Using Vertex Shaders for Hyperbolic Geometry

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Zooming and Panning Around a Complex 2D Display

- Standard (Euclidean) geometry zooming forces much of the information off the screen
- This eliminates the context from the zoomed-in display
- This problem can be solved using hyperbolic methods if we are willing to give up Euclidean geometry
- At one time, this would have also meant severely giving up graphics performance, but not now

Zooming in Euclidean Space

123,101 line strips
446,585 points

Zooming in Polar Hyperbolic Space

Coordinates moved to outer edge when $K = 0$
Coordinates moved to center when $R = \infty$

Polar Hyperbolic Equations

Overall theme: something divided by something a little bigger

$R' = R / (R+K)$

$X' = R' \cos \Theta$
$Y' = R' \sin \Theta$

$\lim_{K \to 0} R' = 1$
$\lim_{K \to \infty} R' = 0$

$\Theta = \tan^{-1} \left( \frac{Y}{X} \right)$

$R = \sqrt{X^2 + Y^2}$

$X' = \frac{R'}{R + K} \cdot X$
$Y' = \frac{R'}{R + K} \cdot Y$

Polar Hyperbolic Equations
Cartesian Hyperbolic Equations

\[
\begin{align*}
X &= \frac{X}{\sqrt{1 + K}} \\
Y &= \frac{Y}{\sqrt{1 + K}}
\end{align*}
\]

Polar

\[
\begin{align*}
X &= \frac{X}{\sqrt{1 + K}} \\
Y &= \frac{Y}{\sqrt{1 + K}}
\end{align*}
\]

Cartesian

\[
\begin{align*}
X &= \frac{X}{\sqrt{1 + K^2}} \\
Y &= \frac{Y}{\sqrt{1 + K^2}}
\end{align*}
\]

Zooming in Cartesian Hyperbolic Space

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hyper.vert

```plaintext
#version 330 compatibility
uniform bool uPolar;
uniform float uK;
uniform float uTransX;
uniform float uTransY;
out vec4 vColor;

void main( void )
{
    vColor = aColor;
    vec2 pos = (uModelViewMatrix * aVertex).xy;
    float r = length(pos.xyz);
    vec4 pos2 = vec4(0., 0., -5., 1.);
    if (uPolar)
    {
        pos2.xy = pos / (r + uK);
    }
    else
    {
        pos2.xy = pos / (pos * pos + uK * uK);
    }
    gl_Position = uProjectionMatrix * pos2;
}
```

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hyper.frag

```plaintext
#version 330 compatibility
in vec4 vColor;
out vec4 fFragColor;

void main(  )
{
    fFragColor = vColor;
}
```