Hyperbolic Geometry for Visualization

Mike Bailey
mjb@cs.oregonstate.edu

Zooming and Panning Around a Complex 2D Display

- Standard (Euclidean) geometry zooming forces much of the information off the screen
- This eliminates the context from the zoomed-in display
- This problem can be solved with hyperbolic methods if we are willing to give up Euclidean geometry

Usual Zooming in Euclidean Space

Zooming in Polar Hyperbolic Space

Polar Hyperbolic Equations

Overall theme: something divided by something else a little bigger

Because \( R' = \frac{R}{R+K} \)

Coordinates moved to outer edge when \( K = 0 \)
Coordinates moved to center when \( K = \infty \)

Polar Hyperbolic Equations Don’t Actually Need to use Trig

\[
R = \sqrt{X^2 + Y^2} \\
\theta = \tan^{-1}\left(\frac{Y}{X}\right) \\
X' = R' \cos \theta = \frac{R \cos \theta}{R + K} = \frac{X}{R + K} \\
Y' = R' \sin \theta = \frac{R \sin \theta}{R + K} = \frac{Y}{R + K}
\]
### Cartesian Hyperbolic Equations – Treat X and Y Independently

#### Polar

\[
\begin{align*}
X' &= \frac{X}{R + K} \\
Y' &= \frac{Y}{R + K}
\end{align*}
\]

- Coordinates move to outer edge when \( K = 0 \)
- Coordinates move to center when \( K = \infty \)

#### Cartesian

\[
\begin{align*}
X' &= \frac{X}{\sqrt{X^2 + Y^2}} \\
Y' &= \frac{Y}{\sqrt{Y^2 + K^2}}
\end{align*}
\]

- Coordinates move to outer edge when \( K = 0 \)
- Coordinates move to center when \( K = \infty \)

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### Zooming in Cartesian Hyperbolic Space

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### The Problem with T-Intersections

Your code computes the hyperbolic transformation here and here, and OpenGL draws a straight line between them. But, that point had its hyperbolic transformation computed separately, and doesn’t match up with the straight line.

This kind of situation is called a T-intersection, and crops up all the time in computer graphics, even though we don’t want it to.

### A Solution to the T-Intersection Problem

Break this line up into several (many?) sub-pieces, and perform the Hyperbolic Transformation on each intermediate point.

\[
P(t) = (1 - t)P_0 + tP_1
\]

\( t = 0, .01, .02, .03, \ldots \)

This makes that straight line into a curve, as it should be. But, how many line segments should we use?

### A More Elegant Approach is to Recursively Subdivide

#### Code

```c
void DrawHyperbolicLine( P0, P1 )
{
  Compute point A to Hyperbolic Coordinates, calling it A'
  Convert P0 and P1 to Hyperbolic Coordinates P0', P1'
  Compute point B' = \frac{P1' - P0'}{2}
  Compare A' and B'
  if they are "close enough"
  { Draw the line P0', P1'
    } else
  { DrawHyperbolicLine( P0, A' );
    DrawHyperbolicLine( A, P1 );
  }
}
```

Subdividing to render a curve or surface correctly is a recurring theme in computer graphics.
Hyperbolic Corvallis (Streets, Buildings, Parks)

Kelley Engineering Center

http://www.sott.net/articles/show/215021-Hyperbolic-map-of-the-internet-will-save-it-from-COLLAPSE