Scalar Visualization

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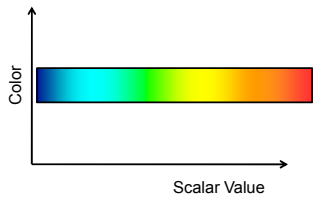
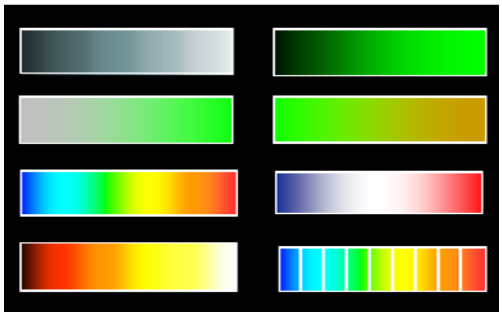
In Visualization, we Use the Concept of a *Transfer Function* to set Color as a Function of Scalar Value



A Gallery of Color Scale Transfer Function Possibilities



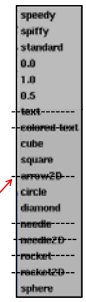
We will cover this in more detail in the color notes.

Glyphs

Glyphs are small symbols that can be placed at the location of data points. In 2D, we often call this a ***scatterplot***. The glyph itself can convey information using properties such as:

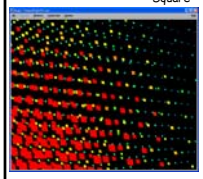
- Type
- Color
- Size
- Orientation
- Transparency
- Features

The OpenDX *AutoGlyph* function gives you these *type* options:

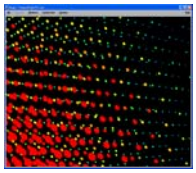


OpenDX Scalar Glyphs

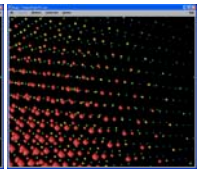
Square



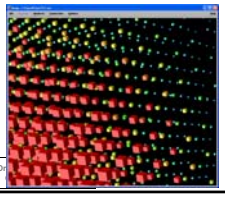
Circle



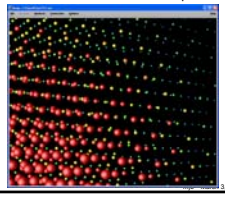
Diamond



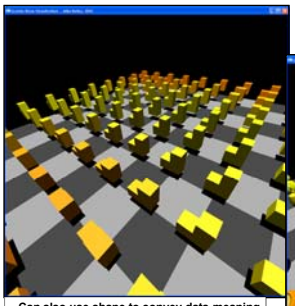
Cube



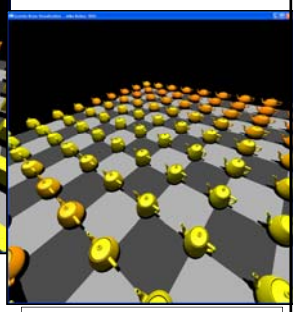
Sphere



LIGO Gravity Glyphs



Can also use shape to convey data-meaning

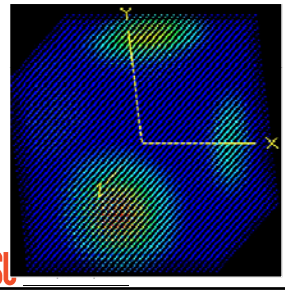


Hitting the secret Easter Egg key

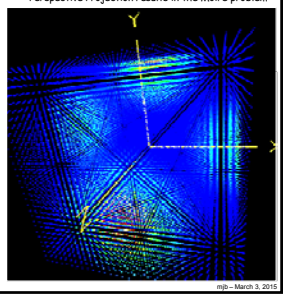
Using 3D Glyphs is called a Point Cloud

Good for overall patterns -- bad for detail

Orthographic Projection results in the row-of-corn problem



Perspective Projection results in the Moiré problem



A Simple Point Cloud Data Structure

```
struct node
{
        float x, y, z;
        float s;
        float r, g, b;
};


struct node   Nodes[NX][NY][NZ];
```

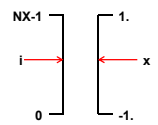

In OpenGL . . .

```
float delx = ( XMAX – XMIN ) / (float)(NX-1);
float dely = ( YMAX – YMIN ) / (float)(NY-1);
float delz = ( ZMAX – ZMIN ) / (float)(NZ-1);

glPointSize( 2. );
glBegin( GL_POINTS );

float x = XMIN;
for( int i=0; i < NX; i++, x += delx )
{
        float y = YMIN;
        for( int j=0; j < NY; j++, y += dely )
        {
                float z = ZMIN;
                for( int k=0; k < NZ; k++, z += delz )
                {
                        float scalar = Nodes[i][j][k].s;
                        float r = ???;
                        float g = ???;
                        float b = ???;
                        glColor3f( r, g, b );
                        glVertex3f( x, y, z );
                }
        }
}

glEnd( );
```

NX = 4 means that we have 3 gaps

Computing x, y, and z

Note that x, y, and z can be computed at each node point by just keeping track of them and incrementing them each time through their respective loop, as shown on the previous page. They can also be computed from the loop index like this:

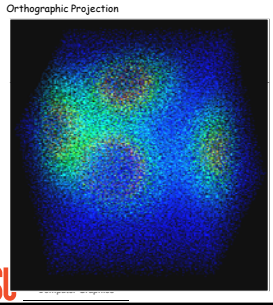


$$\frac{i-0}{(NX-1)-0} = \frac{x-(-1.)}{1.(-1.)}$$

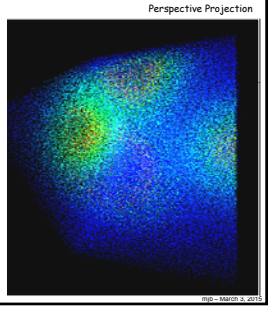
```
float x = -1. + 2. * (float)i / (float)(NX-1);

float y = -1. + 2. * (float)j / (float)(NY-1);

float z = -1. + 2. * (float)k / (float)(NZ-1);
```


Jitter Gives a Better Point Cloud Display

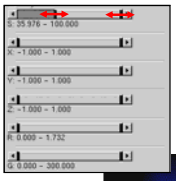
Orthographic Projection



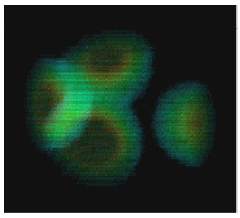
Perspective Projection



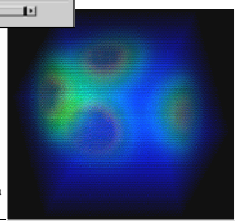
Point Cloud Culling Using Range Sliders



Low values culled



Full data



Using Range Sliders

```
#define S      0

const char *SFORMAT = { "S: %.3f - %.3f" };

float   SLowHigh[2];

GLUI_StaticText *SLabel;
```

```
slider = Glui->add_slider( true, GLUI_HSLIDER_FLOAT, SLowHigh, S, (GLUI_Update_CB) Sliders );

        slider->set_float_limits( SLowHigh[0], SLowHigh[1] );
        slider->set_slider_val(   SLowHigh[0], SLowHigh[1] );
        slider->set_w( SLIDERWIDTH );

        sprintf( str, SFORMAT, SLowHigh[0], SLowHigh[1] );
        SLabel = Glui->add_statictext_to_panel( rollout, str );
```

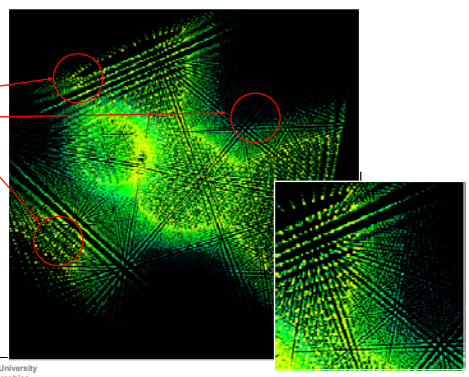
```
void
Sliders( int id )
{
        char str[256];
        switch( id )
        {
                case S:
                        sprintf( str, SFORMAT, SLowHigh[0], SLowHigh[1] );
                        SLabel->set_text( str );
                        break;
```


Drawing the Range Slider-Filtered Point Cloud

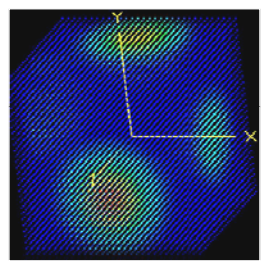
```
float x = XMIN;
for( int i=0; i < NX; i++, x += delx )
{
        if( x < XLowHigh[0]  ||  x > XLowHigh[1] )
                continue;

        float y = YMIN;
        for( int j=0; j < NY; j++, y += dely )
        {
                if( y < YLowHigh[0]  ||  y > YLowHigh[1] )
                        continue;

                float z = ZMIN;
                for( int k=0; k < NZ; k++, z += delz )
                {
                        if( z < ZLowHigh[0]  ||  z > ZLowHigh[1] )
                                continue;

                        if( Nodes[i][j][k].s < SLowHigh[0]  ||  Nodes[i][j][k].s > SLowHigh[1] )
                                continue;

                        . . .

                        glColor3f( r, g, b );
                        glVertex3f( x, y, z);
                }
        }
}

glEnd( );
```

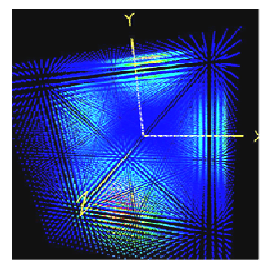

Enhanced Point Clouds

- Color
- Alpha
- Pointsize



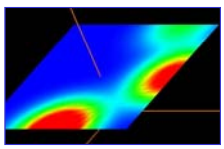
Point Clouds are nice, but they only tell us about the gross patterns. We want more detail !

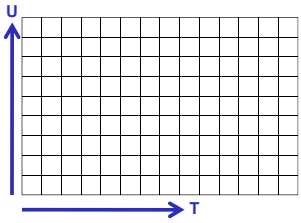




2D Interpolated Color Plots

Here's the situation: we have a 2D grid of data points. At each node, we have an X, Y, Z, and a scalar value S. We know S_{min}, S_{max}, and the Transfer Function.

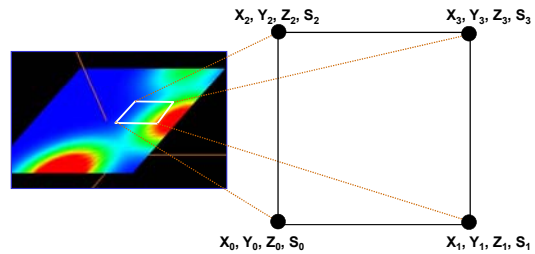




Even though this is a 2D technique, we keep around the X, Y, and Z coordinates so that the grid doesn't have to lie in any particular plane.

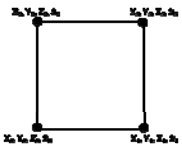
2D Interpolated Color Plots

We deal with one square of the mesh at a time:



2D Interpolated Color Plots

Within that one square, we let OpenGL do the color interpolation for us



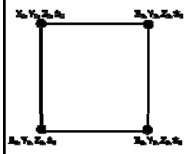

```
void
ColorSquare( . . . )
{
        Compute an r, g, b for S0
        glColor3f( r, g, b );
        glVertex3f( X0, Y0, Z0 );

        Compute an r, g, b for S1
        glColor3f( r, g, b );
        glVertex3f( X1, Y1, Z1 );

        Compute an r, g, b for S3
        glColor3f( r, g, b );
        glVertex3f( X3, Y3, Z3 );

        Compute an r, g, b for S2
        glColor3f( r, g, b );
        glVertex3f( X2, Y2, Z2 );
}
```

Use the Transfer Function

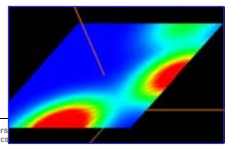
Note the order: 0-1-3-2 !

2D Interpolated Color Plots

Then we loop through all squares:

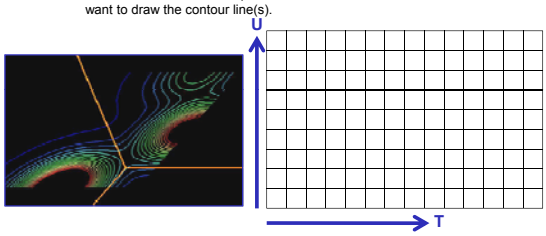



```
glShadeModel( GL_SMOOTH );
glBegin( GL_QUADS );
for( int  i = 0;  i < numT - 1;  i++ )
{
        for( int j = 0;  j < numU-1;  j++ )
        {
                ColorSquare( i, j, … );
        }
}
glEnd( );
```



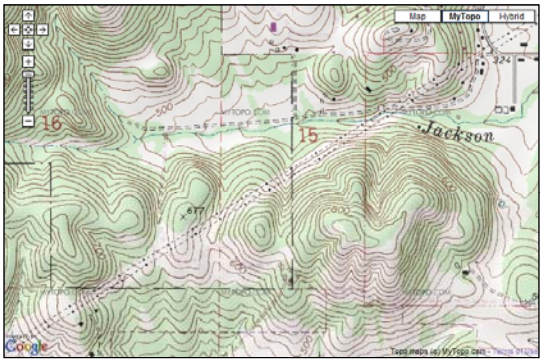
2D Contour Lines

Here's the situation: we have a 2D grid of data points. At each node, we have an X, Y, Z, and a scalar value S. We know the Transfer Function. We also have a particular scalar value, **S***, at which we want to draw the contour line(s).



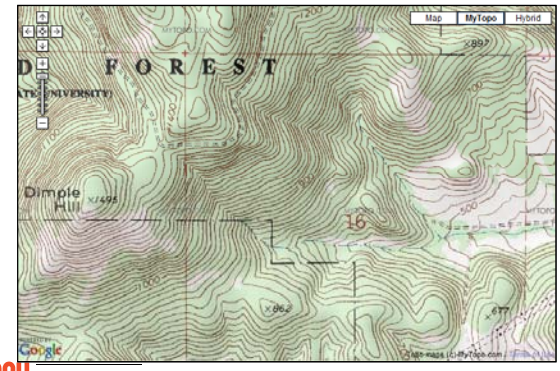
Even though this is a 2D technique, we keep around the X, Y, and Z coordinates so that the grid doesn't have to lie in any particular plane.

Hiking Maps are a Great Use for Contour Lines



http://www.digital-topo-maps.com

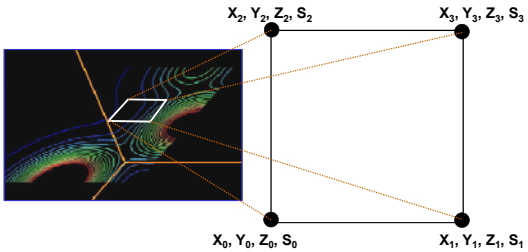
Hiking Maps are a Great Use for Contour Lines



http://www.digital-topo-maps.com

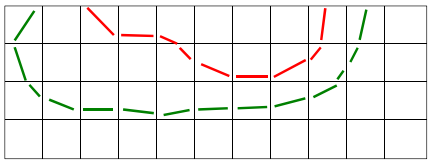
2D Contour Lines: Marching Squares

Rather than deal with the entire grid, we deal with one square at a time, marching through them all. For this reason, this method is called the ***Marching Squares***.

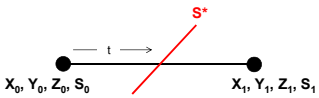


Marching Squares: A Cluster of Connected Line Segments

What's really happening is that we are not creating contours by connecting points into a complete curve. We are creating contours by drawing a collection of 2-point line segments, safe in the knowledge that those line segments will align across square boundaries.



Does S* cross any edges of this square?



Linearly interpolating the scalar value from node 0 to node 1 gives:

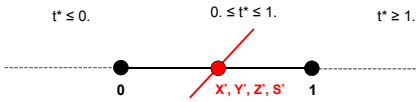
$$S = (1-t)S_0 + tS_1 = S_0 + t(S_1 - S_0)$$

where $0. \le t \le 1.$

Setting this interpolated S equal to S* and solving for t gives:

$$t^* = \frac{S^* - S_0}{S_1 - S_0}$$

Interpreting t*: Where does S* cross the edge?



$$t^* = \frac{S^* - S_0}{S_1 - S_0}$$

If $0. \le t \le 1.$, then S* crosses this edge. You can compute where S* crosses the edge by using the same linear interpolation equation you used to compute t*:

$$X^* = X_0 + t^*(X_1 - X_0)$$

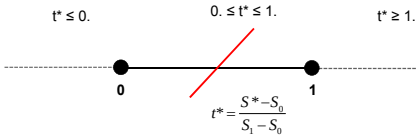
$$Y^* = Y_0 + t^*(Y_1 - Y_0)$$

$$Z^* = Z_0 + t^*(Z_1 - Z_0)$$

Do this for all 4 edges – when you are done, there are 5 possible ways this could have turned out

Situation	Action
# of intersections = **0**:	Do nothing 
# of intersections = **2**:	Draw a line from the first intersection to the second 
# of intersections = **1**:	Error! This implies that the contour line got into the square and never got out
# of intersections = **3**:	Error! This implies that the contour line got into the square and never got out
# of intersections = **4**:	Coming up shortly

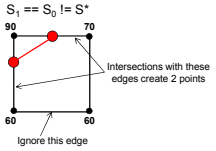
What if $S_1 == S_0$ (i.e., t*=∞?)



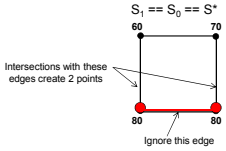
$$t^* = \frac{S^* - S_0}{S_1 - S_0}$$

Surprisingly, you just ignore this edge. Why? There are 2 possibilities. Let S* = 80

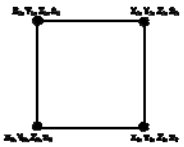
$S_1 == S_0 \; != S^*$



$S_1 == S_0 == S^*$

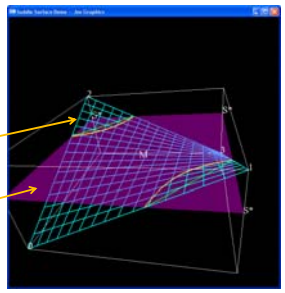


The 4-intersection Case



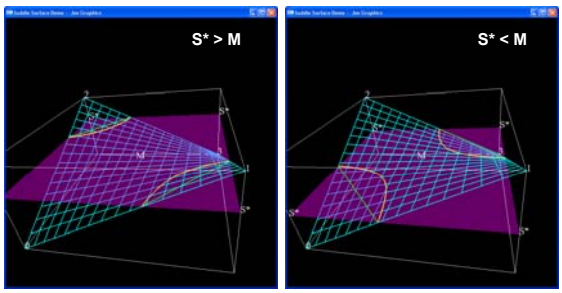
If there are 4 edge intersections with S*, then this must mean that, going around the square, the nodes are >S*, <S*, >S*, and <S* in that order. This gives us a **saddle function**, shown here in cyan.

If we think of the scalar values as terrain heights, then we can think of S* as the height of water that is flooding the terrain, as shown here in magenta.



My Favorite Saddle Function :-)

The 4-intersection Case



The exact contour curve is shown in yellow, The Marching Squares contour line is shown in green. Notice what happens as we lower S* -- there is a change in which sides of the square get connected. That change happens when S* > M becomes S* < M (where M is the middle scalar value).

The 4-intersection Case: Computing the middle scalar value, M

Let's linearly interpolate scalar values along the 0-1 edge, and along the 2-3 edge:

$$S_{01}(t) = (1-t)S_0 + tS_1$$
$$S_{23}(t) = (1-t)S_2 + tS_3$$

Now linearly these two linearly-interpolated scalar values:

$$S(t,u) = (1-u)S_{01} + uS_{23}$$

Expanding gives:

$$S(t,u) = (1-t)(1-u)S_0 + t(1-u)S_1 + (1-t)uS_2 + tuS_3$$

This is the bilinear interpolation equation. Notice the similarity to the linear equation.

The 4-intersection Case: Computing the middle scalar value, M

The middle scalar value, M, is what you get when you set t = .5 and u = .5:

$$S(t,u) = (1-t)(1-u)S_0 + t(1-u)S_1 + (1-t)uS_2 + tuS_3$$

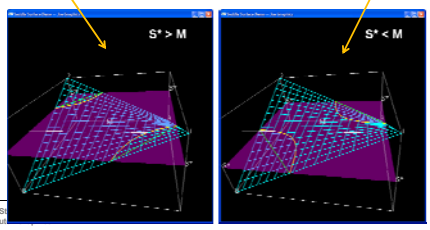
$$M = S(\frac{1}{2},\frac{1}{2}) = \frac{1}{4}S_0 + \frac{1}{4}S_1 + \frac{1}{4}S_2 + \frac{1}{4}S_3 = \frac{S_0 + S_1 + S_2 + S_3}{4}$$

Thus, M is the average of the corner scalar values. (We could maybe have come to this intuitively, but it was worthwhile to actually prove it.)

The 4-intersection Case

The logic for the 4-intersection case is as follows:

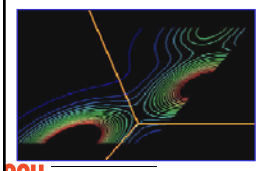
1. Compute M
2. If S_0 is on the same side of M as S* is, then connect the 0-1 and 0-2 intersections, and the 1-3 and 2-3 intersections
3. Otherwise, connect the 0-1 and 1-3 intersections, and the 0-2 and 2-3 intersections



Overall Logic for a Set of Contour Lines

```
for( float S* = S_min ; S* <= S_max ; S* += ΔS )
{
        Set color for S*
        glBegin( GL_LINES );
                for( int i = 0; i < numT - 1; i++ )
                {
                        for( int j = 0; j < numU-1; j++ )
                        {
                                Process the square whose corner is at (i,j)
                        }
                }
        glEnd( );
}
```

Bad programming practice!



Note that it is bad programming practice to use a floating-point variable to index the S* for-loop!

This has been done just to illustrate the concept. Instead do this:

```
int is;
float S*;
for( is = 0, S* = S_min ; is < numS ; is++, S* += ΔS )
{
        . . .
```

Artifacts?

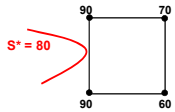
What if the distribution of scalar values along the square edges isn't linear?

We have no basis to assume *anything*, actually. So linear is as good as any other guess, and lets us consider just one square by itself. Some people like looking at adjacent nodes and using quadratic or cubic interpolation on the edge. This is harder to deal with computationally, and is also making an assumption for which there is no evidence..

What if you have a contour that really looks like this?

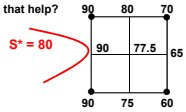
You'll never know. We can only deal with what data we've been given.

There is no substitute for having an adequate number of data points.

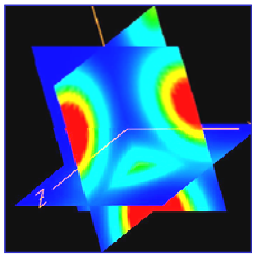


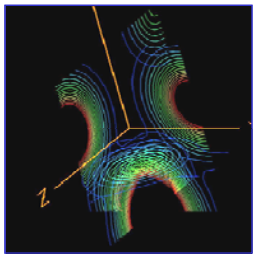
What if we subdivide the square and interpolate values? Does that help?

No. We can only deal with what data we've been given.



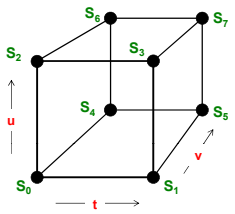
And, of course, if you can do it in one plane, you can do it in multiple planes





Remember this! In a moment, we are going to put this to use in a different way, to create wireframe isosurfaces . . .

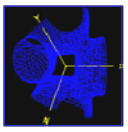
While we're at it: Trilinear interpolation

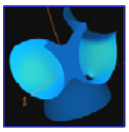


$$S(t,u,v) = (1-t)(1-u)(1-v)S_0 + t(1-u)(1-v)S_1 + (1-t)u(1-v)S_2 + tu(1-v)S_3 + (1-t)(1-u)vS_4 + t(1-u)vS_5 + (1-t)uvS_6 + tuvS_7$$

This is useful, for example, if we have passed an oblique cutting plane through a 3D mesh of points and are trying to interpolate scalar values from the 3D mesh to the 2D plane.

Isosurfaces

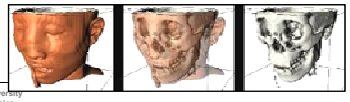




A contour line is often called an ***isoline***, that is a line of equal value. When hiking, for example, if you could walk along a single contour line of the terrain, you would remain at the same elevation.

An isosurface is the same idea, only in 3D. It is a surface of equal value. If you could be a fly walking on the isosurface, you would always experience the same scalar value (e.g., temperature).

Sometimes the shapes of the isosurfaces have a physical meaning, such as with bone, skin, clouds, etc. Sometimes the shape just helps turn an abstract notion into something physical to help us gain insight.



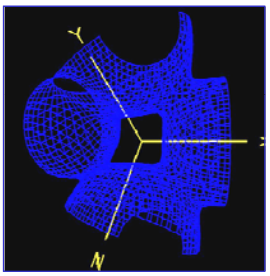
Wireframe Isosurfaces

Here's the situation: we have a 3D grid of data points. At each node, we have an X, Y, Z, and a scalar value S. We know the Transfer Function. We also have a particular scalar value, **S***, at which we want to draw the isosurface(s).

Once you have done Marching Squares for contour lines, doing wireframe isosurfaces is amazingly easy. If I had to come up with a name for this, I'd call it **Marching Planes**.

The strategy is that you pick your S*, then draw S* contours on all the parallel **XY** planes. Then draw S* contours on all the parallel **XZ** planes. Then draw S* contours on all the parallel **YZ** planes. And, then you're done.

What you have looks like it is a connected surface mesh, but in fact it is just independent curves. It is easy to program (once you've done Marching Squares at least), and looks good. Also, it's fast to compute.

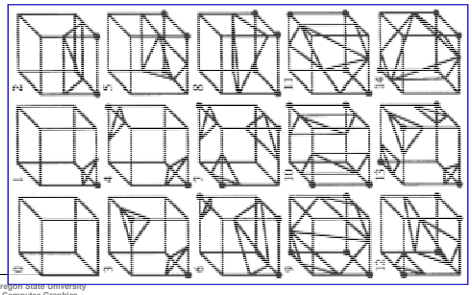


Overall Logic for a Wireframe Isosurface

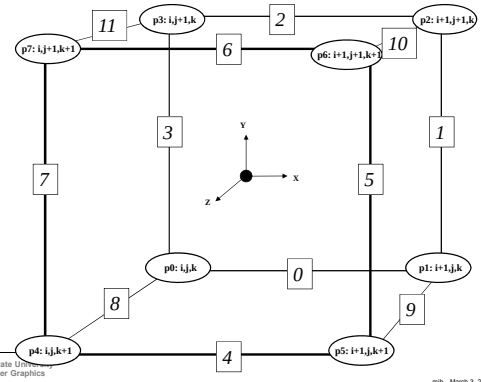
```
Set color for S*
glBegin( GL_LINES );

for( int k = 0;  k < numV;  k++ )
{
        for( int i = 0;  i < numT - 1;  i++ )
        {
                for( int j = 0;  j < numU-1;  j++ )
                {
                        Process square whose corner is at (i,j,k) in TU plane
                }
        }
}

for( int i = 0;  i < numT ;  i++ )
{
        for( int k = 0;  k < numV-1;  k++ )
        {
                for( int j = 0;  j < numU-1;  j++ )
                {
                        Process square whose corner is at (i,j,k) in UV plane
                }
        }
}

for( int j = 0;  j < numU;  j++ )
{
        for( int i = 0;  i < numT - 1;  i++ )
        {
                 for( int k = 0;  k < numV-1;  k++ )
                {
                        Process square whose corner is at (i,j,k) in TV plane
                }
        }
}
glEnd( );
```

Polygonal Isosurfaces

The original polygonal isosurface Marching Cubes algorithm used the observation that when classifying each corner node as <S* or >S*, there were 2^8 = 256 possible ways that it could happen, but of those 256, there were only 15 unique cases which needed to be handled. Even so, this is difficult, and so we will look at another approach.



Polygonal Isosurfaces: Data Organization Diagram



Polygonal Isosurfaces: Data Structures

`bool FoundEdgeIntersection[12]`

One entry for each of the 12 edges.

false means S* did not intersect this edge

true means S* did intersect this edge

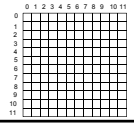
`Node EdgeIntersection[12]`

If an intersection did occur on edge #i, Node[i] will contain the interpolated x, y, z, nx, ny, and nz.

`bool FoundEdgeConnection[12][12]`

A *true* in entry [i][j] or [j][i] means that Marching Squares has decided there needs to be a line drawn from Cube Edge #i to Cube Edge #j

Both entry [i][j] and [j][i] are filled so that it won't matter which order you search in later.



Polygonal Isosurfaces: Algorithm

Strategy in ***`ProcessCube()`***:

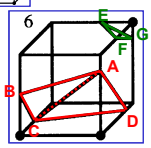
1. Use **`ProcessCubeEdge()`** 12 times to find which cube edges have S* intersections.
2. Return if no intersections were found anywhere.
3. Call **`ProcessCubeQuad()`** 6 times to decide which cube edges will need to be connected. This is Marching Squares like we did it before, but it doesn't need to re-compute intersections on the cube edges in common. **`ProcessCubeEdge()`** already did that. This leaves us with the **`FoundEdgeConnection[][]`** array filled.
4. Call **`DrawCubeTriangles()`** to create triangles from the connected edges.

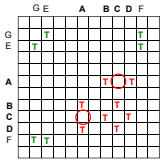
Polygonal Isosurfaces: Algorithm

Strategy in ***`DrawCubeTriangles()`***:

1. Look through the **`FoundEdgeConnection[][]`** array for a Cube Edge #A and a Cube Edge #B that have a connection between them.
2. If can't find one, then you are done with this cube.
3. Now look through the **`FoundEdgeConnection[][]`** array for a Cube Edge #C that is connected to Cube Edge #B. If you can't find one, something is wrong.
4. Draw a triangle using the **`EdgeIntersection[]`** nodes from Cube Edges #A, #B, and #C. Be sure to use **`glNormal3f()`** in addition to **`glVertex3f()`**.
5. Turn to *false* the **`FoundEdgeConnection[][]`** entries from Cube Edge #A to Cube Edge #B.
6. Turn to *false* the **`FoundEdgeConnection[][]`** entries from Cube Edge #B to Cube Edge #C.
7. ***Toggle*** the **`FoundEdgeConnection[][]`** entries from Cube Edge #C to Cube Edge #A. If this connection was there before, we don't need it anymore. If it was not there before, then we just invented it and we will need it again.

Polygonal Isosurfaces: Why Does This Work?





Take this case as an example. The intersection points A, B, C, and D were found and the lines AB, BC, CD, and DA were found because Marching Squares will have been performed on each of the cube's 6 faces.

At this point, we could just draw the quadrilateral ABCD, but this will likely go wrong because it is surely non-planar. So, starting at A, we break out a triangle from the edges AB and BC (which exist) and the edge CA (which doesn't exist, but we need it anyway to complete the triangle).

When we toggle the **`FoundEdgeConnection[][]`** entries for AB and BC, they turn from *true* to *false*. When we toggle the **`FoundEdgeConnection[][]`** for CA, it turns from *false* to *true*.

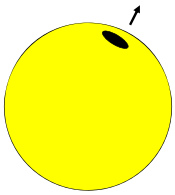
This leaves the **`FoundEdgeConnection[][]`** for CA, CD, and AD all set to *true*, which will cause the algorithm to find them and connect them into a triangle next.

Note that this algorithm will eventually find and properly connect the little triangle in the upper-right corner, even though it has no connection with A-B-C-D.

Polygonal Isosurfaces: Pseudo-Surface Normals

We would very much like to use lighting when displaying polygonal isosurfaces, but we need surface normals at all the triangle vertices. Because there really isn't a surface there, this would seem difficult, but it's not.

Envision a balloon with a dot painted on it. Think of this balloon as an isosurface. Blow up the balloon a little more. This is like changing S*, resulting in a different isosurface. Where does the dot end up?



The dot moves in the direction of the changing isosurface, which is the normal to the balloon surface .

Now, turn that sentence around:

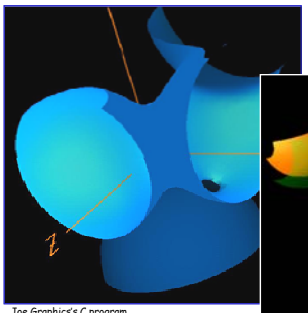
The normal to the isosurface is a vector that shows how the isosurface is changing.

How "something is changing" is called the *gradient*. So, the surface normal to a volume is:

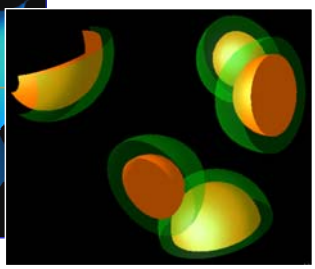
$$\vec{n} = \left(\frac{dS}{dx}, \frac{dS}{dy}, \frac{dS}{dz}\right) = \nabla S$$

Prior to the isosurface calculation, you compute the surface normals for all the nodes in the 3D mesh. You then interpolate them along the cube edges when you create the isosurface triangle vertices.

Polygonal Isosurfaces



Joe Graphics's C program



Joe Graphics using OpenDX

Putting It All Together