
- Comes with a lot of great guarantees:
  - Regardless of dimensionality, the number of elements equals the product of the dimensions
  - Every inner element has exactly $2^D$ neighbors, 2 along each dimension in opposing directions
  - Can act as a discrete approximation of a manifold
  - Can easily be subdivided by cutting elements in half along each dimension
Grid Class

- Template class that stores an N-dimensional grid of a given data type
- Automates heavy lifting for processing data
  - Mapping data with function pointers
  - Subdividing grids
  - Creating triangle arrays, vertex arrays, and index arrays
Grid Subdivision

- Can subdivide a grid into interpolations of the original elements
- Function used to interpolate must have an order-independent guarantee

\[
\begin{align*}
f(f(X, Y), f(Z, W)) &= f(f(X, Z), f(Y, W)) \\
 &= f(f(Y, X), f(W, Z)) \\

A &\quad B &\quad C \\
D &\quad E &\quad F \\
G &\quad H &\quad I
\end{align*}
\]
Brownian Surfaces

- A surface with random perturbations
- Any two points on the surface have an average magnitude of vertical displacement proportional to their horizontal distance
- In essence, the further apart points are, the more likely they are to differ by a larger vertical position

$H = 0.2$  $H = 0.5$  $H = 0.8$
Water

- Stack of sine waves phase shifted by X and Z position
- Easily derivable
- Normals calculated by cross product of tangent vectors