Hyperbolic Geometry for Visualization

Zooming and Panning Around a Complex 2D Display

- Standard (Euclidean) geometry zooming forces much of the information off the screen
- This eliminates the context from the zoomed-in display
- This problem can be solved with hyperbolic methods if we are willing to give up Euclidean geometry
Usual Zooming in Euclidean Space

Zooming in Polar Hyperbolic Space
Polar Hyperbolic Equations

Overall theme: something divided by something else a little bigger

\[ R' = \frac{R}{R+K} \]

\[ X' = R' \cos \Theta' \]
\[ Y' = R' \sin \Theta' \]

Because \( R' = \frac{R}{R+K} \) then:
\[ \lim_{K \to 0} R' = 1 \]
\[ \lim_{K \to \infty} R' = 0 \]

Polar Hyperbolic Equations Don't Actually Need to use Trig

\[ R = \sqrt{X^2 + Y^2} \]
\[ \Theta = \tan^{-1} \left( \frac{Y}{X} \right) \]
\[ R' = \frac{R}{R+K} \]

\[ X' = R' \cos \Theta = \frac{R}{R+K} \times \frac{X}{R} = \frac{X}{R+K} \]
\[ Y' = R' \sin \Theta = \frac{R}{R+K} \times \frac{Y}{R} = \frac{Y}{R+K} \]

Coordinates moved to outer edge when \( K = 0 \)
Coordinates moved to center when \( K = \infty \)
Cartesian Hyperbolic Equations – Treat X and Y Independently

Polar

\[ \begin{align*}
X' &= \frac{X}{R + K} \\
Y' &= \frac{Y}{R + K}
\end{align*} \]

Cartesian

\[ \begin{align*}
X' &= \frac{X}{\sqrt{X^2 + K^2}} \\
Y' &= \frac{Y}{\sqrt{Y^2 + K^2}}
\end{align*} \]

Coordinates moved to outer edge when \( K = 0 \)
Coordinates moved to center when \( K = \infty \)

Zooming in Cartesian Hyperbolic Space
The Problem with T-Intersections

Your code computes the hyperbolic transformation here and here, and OpenGL draws a straight line between them. But, this point had its hyperbolic transformation computed separately, and doesn’t match up with the straight line.

This kind of situation is called a T-intersection, and crops up all the time in computer graphics, even though we don’t want it to.
A Solution to the T-Intersection Problem

Break this line up into several (many?) sub-pieces, and perform the Hyperbolic Transformation on each intermediate point.

\[ P(t) = (1 - t)P_0 + tP_1 \]

\[ t = 0, .01, .02, .03, ... \]

This makes that straight line into a curve, as it should be. But, how many line segments should we use?

A More Elegant Approach is to Recursively Subdivide

```c
void DrawHyperbolicLine( P0, P1 )
{
    Compute point \( A = \frac{P_0 + P_1}{2} \).
    Convert point A to Hyperbolic Coordinates, calling it \( A' \).
    Convert \( P_0 \) and \( P_1 \) to Hyperbolic Coordinates \( P_0', P_1' \).
    Compute point \( B' = \frac{P_0' + P_1'}{2} \).
    Compare \( A' \) and \( B' \).
    if( they are "close enough" )
    {
        Draw the line \( P_0'-P_1' \).
    } else
    {
        DrawHyperbolicLine( P0, A );
        DrawHyperbolicLine( A, P1 );
    }
}
```

Subdividing to render a curve or surface correctly is a recurring theme in computer graphics.
Hyperbolic Corvallis (Streets, Buildings, Parks)

Kelley Engineering Center

http://www.sott.net/articles/show/215021-Hyperbolic-map-of-the-internet-will-save-it-from-COLLAPSE