Vector Visualization

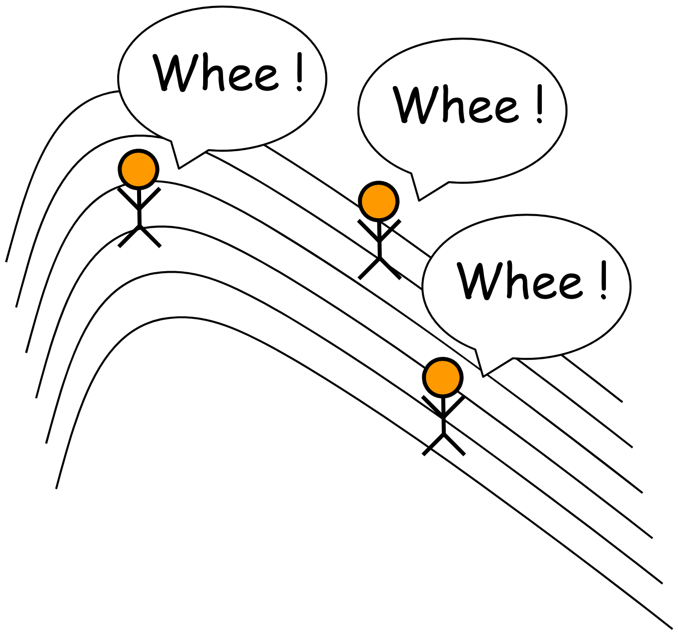
Mike Bailey

mjb@cs.oregonstate.edu

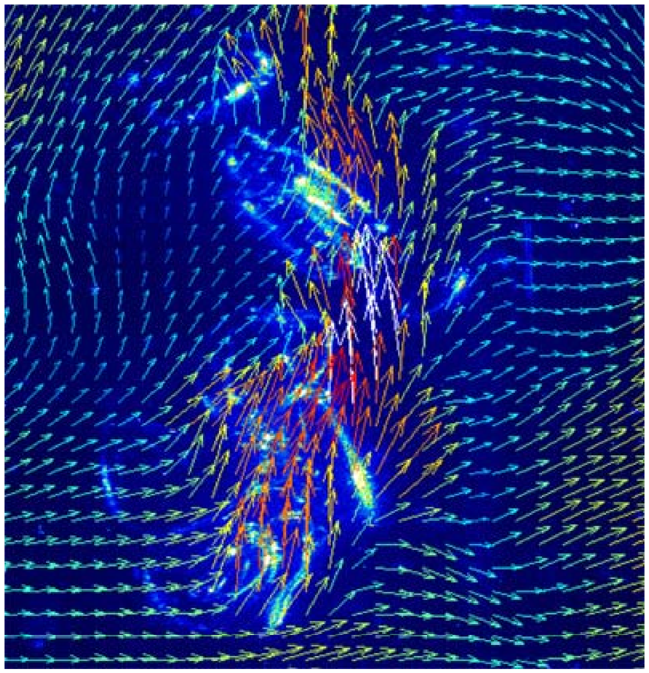
Oregon State University

What is a Vector Visualization Problem?

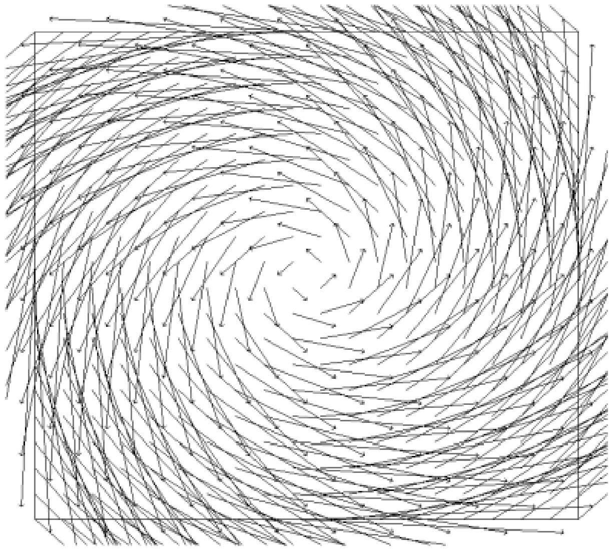
A vector has direction and magnitude. Typically science and engineering problems that work this way are those involving fluid flow through a velocity field.



Examples

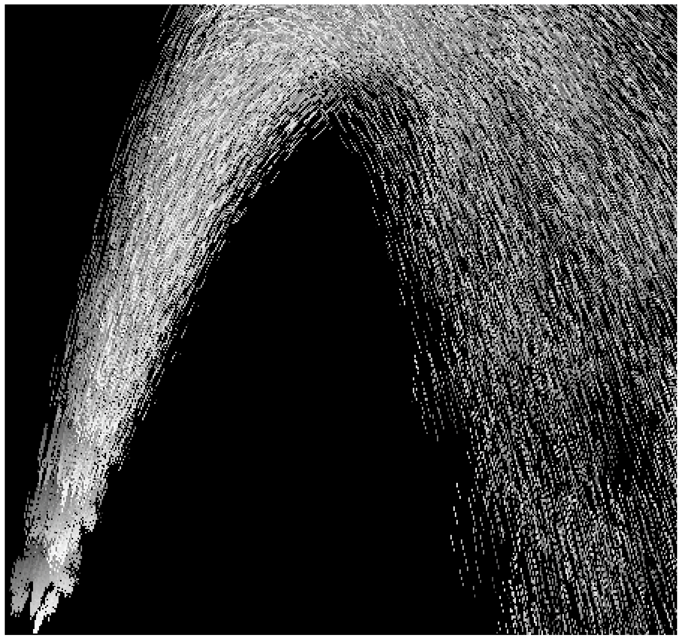


http://mathforum.org/mathimages/index.php/Vector_Fields



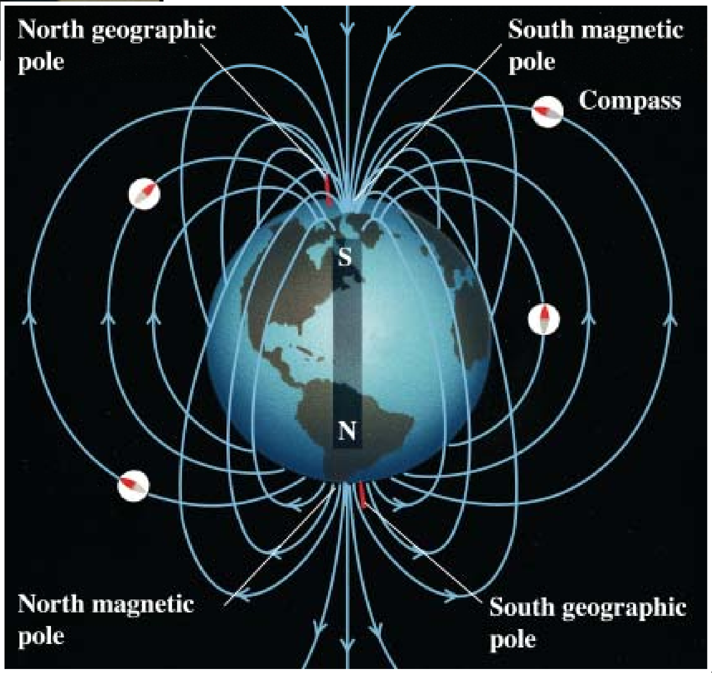
http://www.physicsdaily.com/physics/Image:Vectorfield_jaredwf.png

Chuck Evans



It Doesn't Always Have to be *Physically-Moving* Flow: Things Like Magnetic Fields Count Too

This very cool magnetic wand toy consists of a small bar magnet on a universal joint, and is a good way to give a physical feel to magnetic fields. (You can get these from Edmund Scientific.)



http://www.physics.sjsu.edu/becker/physics51/mag_field.htm

Two Types of Vector Visualization

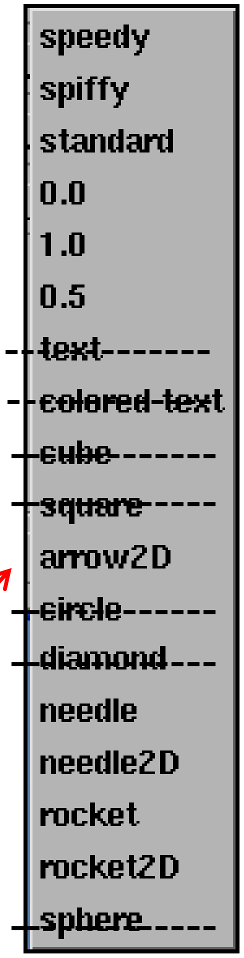
1. What does the field itself look like?

2. What do things placed in the field do?

What Does the Field Look Like? Glyphs

The most straightforward way to see what the field looks like is to place glyphs throughout the field.

The OpenDX *AutoGlyph* function gives you these type options:


speedy
spiffy
standard
0.0
1.0
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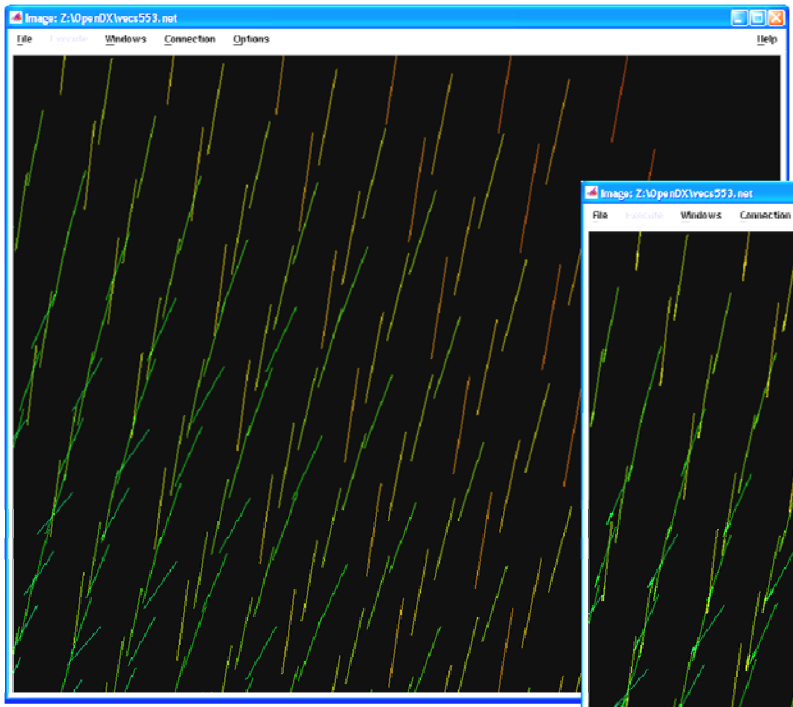
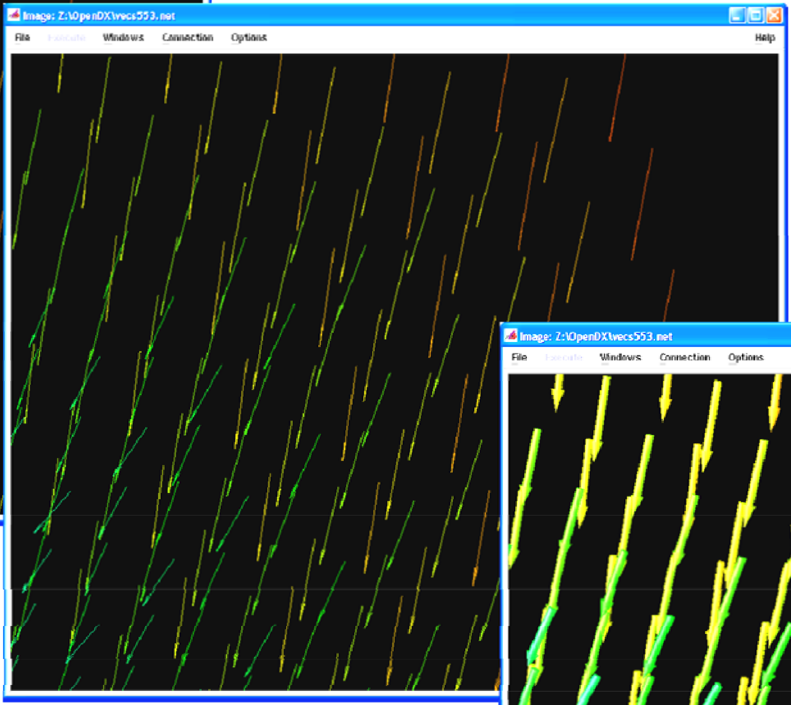
~~text~~
~~colored-text~~
~~cube~~
~~square~~
arrow2D
~~circle~~
~~diamond~~
needle
needle2D
rocket
rocket2D
~~sphere~~

OpenDX Vector Glyphs

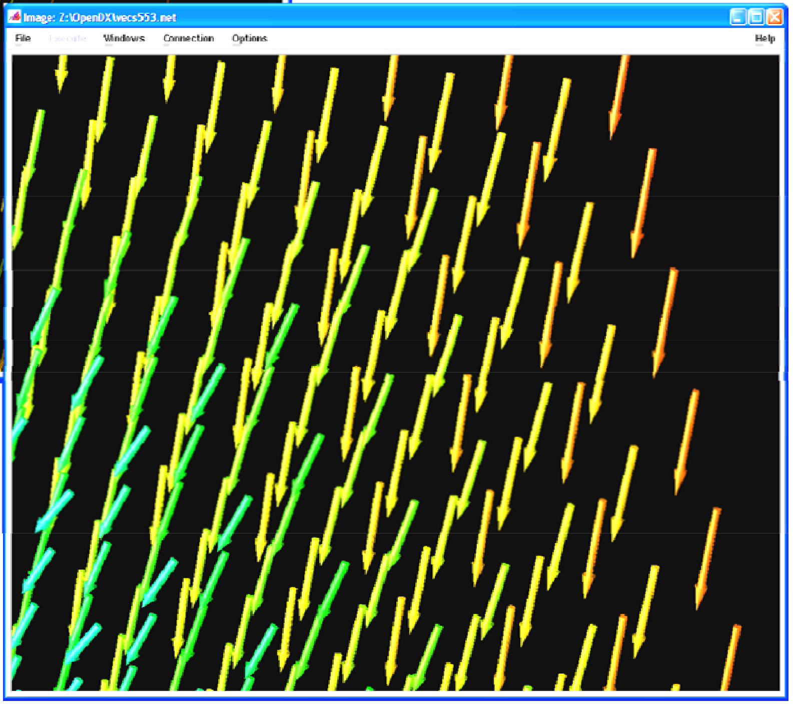
Needle



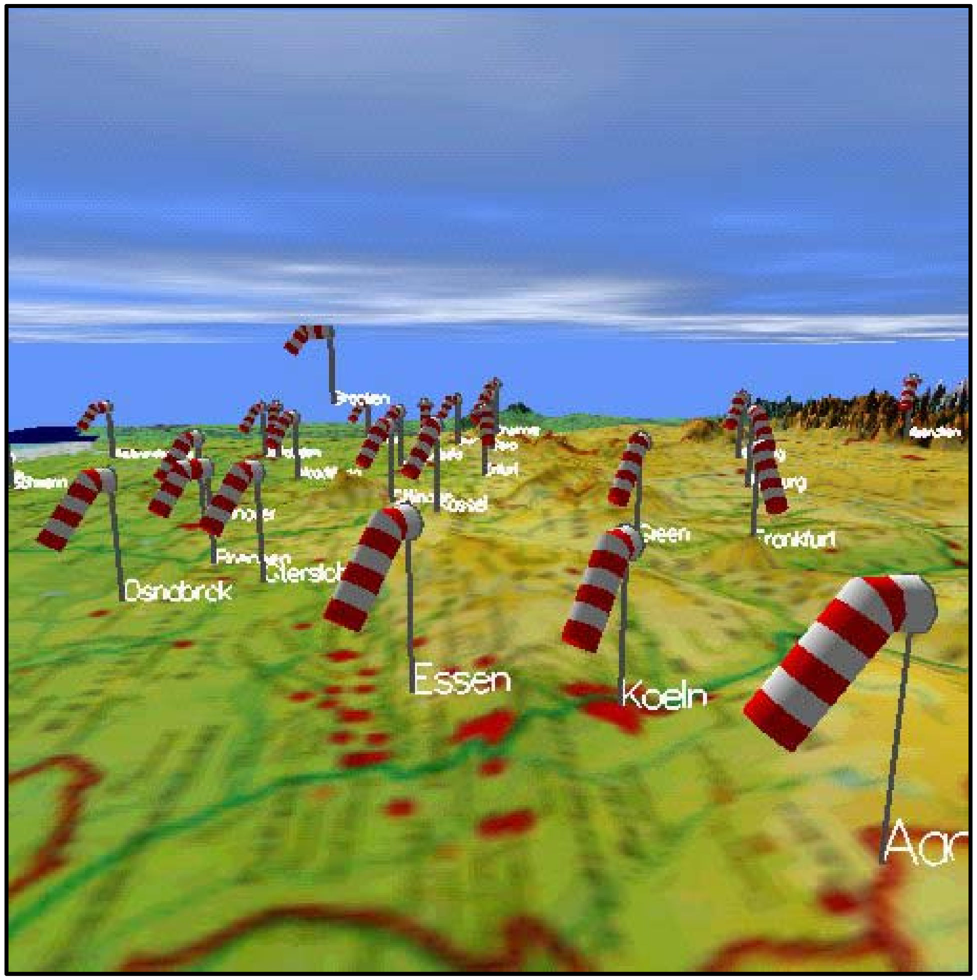
Arrow



Rocket



A Cool Wind Speed and Direction Glyph



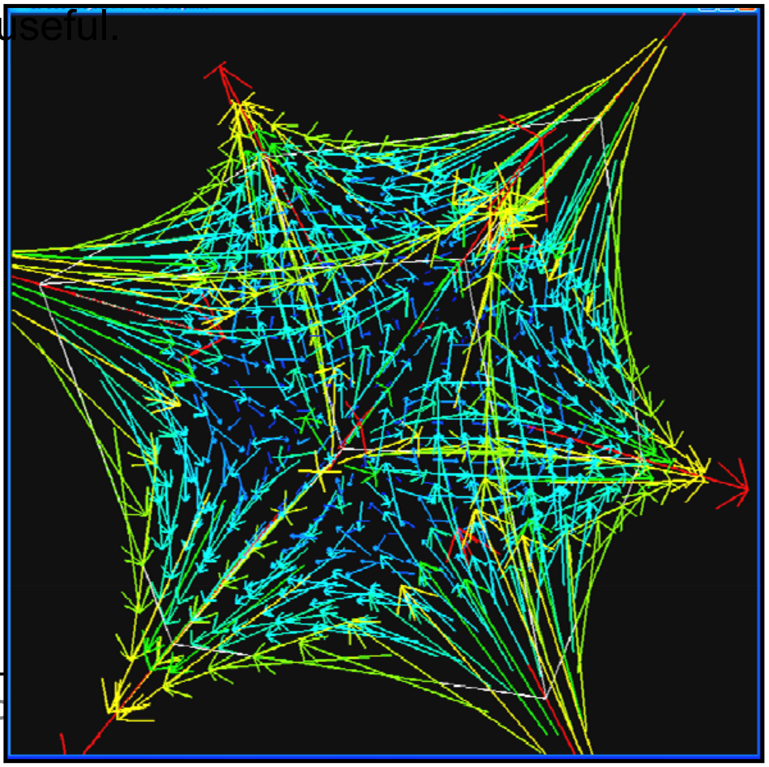
Wolfgang Bloem

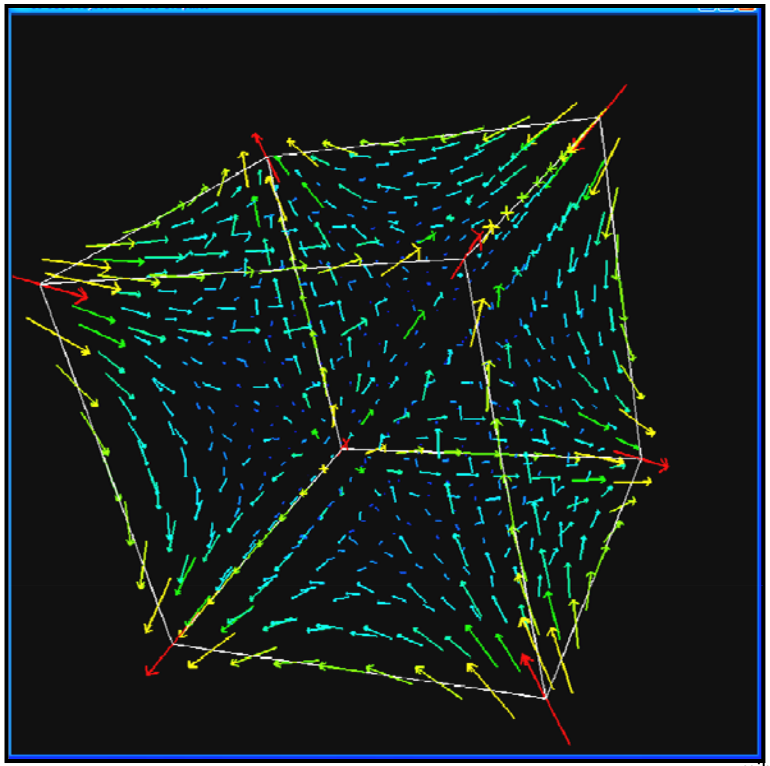
What Does the Field Look Like? Glyphs as Vector Clouds

In the same way that a point cloud was a simple way to visualize a scalar field, a Vector Cloud is a simple way to visualize a vector field. Go to selected points in the data volume, look up the velocity vector there (v_x, v_y, v_z), and draw an arrow centered around that point. The arrow's direction shows the direction of flow. The arrow's length shows the magnitude of the velocity field (i.e., its speed).

Nuance alert: the size of the arrow comes out in whatever units the velocity field is defined in, and might be too small to be seen or so large that it clutters the screen. You typically have to uniformly scale all the arrows to make the display useful.

Bad





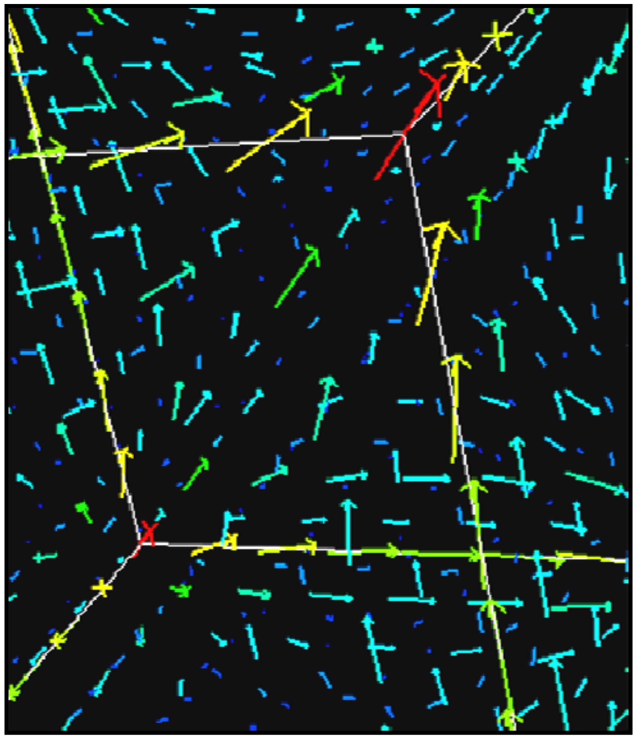
Better

OSU

Drawing an Arrow

It's surprisingly involved to draw a good-looking 3D arrow. So, you've been given a C/C++ function to do it for you. Use it like this:

```
float  tail[3], head[3];
. . .
// Center a 3D arrow at the point (x,y,z) indicating a
// velocity there of (vx,vy,vz):

tail[0] = x – Scale*vx/2.;
tail[1] = y – Scale*vy/2.;
tail[2] = z – Scale*vz/2.;

head[0] = x + Scale*vx/2.;
head[1] = y + Scale*vy/2.;
head[2] = z + Scale*vz/2.;

Arrow( tail, head );
```

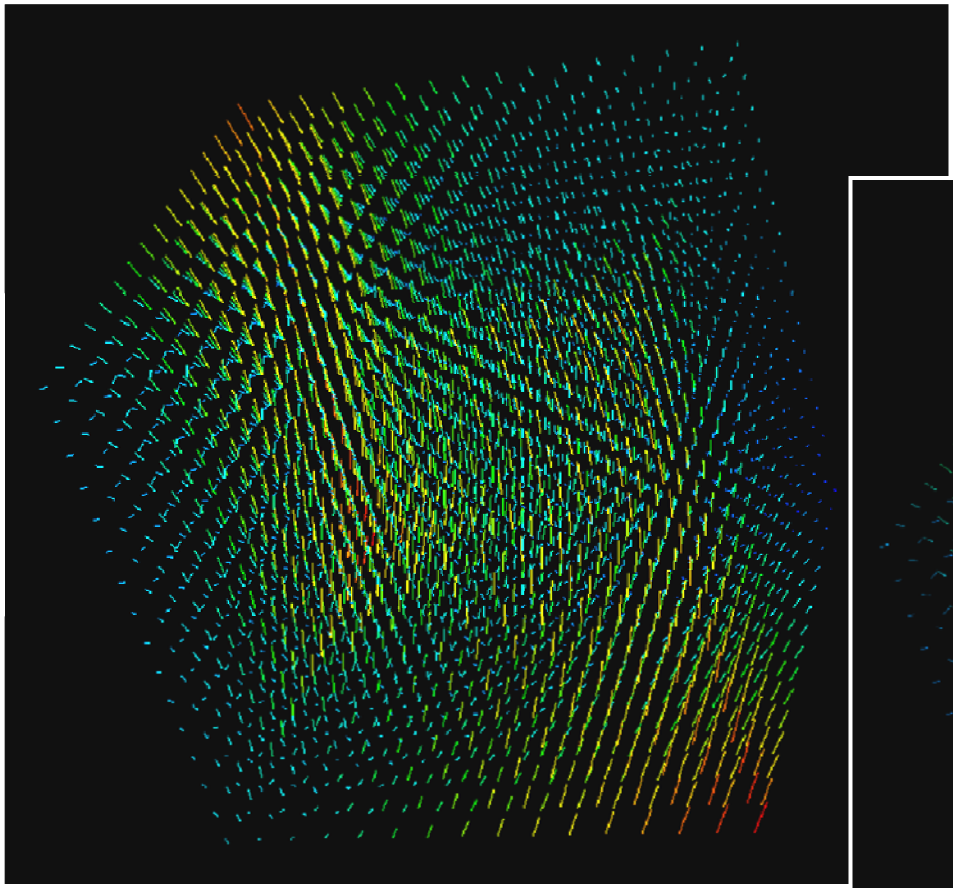


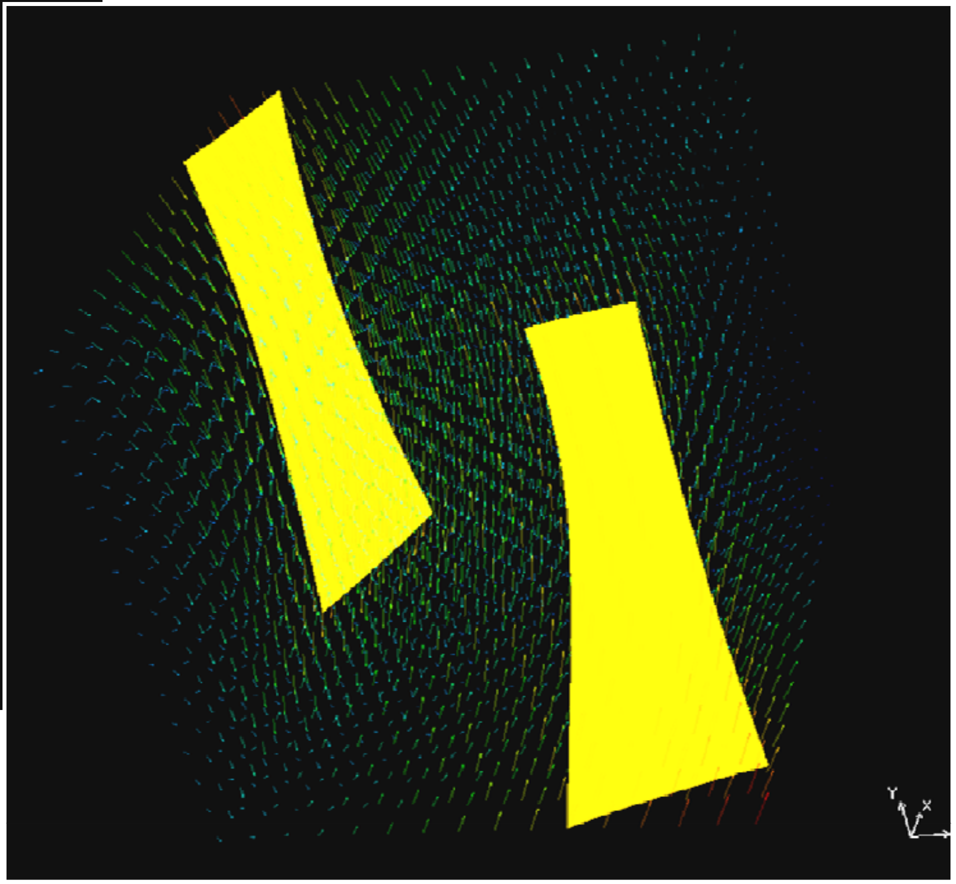
Arrow( ) uses OpenGL lines, so the current line width and current color can be set as usual:

```
glLineWidth( w );                  // number of pixels wide (floating point), ≥ 1.
glColor3f( r, g, b );              // red, green, blue in the range 0.-1.
Arrow( tail, head );
```

The arrows also get transformed along with everything else.

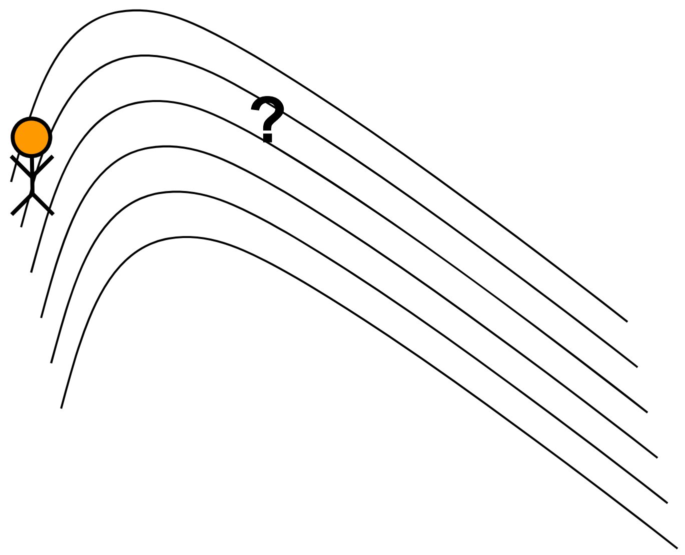
Turning it Into a Scalar Problem: Magnitude Isosurfaces





Particle Advection

Vector Clouds are OK, but we can do more. The next step is to think about what would happen if we released an imaginary massless ping-pong ball somewhere in the velocity field. Where would it go? This is called **Particle Advection**.

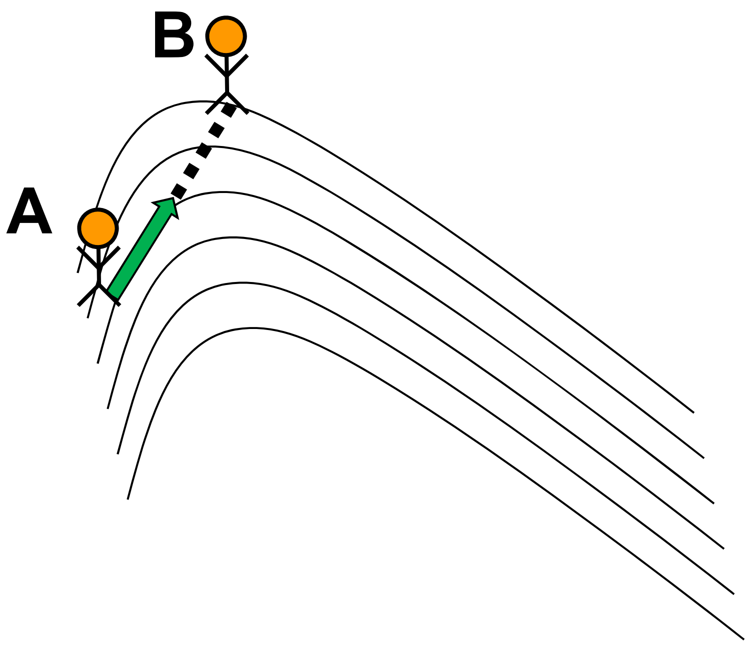


Taking a First Order Step

If we are at Point A, and the velocity field is as shown, how do we know where we end up next?

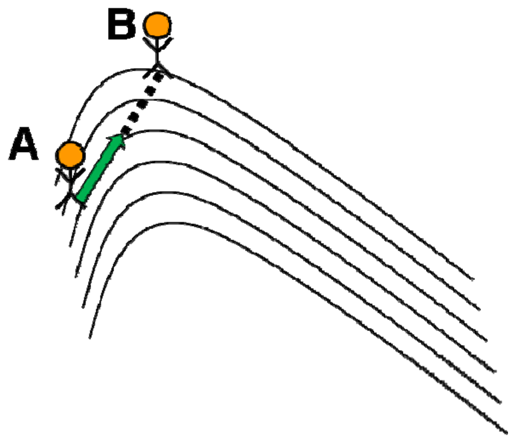
Easy, right? We look at the velocity field at Point A, and take a step in that direction, ending up at Point B.

This is called a **First Order Step**. It is also sometimes called **Euler's Method.**



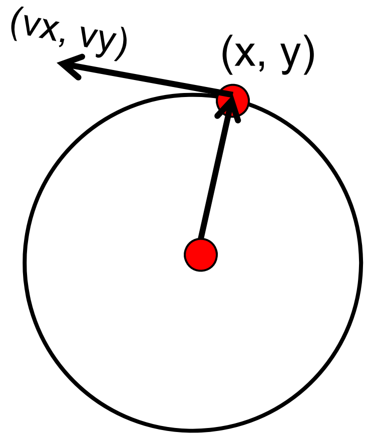
Taking a First Order Step

```
void
Advect( float *x, float *y, float *z )
{
            xa = *x;    ya = *y;    za = *z;

            GetVelocity( xa, ya, za,    &vxa, &vya, &vza );

            xb = xa + TimeStep*vxa;
            yb = ya + TimeStep*vya;
            zb = za + TimeStep*vza;

            *x = xb;    *y = yb;    *z = zb;
}
```

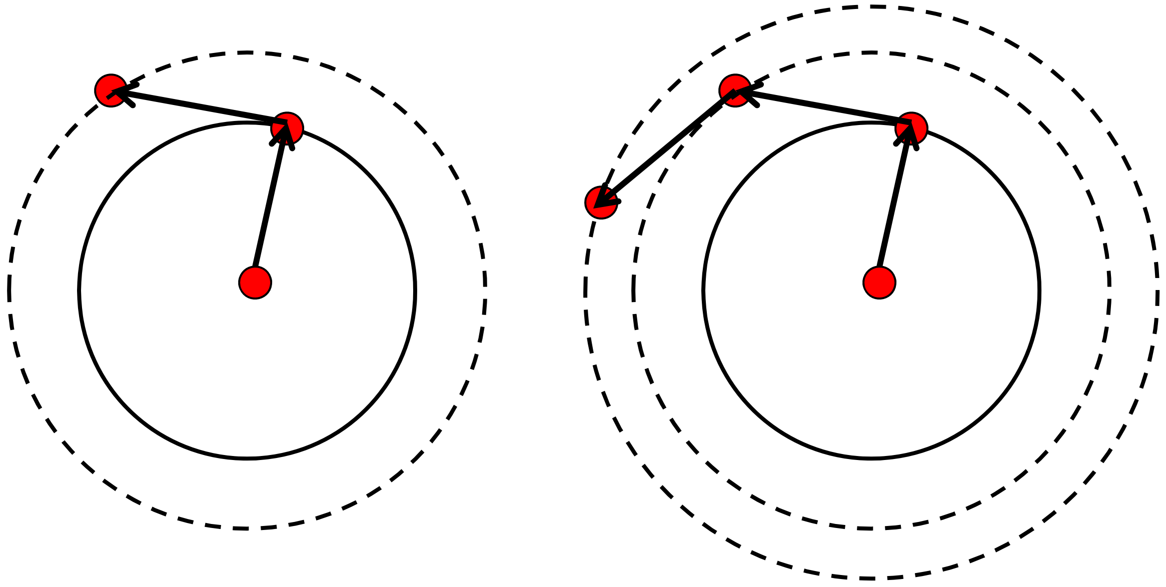


All is Not Right: the Spiral Problem

Assume we have a vector field
that moves in a circle, that is:
vx = - y
vy =   x



Now, take a First Order step, and you move like this:

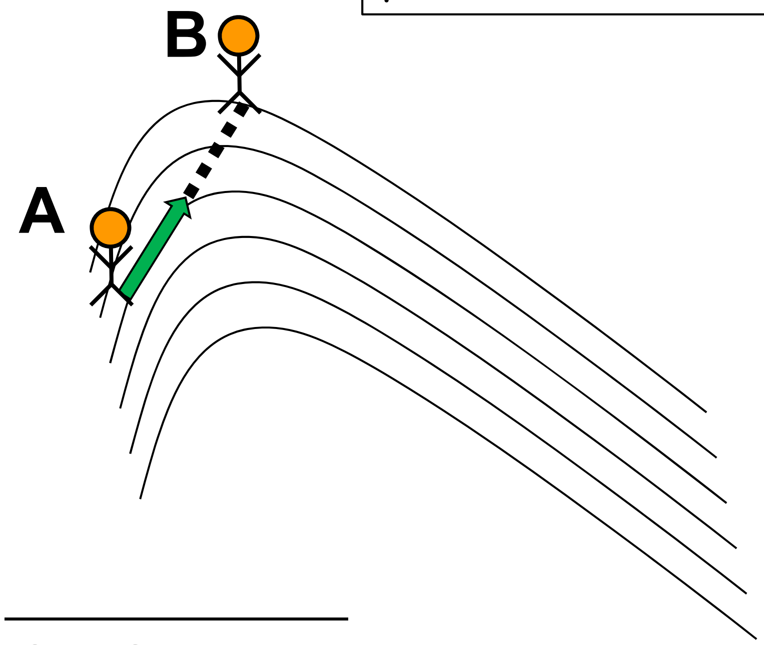


Which puts you on a larger radius. The next First Order step puts you on an even larger radius. And so on, and so on. What should be circular motion has now become spiraling-out motion.

All is Not Right with First Order

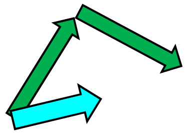
Clearly something is not right. While we were taking that straight-line step, the velocity field was changing underneath us, and we weren't taking it into account.

Obviously, we could simply take smaller time steps, but that wouldn't solve the problem, just make it smaller. And, in the process, it could take lots longer to compute.

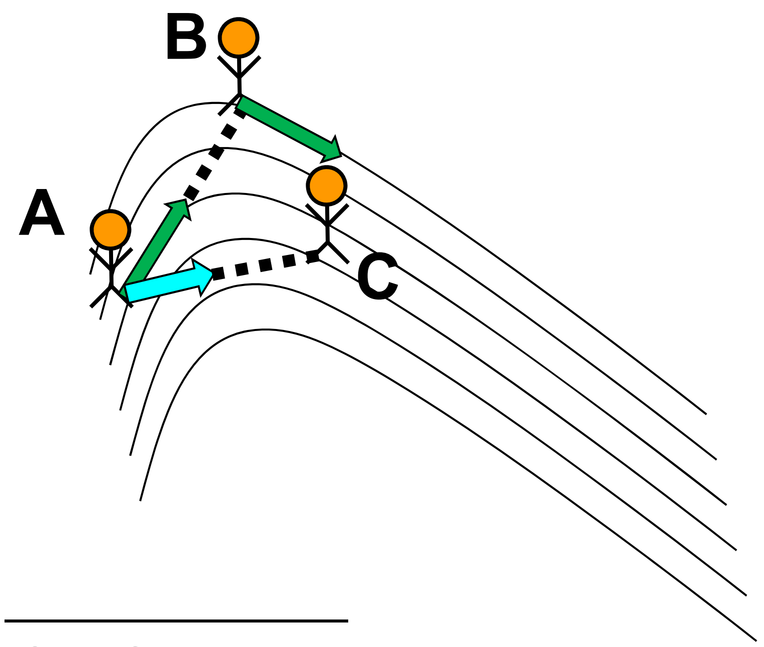


Taking a Second Order Step

Here's another approach. Let's assume that the field change during the time step is linear so that the average velocity vector during the step is the average of the velocity vector at **A** and the velocity vector at **B**. You do this by adding up the individual x, y, and z vector components and dividing by 2:



This is called a **Second Order Step**.



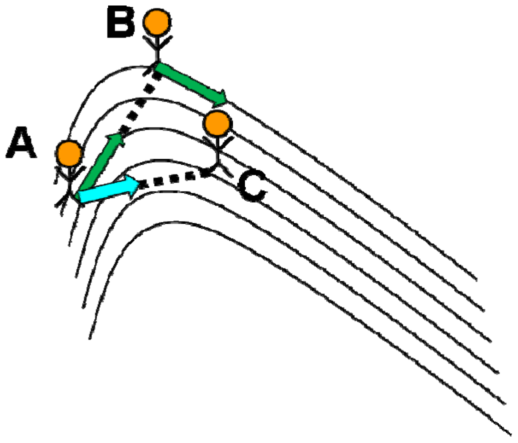
And, of course, you can continue this way even more.

Average the velocity vectors at A and C, take a step in that direction, and call it D.

Average the velocity vectors at A and D, take a step in that direction, and call it E.

Taking a Second Order Step

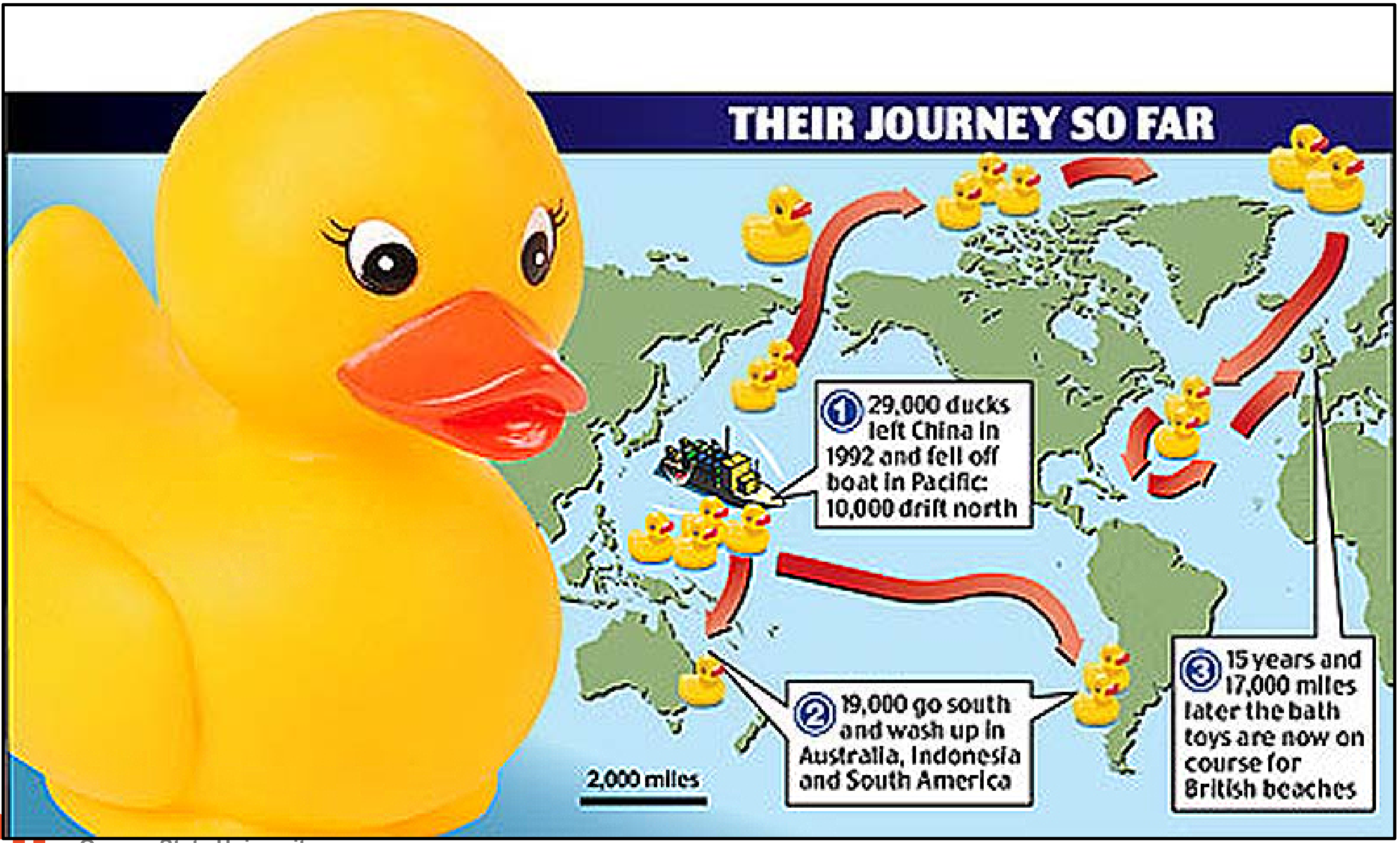
```
void
Advect( float *x, float *y, float *z )
{
            xa = *x;     ya = *y;     za = *z;

            GetVelocity( xa, ya, za,   &vxa, &vya, &vza );

            xb = xa + TimeStep*vxa;
            yb = ya + TimeStep*vya;
            zb = za + TimeStep*vza;

            GetVelocity( xb, yb, zb,   &vxb, &vyb, &vzb );

            vx = ( vxa + vxb ) / 2.;
            vy = ( vya + vyb ) / 2.;
            vz = ( vza + vzb ) / 2.;

            xc = xa + TimeStep*vx;
            yc = ya + TimeStep*vy;
            zc = za + TimeStep*vz;

            *x = xc;     *y = yc;     *z = zc;
}
```



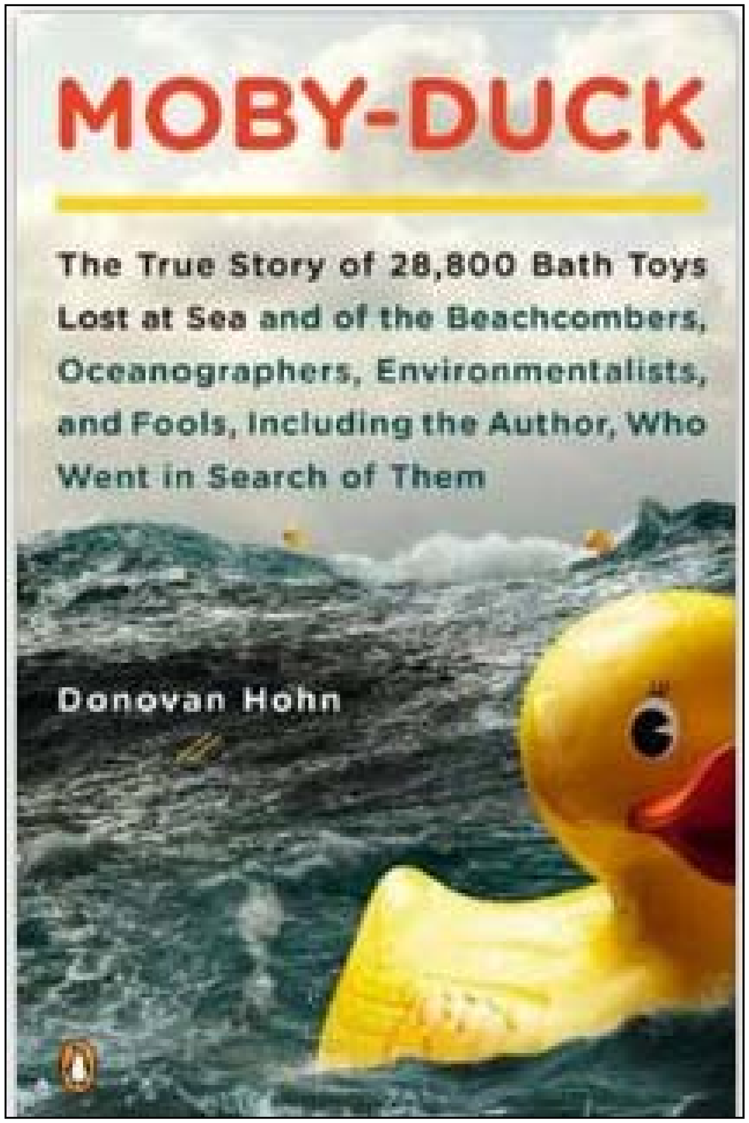
First Order Code

```
Advect( float *x, float *y, float *z )
{
            xa = *x;     ya = *y;     za = *z;

            GetVelocity( xa, ya, za,   &vxa, &vya, &vza );

            xb = xa + TimeStep*vxa;
            yb = ya + TimeStep*vya;
            zb = za + TimeStep*vza;

            *x = xb;     *y = yb;     *z = zb;
}
```

The World’s Largest Particle Advection Experiment ☺


THEIR JOURNEY SO FAR
1 29,000 ducks left China in 1992 and fell off boat in Pacific: 10,000 drift north
2 19,000 go south and wash up in Australia, Indonesia and South America
3 15 years and 17,000 miles later the bath toys are now on course for British beaches
2,000 miles

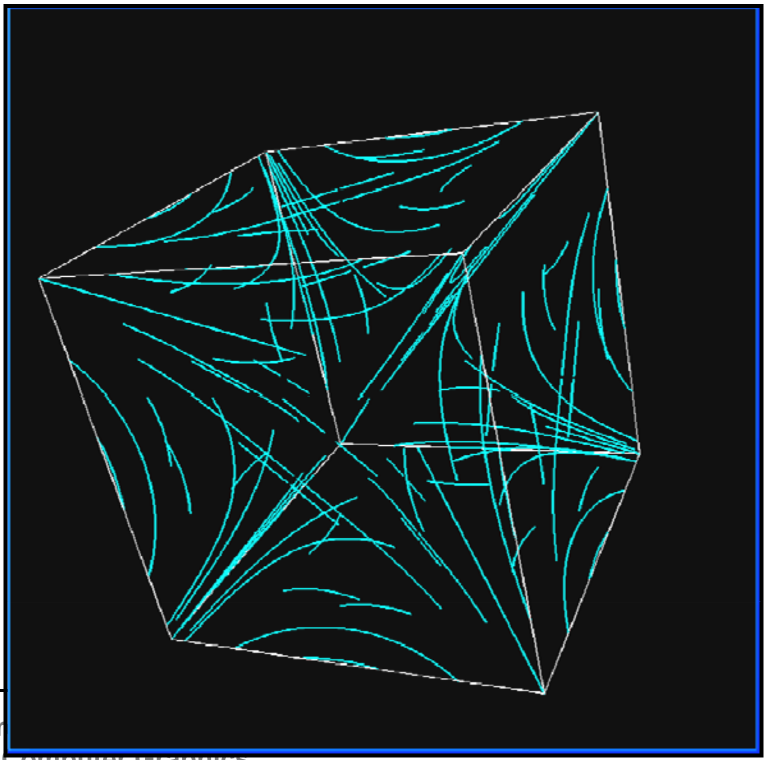
The World’s Largest Particle Advection Experiment ☺

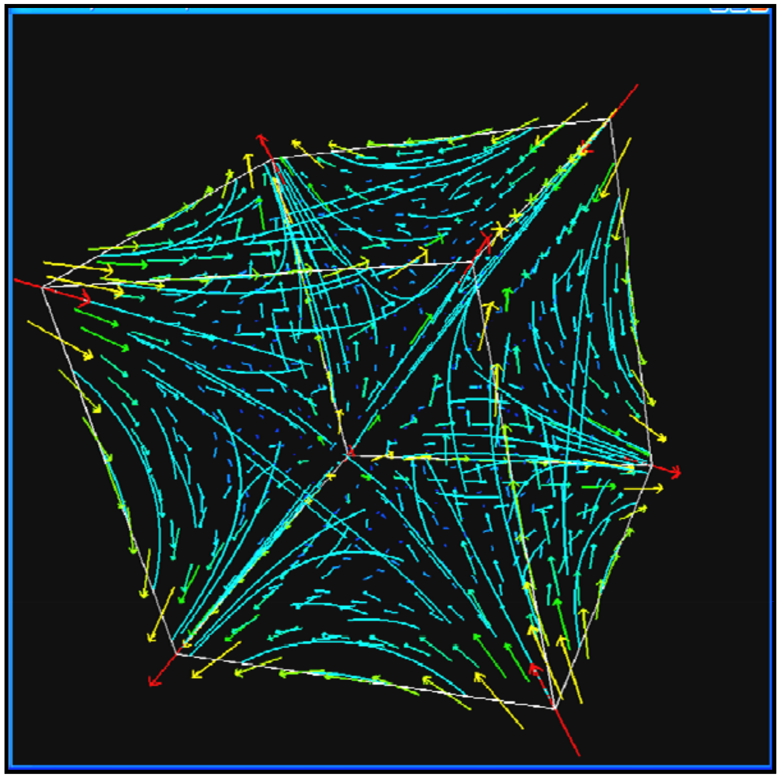

MOBY-DUCK
The True Story of 28,800 Bath Toys Lost at Sea and of the Beachcombers, Oceanographers, Environmentalists, and Fools, Including the Author, Who Went in Search of Them
Donovan Hohn

Streamlines

Using particle advection, we could animate little ping-pong balls flying through the field. We can also take the particle advection idea and create other geometrizations.

In this case, we are going to advect a particle and draw a line between its locations at successive time steps. This is called a **Streamline.** Because of the nature of particle advection, the tangent of the streamline curve always shows the direction of the velocity field there.

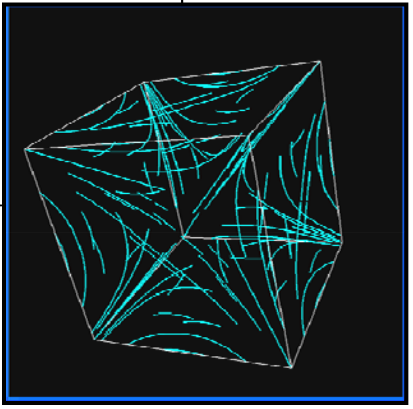




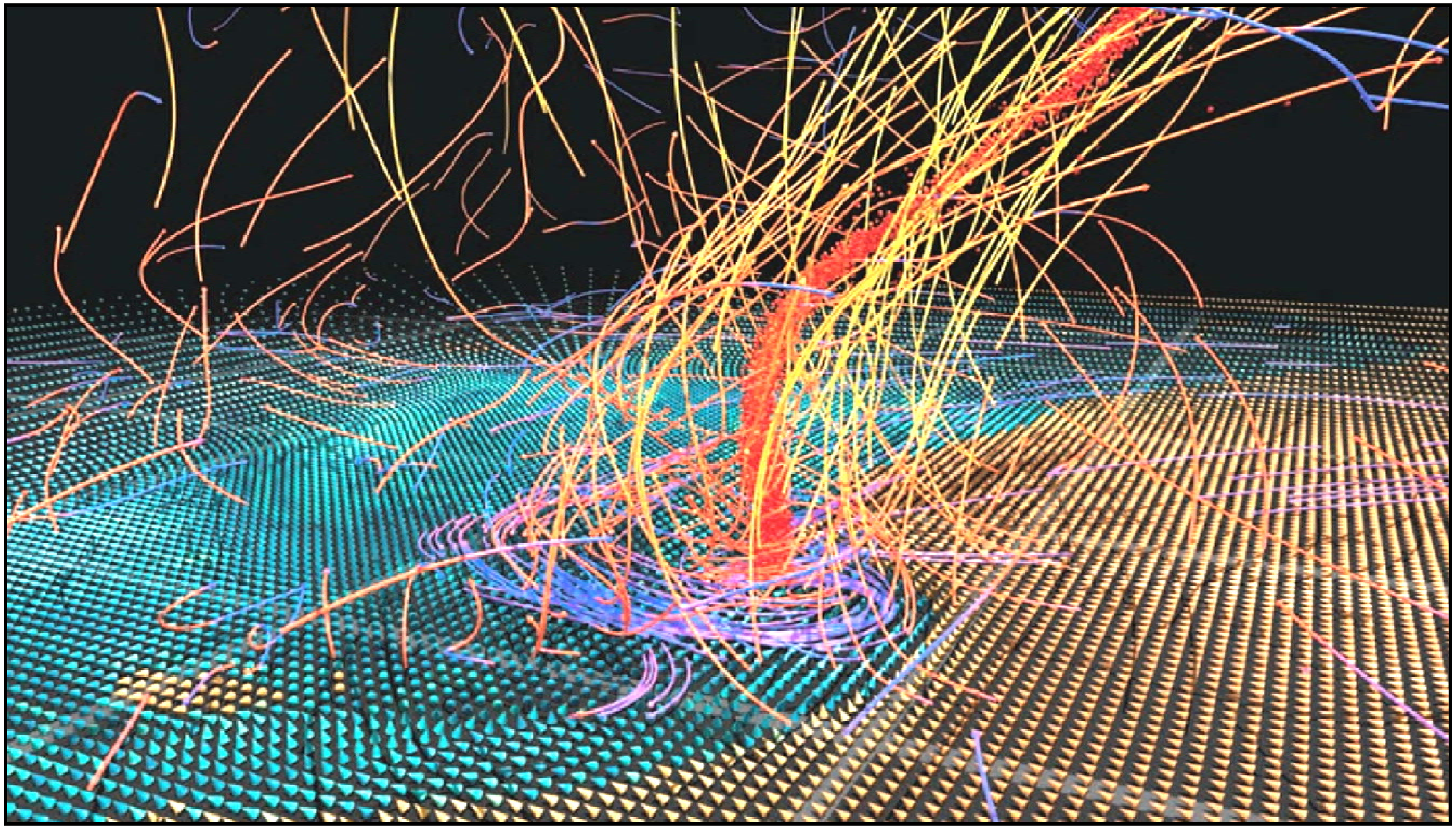
Streamlines

```
void
Streamline( float x, float y, float z )
{
        glLineWidth( 2. );
        glColor3f( ??, ??, ?? );
        glBegin( GL_LINE_STRIP );

        for( int  i = 0; i < MAX_ITERATIONS; i++ )      1
        {
                if( x < Xmin  ||  x > Xmax )          break;
                if( y < Ymin  ||  y > Ymax )          break;      2
                if( z < Zmin  ||  z > Zmax )          break;

                glVertex3f( x, y, z );

                GetVelocity( x, y, z,   &vx, &vy, &vz );
                if(  ||vx,vy,vz||  < SOME_TOLERANCE  )      break;      3

                Advect( &x, &y, &z );
        }
        glEnd(  );
}
```

Three reasons to stop drawing the streamline



Streamlines and Particle Advection



National Center for Supercomputing Applications

Streaklines

So far, we have been treating the flow as if it was steady-state, that is, we are advancing the streamline using a snapshot of the vector field information. What if it's not steady-state?

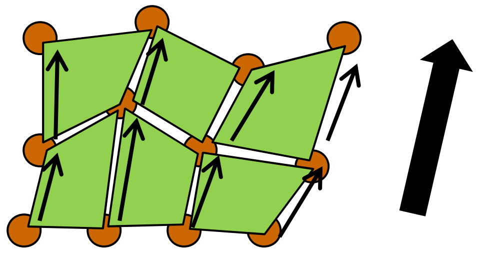
If we follow the same procedure, but use a new time's vector field every time we advance the streamline, then we have what is known as a ***streakline***.

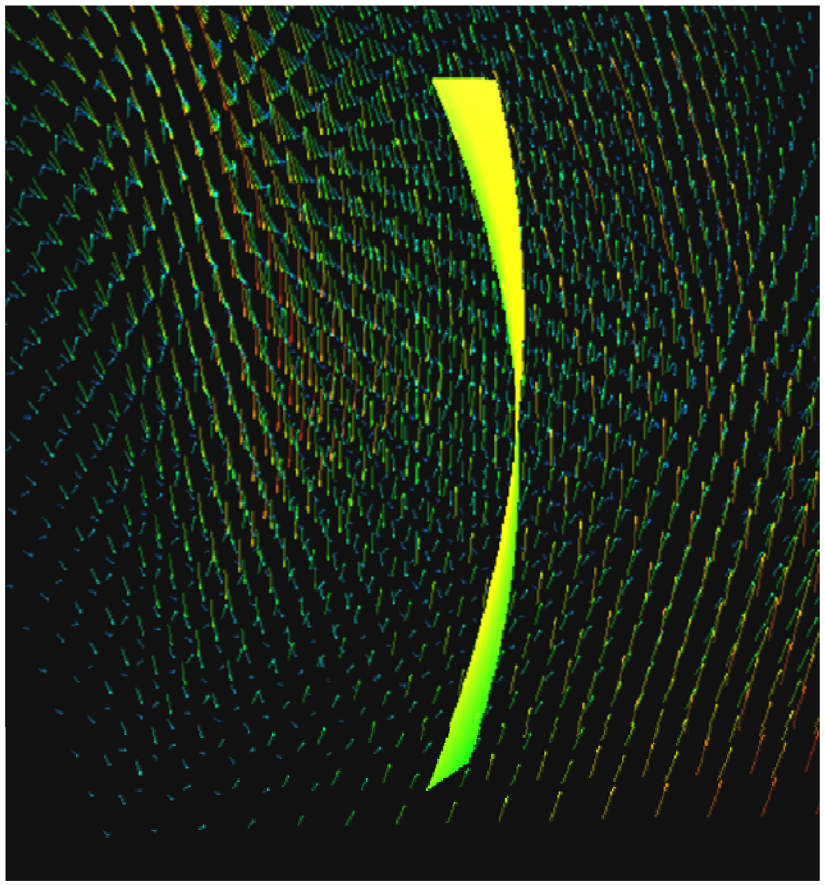
The formal definition of a streakline is the *locus of fluid particles that have passed through a specific starting point*. Perhaps a more intuitive way to think about streaklines is thinking about what would happen if some colored dye was continuously injected into a flow field at a given point.

If the flow is steady-state, streamlines and streaklines are the same things.

Ribbon Traces

Envision a series of streamlines created from a row of starting points. But, every time a time step is taken, the corresponding points on the streamlines are connected and colored in. This is called a ***Ribbon Trace.***

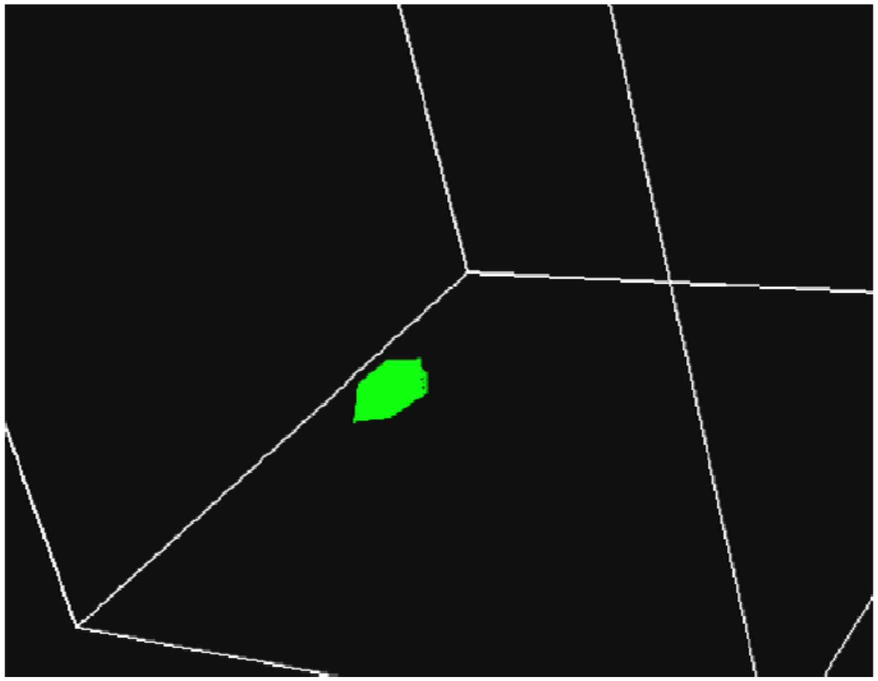


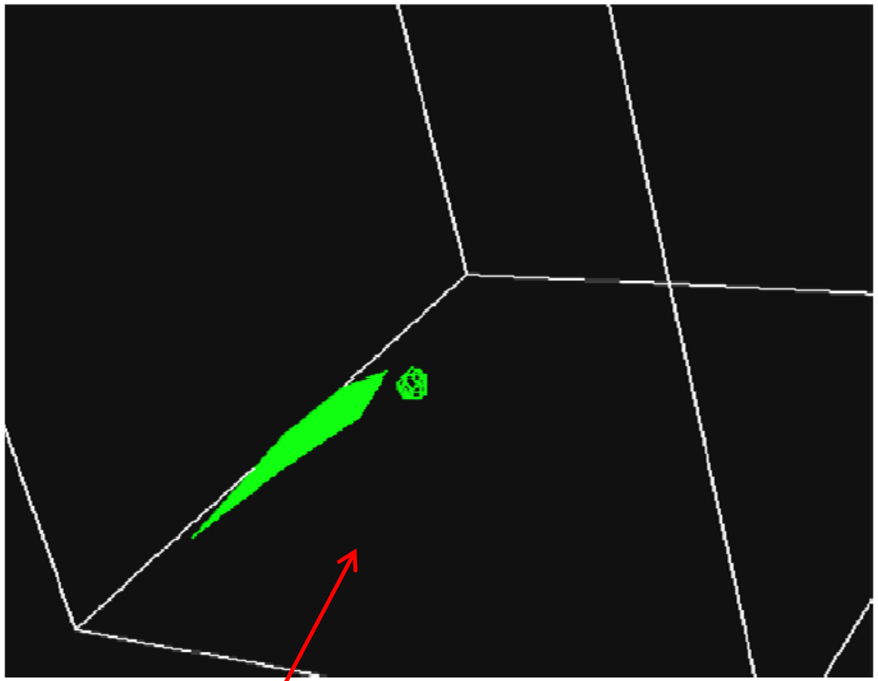


The big advantage of using a ribbon trace is that it can show *twisting motion* in the field (streamlines can't)

Blob Tracing

Idea: start with a 3D shape and particle-advect each vertex. Then connect all the vertices with the same topology that was used for the original 3D object.



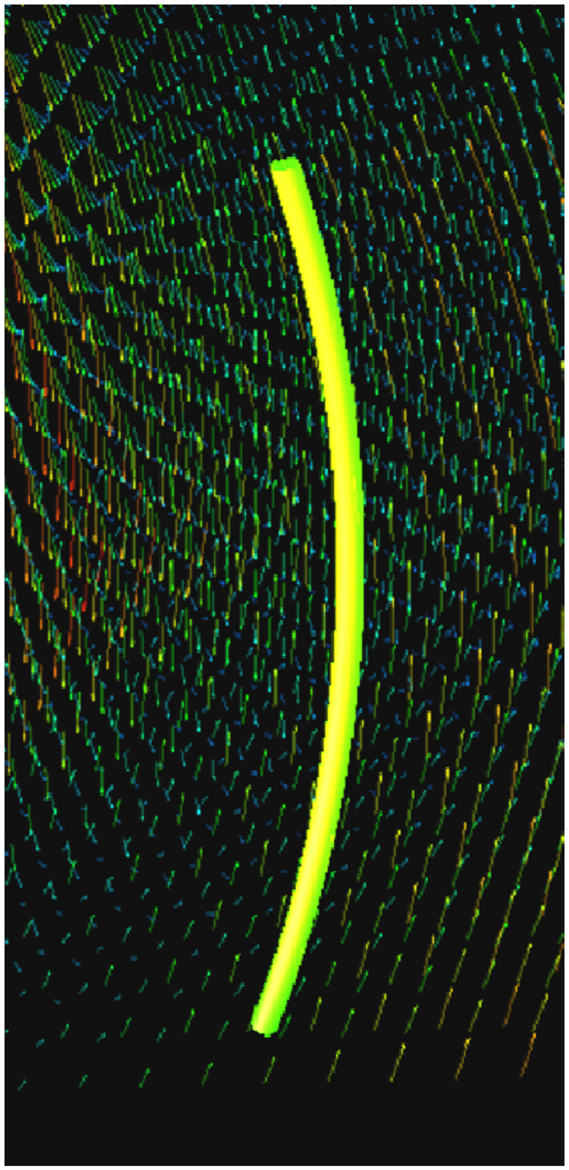


Stretched object shows that the field is *accelerating*.

Streamtubes

A ***Streamtube*** is like a streamline, but with a finite cross sectional area. (Which doesn't have to be a circle – "tube" is just what it is called.)

This makes your streamlines easier to see, and allows you to plot other information in color on the streamtube.



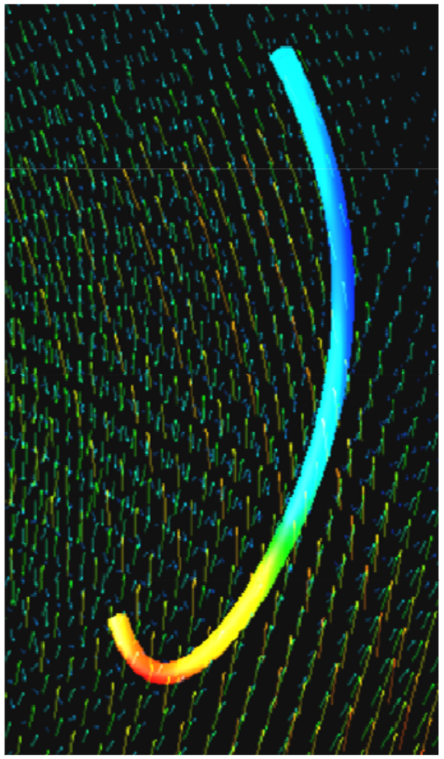
Curl

Curl and Divergence are referred to as “derived quantities” of a velocity field because they are not, generally, part of the original data, but are computed during the visualization.

The **Curl** tells you how much the field is curving. Think of it as the reciprocal of the radius of curvature of a streamline. The equation of the curl looks like this:

$$\vec{\nabla} \times \vec{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) \hat{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) \hat{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \hat{k}$$

This image shows the curl of a velocity field mapped as color to a streamtube.

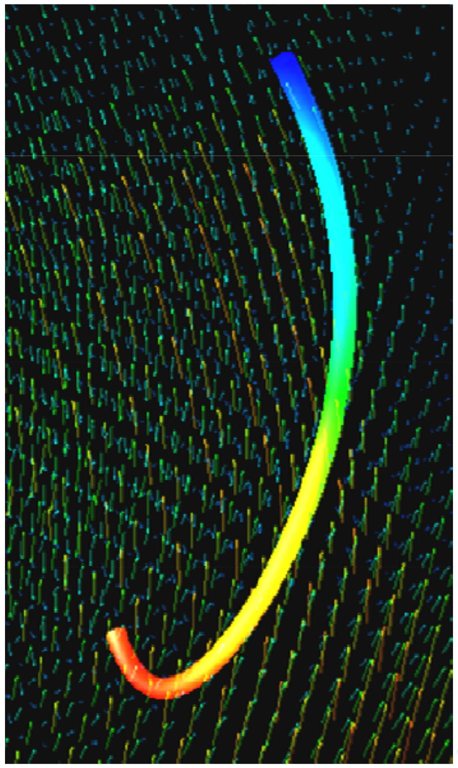


Divergence

The **Divergence** tells you how much the field is spreading out or compressing. The equation of the divergence looks like this:

$$\nabla \bullet V = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

If the fluid that is flowing is incompressible, then the Conservation of Mass law tells us that the divergence is zero everywhere.

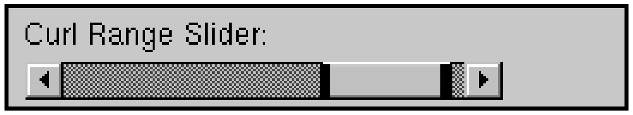


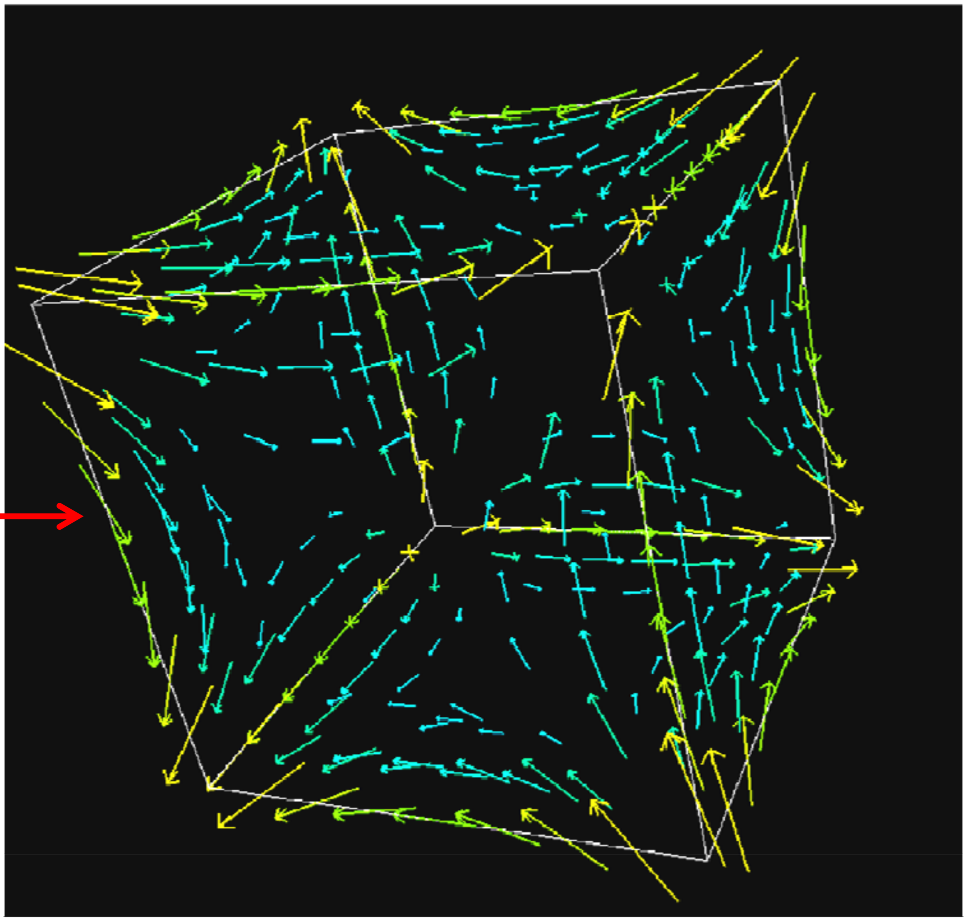
This image shows the divergence of a velocity field mapped as color to a streamtube.

Curl Range Sliders

Idea: Show where the field has a particular range of curls.

Looking at just the large curls

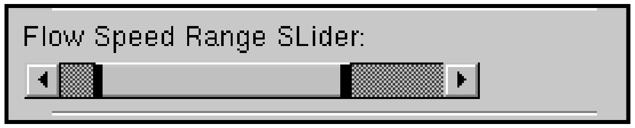


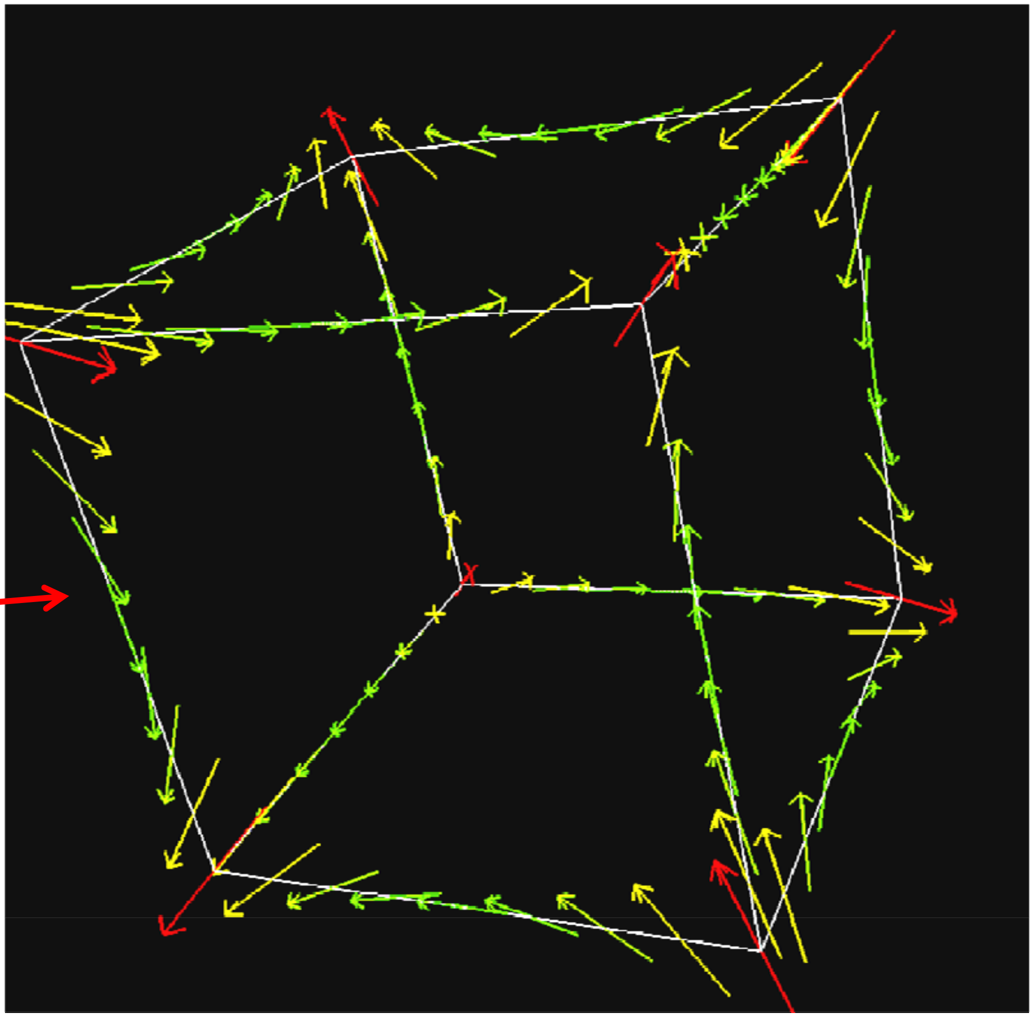


Magnitude Range Sliders

Idea: Show where the field has a particular range of magnitudes.

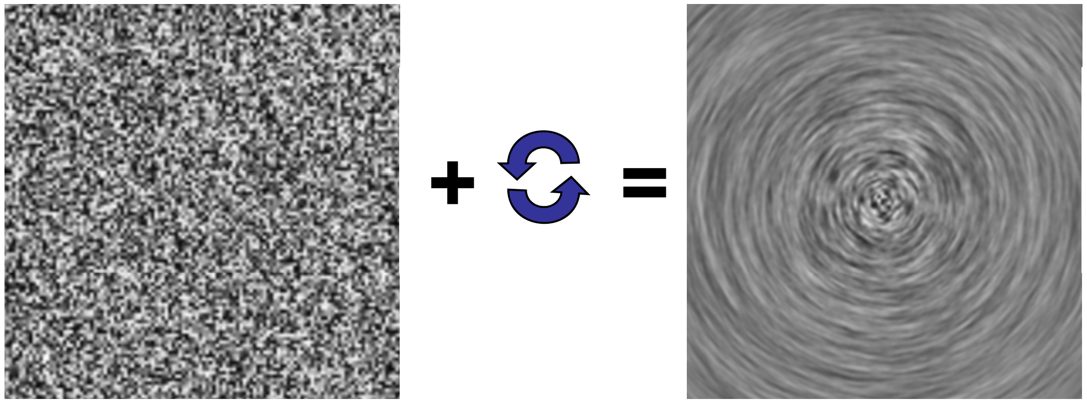
Looking at just the medium speeds





Line Integral Convolution

Line Integral Convolution (LIC) involves taking a white noise image and smearing it in the directions of the flow, in this case, a circular flow:

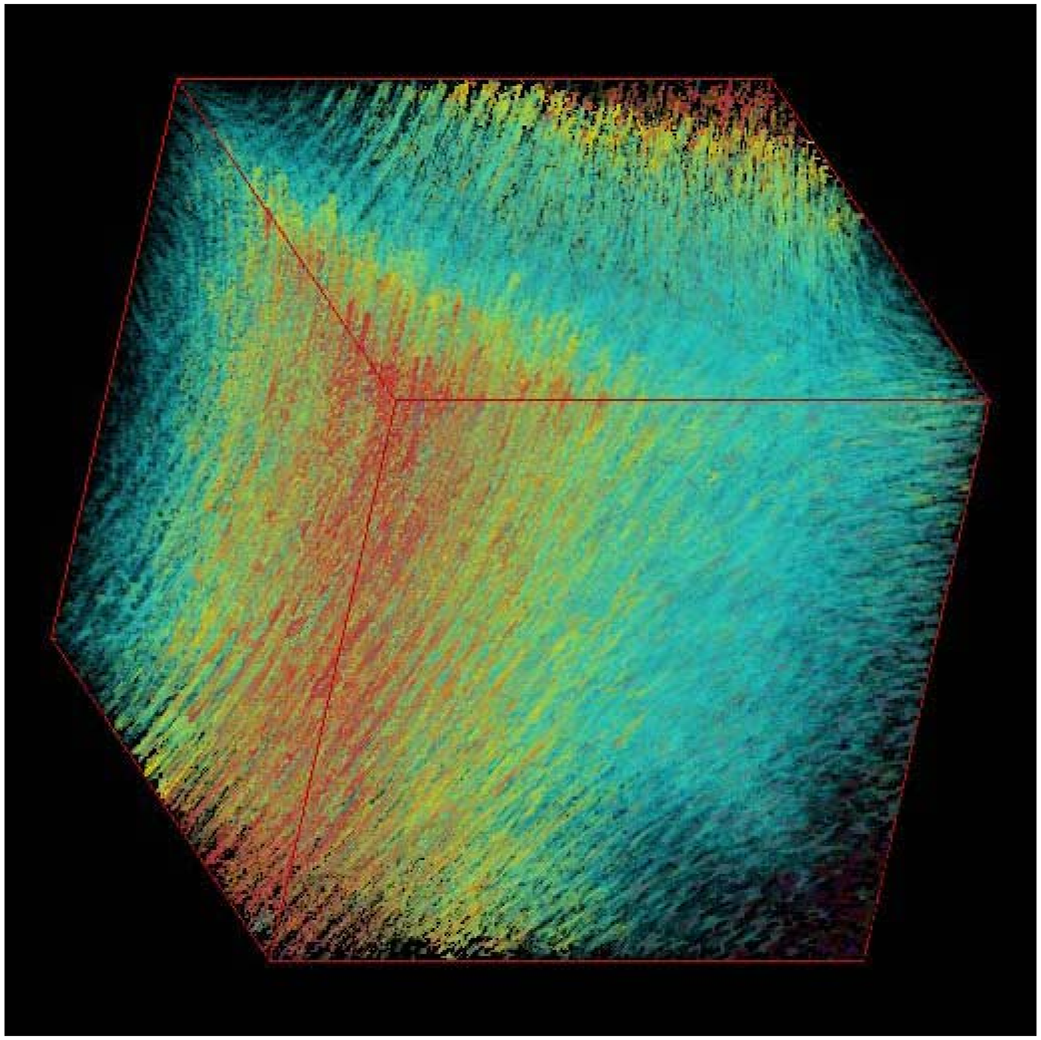


Mathematically, we create each pixel in the output image by following a streamline from that point (forwards and backwards) and performing a weighted average of all pixels that the streamline touches in the white noise image:

$$I'(x,y) = \frac{\sum_{i=-L}^{L} w(i) I(S(i))}{\sum_{i=-L}^{L} w(i)}$$

Where S(i) is the streamline position “i” pixels away from the starting point, I( ) are the contents of the white noise image, w(i) is the weight used for this pixel, and I’( ) is the resulting image.

3D Line Integral Convolution

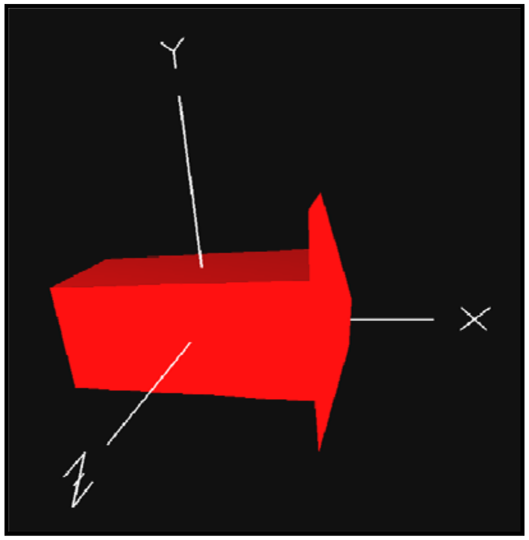


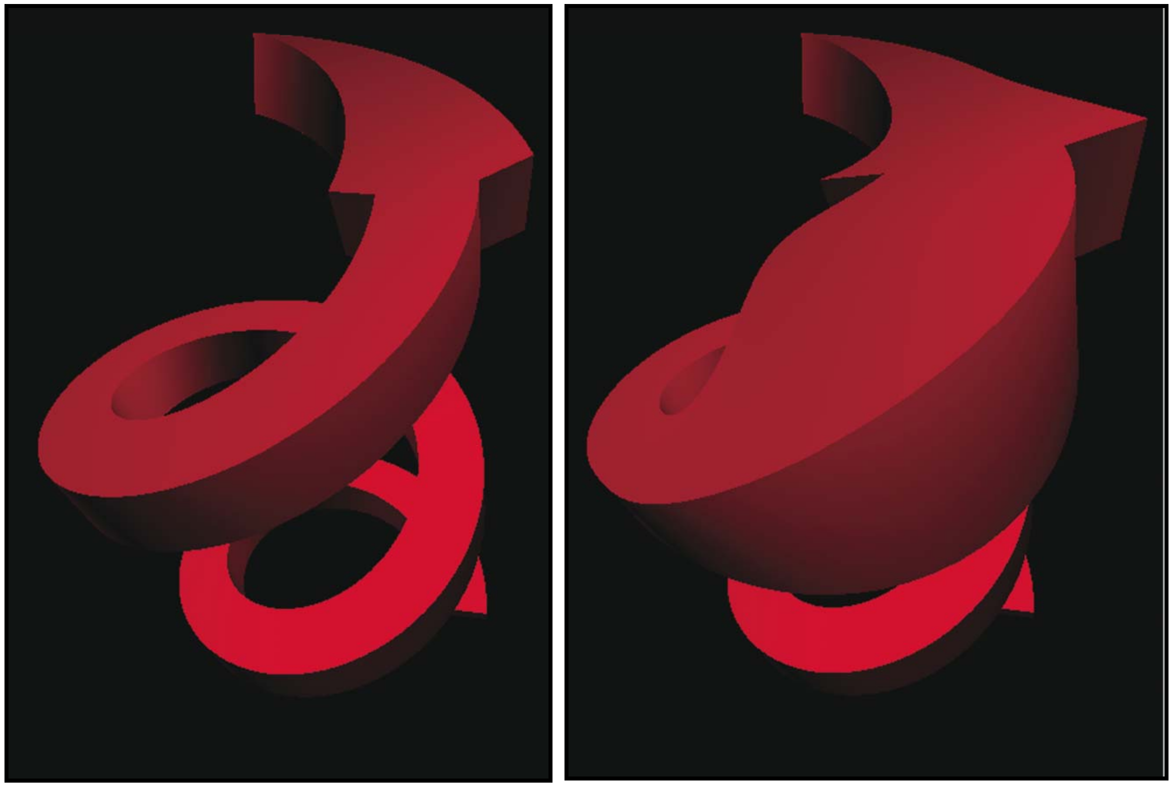
Vasu Lakshmanan

You need to apply some amount of decimation, or you can't see into the volume

Peristalsis

As long as you're extruding some cross section to make a streamtube, you can also animate a moving bulge through it.





How Big Should the Time Step Be?

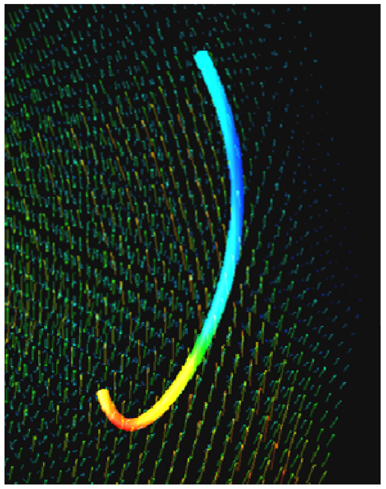
One of the trickiest parts of doing good particle advection for any reason is deciding how large to make the time step, Δt.

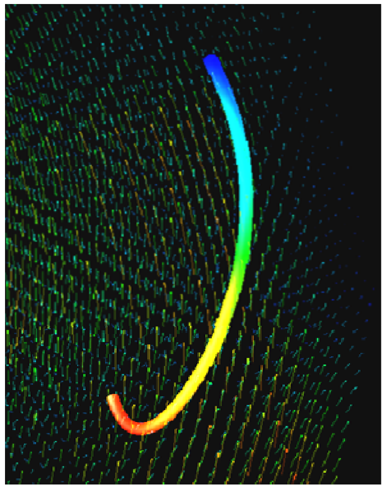
You could make it very, very tiny. That would give you good accuracy results, but poor interaction.

You could make it large. That would give you good interactivity, but at a cost of accuracy.

Clearly you need to find some way to *adapt* the time step to the situation.

One way is to think of the divergence and the curl as a way to measure how much the flow at a certain location is deviating from constant-speed straight-line motion. The larger the divergence and the curl, the smaller the time step should be.





How Big Should the Time Step Be?

Another way to do it is to check what would happen if two half-steps were taken instead of one whole step:

```
void
TakeStep( float Δt,  float * x0,  float * y0, float * z0 )
{
		float xw, yw, zw;
		xw = *x0;   yw = *y0;   zw = *z0;   // one whole step
		Advect(Δt, &xw, &yw, &zw );

		float xh, yh, zh;
		xh = *x0;   yh = *y0;   zh = *z0;   // two half steps
		Advect(Δt,/2.  &xh, &yh, &zh ) ;
		Advect(Δt,/2., &xh, &yh, &zh ) ;

		if( (xh,yh,zh) is "close enough" to (xw,yw,zw) )
		{
				*x0 = xh;   *y0 = yh;   *z0 = zh;
				return;
		}

		TakeStep( Δt, / 2.,  x0, y0, z0 );     // re-try with a smaller time step
		                                       // note: x0,y0,z0 are float pointers
		                                       // x0, y0, z0 come back changed
		TakeStep( Δt, / 2.,  x0, y0, z0 );
}
```