What do we mean by “Modeling”?

How we model geometry depends on what we would like to use the geometry for:

- Looking at its appearance
- Interacting with its shape?
- How does it interact with its environment?
- What is its surface area and volume?
- Will it need to be 3D-printed?
- Etc.

Explicitly Listing Geometry and Topology

Models can consist of thousands of vertices and faces – we need some way to list them efficiently

This is called a **Mesh**.

If it’s in nice neat rows like this, it is called a **Regular Mesh**. If it’s not, it is called an **Irregular Mesh**, or oftentimes called a **Triangular Irregular Network**, or TIN.

The Cube Can Also Be Defined with Triangles

```
GLuint CubeTriangleIndices[ ][3] = {
  { 0, 2, 3 },
  { 0, 3, 1 },
  { 4, 5, 7 },
  { 4, 7, 6 },
  { 1, 3, 7 },
  { 1, 7, 5 },
  { 0, 4, 6 },
  { 0, 6, 2 },
  { 2, 6, 7 },
  { 2, 7, 3 },
  { 0, 1, 5 },
  { 0, 5, 4 }
};
```
3D Printing uses an Irregular Triangular Mesh Data Format

3D Printing uses a Triangular Mesh Data Format

Meshes Can Be Smoothed

Editing the Vertices of a Mesh

Go Beavers — mmmmmm!

Remember Venn Diagrams (2D Boolean Operators) from High School?
Well, Welcome to Venn Diagrams in 3D

Two Overlapping Solids
Union: \( A \cup B \)
Intersection: \( A \cap B \)
Difference: \( A - B \)

This is often called Constructive Solid Geometry, or CSG.

Geometric Modelling Using 3D Boolean Operators

Two Overlapping Solids
Union: \( A \cup B \)
Intersection: \( A \cap B \)
Difference: \( A - B \)

Another way to Model:
Curve Sculpting – Bézier Curve Sculpting

\[
P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3
\]

where \( P \) represents \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} \)

\( t \) goes from 0.0 to 1.0 in whatever increment you’d like

Curve Sculpting – Bézier Curve Sculpting Example

Moving a single control point moves its entire curve

A Small Amount of Input Change Results in a Large Amount of Output Change
The Catmull-Rom curve consists of any number of points. The first point influences how the curve starts. The last point influences how the curve ends. The overall curve goes smoothly through all other points.

To draw the curve, grab points 0, 1, 2, and 3, call them \( P_0, P_1, P_2 \), and \( P_3 \), and loop through the following equation, varying \( t \) from 0. to 1. in an increment of your own choosing:

\[
P(t) = 0.5 \times \left[ 2 \times P_1 + t \times (-P_0 + P_2) + t^2 (2 \times P_0 - 5 \times P_1 + 4 \times P_2 - P_3) + t^3 (-P_0 + 3 \times P_1 - 3 \times P_2 + P_3) \right]
\]

where \( P \) represents \( \left[ \begin{array}{c} P_0 \\ P_1 \\ P_2 \\ P_3 \end{array} \right] \).

For each set of 4 points, this equation just draws the line between the second and third points. That's why you keep having to use subsequent sets of 4 points.

To draw the curve, grab points 0, 1, 2, and 3, call them \( P_0, P_1, P_2 \), and \( P_3 \), and loop through the equation, varying \( t \) from 0. to 1. in an increment of your own choosing:

Then, grab points 1, 2, 3, and 4, call them \( P_0, P_1, P_2 \), and \( P_3 \), and loop through the same equation.

Then, grab points 2, 3, 4, and 5, call them \( P_0, P_1, P_2 \), and \( P_3 \), and loop through the same equation.

And so on...

A Small Amount of Input Change Results in a Large Amount of Output Change

Surface Equations can also be used for Analysis

Showing Contour Lines

Showing Curvature
Another Way to Model: Volume Sculpting

This is often called a "Lattice" or a "Cage".

A Small Amount of Input Change Results in a Large Amount of Output Change

Voxelization as a Way to Model 3D Geometry

Slip a simpler object (e.g., a subdivided cube) around some of the object’s vertices. As you sculpt the simpler object, all these object vertices get sculpted too.

Metaball Objects

The cool thing is that, if you move them close enough together, they will "glom" into a single object.

Metaball Objects Can Be Turned into Meshes for Later Editing

Modeling → Simulation (Explosion)
The object must be a legal solid. It must have a definite inside and a definite outside. It cannot have any missing face pieces.

"Definite inside and outside" is sometimes called "Two-manifold" or "Watertight".

The Simplified Euler’s Formula* for Legal Solids

\[ F - E + V = 2 \]

*sometimes called the Euler-Poincaré formula

For a cube, 6 – 12 + 8 = 2

The full formula is:

\[ F - E + V - L = 2( B - G ) \]

Objects cannot pass through other objects. If you want two shapes together, do a Boolean union on them so that they become one complete object.

Overlapped in 3D – bad

Boolean union – good