The Message Passing Interface (MPI): Parallelism on Distributed CPUs

http://mpi-forum.org
https://www.open-mpi.org/

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Oregon State University
Computer Graphics
**Why Two URLs?**

**http://mpi-forum.org**
This is the definitive reference for the MPI standard. Go here if you want to read the official specification, which, BTW, continues to evolve.

**https://www.open-mpi.org/**
This consortium formed later. This is the open source version of MPI. If you want to start using MPI, I recommend you look here. This is the MPI that the COE systems use

**https://www.open-mpi.org/doc/v4.0/**
This URL is also really good – it is a link to all of the MPI man pages
MPI: The Basic Idea

Programs on different CPUs coordinate computations by passing messages between each other.

Note: Each CPU in the MPI “cluster” must be prepared ahead of time by having the MPI server code installed on it. Each MPI CPU must also have an integer ID assigned to it (called its rank).
This paradigm is how modern supercomputers work!

The Texas Advanced Computing Center’s new *Frontera* supercomputer, currently the 5th fastest in the world
How to SSH to the COE MPI Cluster

```bash
flip3 151% ssh submit-c@hpc.engr.oregonstate.edu
```

```bash
submit-c 142% module load slurm
submit-c 143% module load openmpi/3.1
```

Type these two lines right away to set your paths correctly

BTW, you can find out more about the COE cluster here:

https://it.engineering.oregonstate.edu/hpc

“The College of Engineering HPC cluster is a heterogeneous mix of 202 servers providing over 3600 CPU cores, over 130 GPUs, and over 31 TB total RAM. The systems are connected via gigabit ethernet, and most of the latest servers also utilize a Mellanox EDR InfiniBand network connection. The cluster also has access to 100TB global scratch from the College of Engineering's Dell/EMC Isilon enterprise storage.”
Compiling and Running

% mpicc -o program program.c . . .  

or

% mpic++ -o program program.cpp . . .

All distributed processors execute the same program at the same time

% mpiexec -mca btl self,tcp -np 4 program

# of processors to use

Warning – use mpic++ and mpiexec!

Don’t use g++ and don’t run by just typing the name of the executable!
So, a *bash* Batch Script Would Look Like This

```bash
#!/bin/bash
#SBATCH  -J  Heat
#SBATCH  -A  cs475-575
#SBATCH  -p  class
#SBATCH  -N 8       # number of nodes
#SBATCH  -n 8       # number of tasks
#SBATCH  -o  heat.out
#SBATCH  -e  heat.err
#SBATCH  --mail-type=END,FAIL
#SBATCH  --mail-user=mjb@cs.oregonstate.edu
mpic++ heat.cpp -o heat -lm
mpiexec -mca btl self,tcp -np 4 heat
```

**submit-c 143% sbatch submit.bash**
Submitted batch job 258759
#SBATCH --mail-user=joeparallel@oregonstate.edu

You don’t have to ask for email notification, but if you do, please, please be sure you get your email address right!

The IT people are getting real tired of fielding the bounced emails when people spell their own email address wrong.
Use slurms’s `scancel` if your Job Needs to Be Killed

submit-c 143% sbatch submit.bash
Submitted batch job 258759

submit-c 144% scancel 258759
#include <mpi.h>

int 
main( int argc, char *argv[ ] )
{
    MPI_Init( &argc, &argv );

    MPI_Finalize( );
    return 0;
}

You don’t need to process command line arguments if you don’t need to. You can also call it as:

MPI_Init( NULL, NULL );
A *communicator* is a collection of CPUs that are capable of sending messages to each other.

Getting information about our place in the *communicator*:

```c
int numCPUs; // total # of cpus involved
int me;     // which one I am

MPI_Comm_size( MPI_COMM_WORLD, &numCPUs );
MPI_Comm_rank( MPI_COMM_WORLD, &me );
```

- **Size**, i.e., how many altogether?
- **Rank**, i.e., which one am I?

It is then each CPU’s job to figure out what piece of the overall problem it is responsible for and then go do it.
A First Test of MPI

```c
#include <stdio.h>
#include <math.h>
#include <mpi.h>

#define BOSS 0

int main( int argc, char *argv[] )
{
    MPI_Init( &argc, &argv );

    int numCPUs;       // total # of cpus involved
    int me;            // which one I am

    MPI_Comm_size( MPI_COMM_WORLD, &numCPUs );
    MPI_Comm_rank( MPI_COMM_WORLD, &me );

    if( me == BOSS )
        fprintf( stderr, "Rank %d says that we have a Communicator of size %d\n", BOSS, numCPUs );
    else
        fprintf( stderr, "Welcome from Rank %d\n", me );

    MPI_Finalize( );
    return 0;
}
```

submit-c 165% mpiexec -np 16 .first
Welcome from Rank 13
Welcome from Rank 15
Welcome from Rank 3
Welcome from Rank 7
Welcome from Rank 5
Welcome from Rank 8
Welcome from Rank 9
Welcome from Rank 11
Rank 0 says that we have a Communicator of size 16
Welcome from Rank 1
Welcome from Rank 12
Welcome from Rank 14
Welcome from Rank 6
Welcome from Rank 2
Welcome from Rank 10
Welcome from Rank 4

submit-c 166% mpiexec -np 16 .first
Welcome from Rank 13
Welcome from Rank 15
Welcome from Rank 5
Welcome from Rank 7
Welcome from Rank 9
Welcome from Rank 11
Welcome from Rank 13
Welcome from Rank 15
Rank 0 says that we have a Communicator of size 16
Welcome from Rank 2
Welcome from Rank 3
Welcome from Rank 4
Welcome from Rank 6
Welcome from Rank 8
Welcome from Rank 12
Welcome from Rank 14
Welcome from Rank 10

submit-c 167% mpiexec -np 16 .first
Welcome from Rank 9
Welcome from Rank 11
Welcome from Rank 13
Welcome from Rank 7
Welcome from Rank 1
Welcome from Rank 3
Welcome from Rank 10
Welcome from Rank 15
Welcome from Rank 4
Welcome from Rank 5
Rank 0 says that we have a Communicator of size 16
Welcome from Rank 2
Welcome from Rank 6
Welcome from Rank 14
Welcome from Rank 12
Welcome from Rank 10

submit-c 168% mpiexec -np 16 .first
Welcome from Rank 13
Welcome from Rank 15
Welcome from Rank 7
Welcome from Rank 3
Welcome from Rank 5
Welcome from Rank 9
Welcome from Rank 11
Welcome from Rank 1
Welcome from Rank 12
Welcome from Rank 14
Welcome from Rank 4
Welcome from Rank 2
Rank 0 says that we have a Communicator of size 16
Welcome from Rank 8
Welcome from Rank 10
Welcome from Rank 6
So, we have a group (a “communicator”) of distributed processors. How do they communicate about what work they are supposed to do?

Example: You could coordinate the units of our DGX system using MPI

Who am I?  
Where am I?  
What am I supposed to be doing?  
Hello? Is anyone listening?
A Good Place to Start: MPI Broadcasting

MPI_Bcast( array, count, type, src, MPI_COMM_WORLD );

Address of data to send from if you are the src node; Address of the data to receive into if you are not

# elements

MPI_CHAR
MPI_INT
MPI_LONG
MPI_FLOAT
MPI_DOUBLE

• • •

rank of the CPU doing the sending

Broadcast

src node

≠ src nodes

Both the sender and receivers need to execute MPI_Bcast – there is no separate receive function
This is our heat transfer equation from before. Clearly, every CPU will need to know this value.

\[ \Delta T_i = \left( \frac{k}{\rho C} \right) \left( \frac{T_{i-1} - 2T_i + T_{i+1}}{(\Delta x)^2} \right) \Delta t \]

```
int numCPUs;
int me;
float k_over_rho_c;  // the BOSS node will know this value, the others won’t (yet)
#define BOSS 0

MPI_Comm_size( MPI_COMM_WORLD, &numCPUs );  // how many are in this communicator
MPI_Comm_rank( MPI_COMM_WORLD, &me );  // which one am I?

if( me == BOSS )
{
    << read k_over_rho_c from the data file >>
}

MPI_Bcast( &k_over_rho_c, 1, MPI_FLOAT, BOSS, MPI_COMM_WORLD );  // send if BOSS, and receive if not
```
Confused? Look at this Diagram

Both the sender and receivers need to execute `MPI_Bcast` – there is no separate receive function

Executable code: k_over_rho_c (set)

Node #BOSS:

```c
MPI_Bcast(&k_over_rho_c, 1, MPI_FLOAT, BOSS, MPI_COMM_WORLD); // send if BOSS, and receive if not
```

All Nodes that are not #BOSS:

<table>
<thead>
<tr>
<th>Executable code</th>
<th>k_over_rho_c (being set)</th>
</tr>
</thead>
<tbody>
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</table>
How Does this Work?
Think Star Trek Wormholes!
Sending Data from One Source CPU to One Destination CPU

\[
\text{MPI\_Send} ( \text{array, numToSend, type, dst, tag, MPI\_COMM\_WORLD} );
\]

- Address of data to send from
- \# elements (note: this is the number of elements, not the number of bytes!)
- MPI\_CHAR
- MPI\_INT
- MPI\_LONG
- MPI\_FLOAT
- MPI\_DOUBLE
- ... An integer to differentiate this transmission from any other transmission (be sure this is unique!)
- Rank of the CPU to send to
- Rules:
  - One message from a specific src to a specific dst cannot overtake a previous message from the same src to the same dst.
  - MPI\_Send() blocks until the transfer is far enough along that array can be destroyed or re-used.
  - There are no guarantees on order from different src’s.

\[\text{src node} \rightarrow \text{dst node}\]
Receiving Data in a Destination CPU from a Source CPU

MPI_Recv( array, maxCanReceive, type, src, tag, MPI_COMM_WORLD, &status );

Rules:

• The receiver blocks waiting for data that matches what it declares to be looking for

• One message from a specific src to a specific dst cannot overtake a previous message from the same src to the same dst

• There are no guarantees on the order from different src’s

• The order from different src’s could be implied in the tag

• status is type MPI_Status – the “&status” can be replaced with MPI_STATUS_IGNORE

src node \[\square\] \rightarrow \text{dst node}
Example

Remember, this *identical code* runs on all CPUs:

```c
int numCPUs;
int me;
#define MYDATA_SIZE 128
char mydata[MYDATA_SIZE];
#define BOSS 0

MPI_Comm_size( MPI_COMM_WORLD, &numCPUs );
MPI_Comm_rank( MPI_COMM_WORLD, &me );

if( me == BOSS ) // the primary
{
    for( int dst = 0; dst < numCPUs; dst++ )
    {
        if( dst != BOSS )
        {
            char *InputData = "Hello, Beavers!";
            MPI_Send( InputData, strlen(InputData)+1, MPI_CHAR, dst, 'B', MPI_COMM_WORLD );
        }
    }
}
else // a secondary
{
    MPI_Recv( myData, MYDATA_SIZE, MPI_CHAR, BOSS, 'B', MPI_COMM_WORLD, MPI_STATUS_IGNORE );
    printf( " '%s' from rank # %d\n", in, me );
}
```

You are highly discouraged from sending to yourself. Because both the send and receive are capable of blocking, the result could be deadlock.
Look at this Diagram

if ( dst != BOSS )
{
    char *InputData = "Hello, Beavers!";
    MPI_Send( InputData, strlen(InputData)+1, MPI_CHAR, dst, 0, MPI_COMM_WORLD );
}

else // a secondary
{
    MPI_Recv( myData, MYDATA_SIZE, MPI_CHAR, BOSS, 0, MPI_COMM_WORLD, MPI_STATUS_IGNORE );
    printf( "%s from rank %d\n" in, me );
}
How does MPI let the Sender perform an MPI_Send( ) even if the Receivers are not ready to MPI_Recv( )?

MPI_Send( ) blocks until the transfer is far enough along that the array can be destroyed or re-used.
Another Example

You typically don’t send the entire workload to each dst – you just send part of it, like this:

```c
#define NUMELEMENTS ????
int numCPUs;
int me;
#define BOSS 0

MPI_Comm_size( MPI_COMM_WORLD, &numCPUs );
MPI_Comm_rank( MPI_COMM_WORLD, &me );

int localSize = NUMELEMENTS / numCPUs; // assuming it comes out evenly
float *myData = new float [ localSize ];

if( me == BOSS ) // the sender
{
    float *InputData = new float [ NUMELEMENTS ];
    << read the full input data into InputData from disk >>
    for( int dst = 0; dst < numCPUs; dst++ )
    {
        if( dst != BOSS )
        {
            MPI_Send( &InputData[dst*localSize], localSize, MPI_FLOAT, dst, 0, MPI_COMM_WORLD );
        }
    }
}else // a receiver
{
    MPI_Recv( myData, localSize, MPI_FLOAT, BOSS, 0, MPI_COMM_WORLD, MPI_STATUS_IGNORE );
    // do something with this subset of the data
}
```
You typically don’t send the entire workload to each dst – you just send part of it, like this:

```c
if ( dst != BOSS )
{
    MPI_Send( &InputData[dst*localSize], localSize, MPI_FLOAT, dst, 0, MPI_COMM_WORLD );
}
else
{
    // a secondary
    MPI_Recv( myData, localSize, MPI_FLOAT, BOSS, 0, MPI_COMM_WORLD, MPI_STATUS_IGNORE );
    // do something with this subset of the data
}
```
In Distributed Computing, You Often Hear About These Design Patterns

- **Broadcast**

- **Scatter**

- **Gather**
Scatter and Gather Usually Go Together

Note surprisingly, this is referred to as Scatter/Gather
MPI Scatter

Take a data array, break it into ~equal portions, and send it to each CPU

```c
MPI_Scatter( snd_array, snd_count, snd_type, rcv_array, rcv_count, rcv_type, src, MPI_COMM_WORLD );
```

- The total large array to split up
- # elements to send per-processor
- Local array to store this processor’s piece in
- # elements to receive per-processor
- This is who is doing the sending – everyone else is receiving

Both the sender and receivers need to execute `MPI_Scatter`. There is no separate receive function.
MPI_Gather( snd_array, snd_count, snd_type, rcv_array, rcv_count, rcv_type, dst, MPI_COMM_WORLD );

The total large array to put the pieces back into

# elements to send back per-processor

Local array that this processor is sending back

# elements to return per-processor

This is who is doing the receiving – everyone else is sending

Both the sender and receivers need to execute MPI_Gather. There is no separate receive function
Remember This? It’s Baaaaaack as a complete Scatter/Gather Example

The **Compute : Communicate Ratio** still applies, except that it is even more important now because there is much more overhead in the Communicate portion.

This pattern of breaking a big problem up into pieces, sending them to different CPUs, computing on the pieces, and getting the results back is *very* common. That’s why MPI has its own scatter and gather functions.
#include <stdio.h>
#include <math.h>
#include <mpi.h>

const float RHO = 8050.;
const float C   = 0.466;
const float K   = 20.;
float k_over_rho_c = K / (RHO*C); // units of m^2/sec  NOTE: this cannot be a const!
// K / (RHO*C) = 5.33x10^-6 m^2/sec

const float DX    = 1.0;
const float DT    = 1.0;

#define BOSS 0

#define NUMELEMENTS  (8*1024*1024)
#define NUM_TIME_STEPS  4
#define DEBUG false

float * NextTemps; // per-processor array to hold computer next-values
int NumCpus; // total # of cpus involved
int PPSize; // per-processor local array size
float * PPTemps; // per-processor local array temperature data
float * TempData; // the overall NUMELEMENTS-big temperature data

void DoOneTimeStep( int );
```c
int main( int argc, char *argv[ ] )
{
    MPI_Init( &argc, &argv );

    int me; // which one I am

    MPI_Comm_size( MPI_COMM_WORLD, &NumCpus );
    MPI_Comm_rank( MPI_COMM_WORLD, &me );

    // decide how much data to send to each processor:
    PPSize = NUMELEMENTS / NumCpus; // assuming it comes out evenly
    PPTemps   = new float [PPSize]; // all processors now have this uninitialized Local array
    NextTemps = new float [PPSize]; // all processors now have this uninitialized local array too

    // broadcast the constant:
    MPI_Bcast( (void *)&k_over_rho_c, 1, MPI_FLOAT, BOSS, MPI_COMM_WORLD );
```
if( me == BOSS ) // this is the data-creator
{
    TempData = new float [NUMELEMENTS];
    for( int i = 0; i < NUMELEMENTS; i++ )
        TempData[ i ] = 0.;
    TempData[NUMELEMENTS/2] = 100.;
}

MPI_Scatter( TempData, PPSize, MPI_FLOAT, PPTemps, PPSize, MPI_FLOAT, BOSS, MPI_COMM_WORLD );
// all the PPTemps arrays have now been filled
// do the time steps:

double time0 = MPI_Wtime( );

for( int steps = 0; steps < NUM_TIME_STEPS; steps++ )
{
    // do the computation for one time step:
    DoOneTimeStep( me );

    // ask for all the data:
#ifdef WANT_EACH_TIME_STEPS_DATA
    MPI_Gather( PPTemps, PPSize, MPI_FLOAT, TempData, PPSize, MPI_FLOAT,
               BOSS, MPI_COMM_WORLD );
#endif

#ifndef WANT_EACH_TIME_STEPS_DATA
    MPI_Gather( PPTemps, PPSize, MPI_FLOAT, TempData, PPSize, MPI_FLOAT,
               BOSS, MPI_COMM_WORLD );
#endif

    double time1 = MPI_Wtime( );
}
if( me == BOSS )
{
    double seconds = time1 - time0;
    double performance =
        (double)NUM_TIME_STEPS * (double)NUMELEMENTS / seconds / 1000000.;
    // mega-elements computed per second
    fprintf( stderr, "%3d, %10d, %8.2lf\n", NumCpus, NUMELEMENTS, performance );
}

MPI_Finalize( );
return 0;
void DoOneTimeStep( int me )
{
    MPI_Status status;

    // send out the left and right end values:
    // (the tag is from the point of view of the sender)
    if( me != 0 ) // i.e., if i'm not the first group on the left
    {
        // send my PPTemps[0] to me-1 using tag 'L'
        MPI_Send( &PPTemps[0], 1, MPI_FLOAT, me-1, 'L', MPI_COMM_WORLD );
        if( DEBUG ) fprintf( stderr, "%3d sent 'L' to %3d\n", me, me-1 );
    }

    if( me != NumCpus-1 ) // i.e., not the last group on the right
    {
        // send my PPTemps[PPSize-1] to me+1 using tag 'R'
        MPI_Send( &PPTemps[PPSize-1], 1, MPI_FLOAT, me+1, 'R', MPI_COMM_WORLD );
        if( DEBUG ) fprintf( stderr, "%3d sent 'R' to %3d\n", me, me+1 );
    }
}
float left = 0.;
float right = 0.;

if( me != 0 ) // i.e., if i'm not the first group on the left
{
    // receive my "left" from me-1 using tag 'R'
    MPI_Recv( &left, 1, MPI_FLOAT, me-1, 'R', MPI_COMM_WORLD, &status );
    if( DEBUG ) fprintf( stderr, "%3d received 'R' from %3d\n", me, me-1 );
}

if( me != NumCpus-1 ) // i.e., not the last group on the right
{
    // receive my "right" from me+1 using tag 'L'
    MPI_Recv( &right, 1, MPI_FLOAT, me+1, 'L', MPI_COMM_WORLD, &status );
    if( DEBUG ) fprintf( stderr, "%3d received 'L' from %3d\n", me, me+1 );
}
Sharing Values Across the Boundaries

1 sent 'L' to 0
1 sent 'R' to 2
2 sent 'L' to 1
2 sent 'R' to 3
2 received 'R' from 1
0 sent 'R' to 1
0 received 'L' from 1
1 received 'R' from 0
1 received 'L' from 2
3 sent 'L' to 2
3 received 'R' from 2
2 received 'L' from 3
1D Compute-to-Communicate Ratio

Compute : Communicate ratio = $N : 2$

where $N$ is the number of compute cells per processor

In the above drawing, Compute : Communicate is 4 : 2
// first element on the left (0):
{
    float dtemp = ( k_over_rho_c * 2. * PPTemps[0] + PPTemps[1]) / (DX*DX) * DT;
    NextTemps[0] = PPTemps[0] + dtemp;
}

// all the nodes in the middle:
for(int i = 1; i < PPSize-1; i++)
{
    float dtemp = ( k_over_rho_c * 2. * PPTemps[i] + PPTemps[i+1]) / (DX*DX) * DT;
    NextTemps[i] = PPTemps[i] + dtemp;
}

// last element on the right (PPSize-1):
{
    float dtemp = ( k_over_rho_c * 2. * PPTemps[PPSize-2] + PPTemps[PPSize-1] ) / (DX*DX) * DT;
    NextTemps[PPSize-1] = PPTemps[PPSize-1] + dtemp;
}
// update the local dataset:

for( int i = 0; i < PPSize; i++ )
{
    PPTemps[ i ] = NextTemps[ i ];
}

DoOneTimeStep, IV
MPI Performance

Mega-Elements Computed Per Second vs. Number of Elements

Number of Elements

Mega-Elements Computed Per Second

Number of Processors
Low Dataset-Size MPI Performance
Mega-Elements Computed Per Second vs. Number of Processors

Number of Processors

Mega-Elements Computed Per Second

Number of Elements

Oregon State University
Computer Graphics

mjb – May 1, 2021
MPI Reduction

`MPI_Reduce( partialResult, globalResult, count, type, operator, dst, MPI_COMM_WORLD );`

- **Where the partial result is stored on each CPU:**
- **Place to store the full result on the dst CPU:**
- **Number of elements in the partial result:**
- **Operator:**
  - MPI_MIN
  - MPI_MAX
  - MPI_SUM
  - MPI_PROD
  - MPI_MINLOC
  - MPI_MAXLOC
  - MPI_BAND
  - MPI_BOR
  - MPI_LAND
  - MPI_HOR
- **Who is given the final answer:**

This really should be called **Scatter/Gather/Reduction**

Both the sender and receivers need to execute **MPI_Reduce**. There is no separate receive function.
MPI Reduction Example

// gratuitous use of a reduce -- average all the temperatures:

    float partialSum = 0.;
    for( int i = 0; i < PPSize; i++ )
        partialSum += PPTemps[ i ];

    float globalSum = 0.;
    MPI_Reduce( &partialSum, &globalSum, 1, MPI_FLOAT, MPI_SUM, BOSS, MPI_COMM_WORLD );

    if( me == BOSS )
        fprintf( stderr, "Average temperature = %f\n", globalSum/(float)NUMELEMENTS );
MPI Barriers

All CPUs must execute the call to MPI_Barrier() before any of the CPUs can move past it. That is, each CPU’s MPI_Barrier() blocks until all CPUs execute a call to MPI_Barrier().
MPI Derived Types

Idea: In addition to types MPI_INT, MPI_FLOAT, etc., allow the creation of new MPI types so that you can transmit an “array of structures”.

Reason: There is significant overhead with each transmission. Better to send one entire array of structures instead of sending several arrays separately.

\[
\text{MPI\_Type\_create\_struct}(\text{count, blocklengths, displacements, types, datatype});
\]

```
struct point
{
    int    pointSize;
    float x, y, z;
};
```

```
MPI\_Datatype MPI\_POINT;
int blocklengths[ ] = { 1, 1, 1, 1 };
int displacements[ ] = { 0, 4, 8, 12 },
MPI\_type types[ ] = { MPI\_INT, MPI\_FLOAT, MPI\_FLOAT, MPI\_FLOAT };

\text{MPI\_Type\_create\_struct}(4, \text{blocklengths, displacements, types, &MPI\_POINT});
```

You can now use MPI\_POINT everywhere you could have used MPI\_INT, MPI\_FLOAT, etc.
MPI Timing

```c
double MPI_Wtick( );
```

Returns the resolution of the clock, in seconds.

```c
double MPI_Wtime( );
```

Returns the time, in seconds, since “some time in the past”.

**Warning:** the clocks on the different CPUs are not guaranteed to be synchronized!
Autocorrelation – a Piece of the Original Signal

The Original Signal
Autocorrelation – More than Just a Scatter

NUMELEMENTS

Divide NUMELEMENTS into pieces for the NumCpus
(this is what MPI_Scatter does)

But, in the Autocorrelation case, we need MAXSHIFTs more
data values for each CPU
Autocorrelation – How the Shifting Works

Shift = 0

NUMELEMENTS
NumCpus

MAXSHIFTS

Shift = 1

NUMELEMENTS
NumCpus

MAXSHIFTS

Shift = 2

NUMELEMENTS
NumCpus

MAXSHIFTS

Shift = 3

NUMELEMENTS
NumCpus

MAXSHIFTS

Shift = MAXSHIFTS-1

NUMELEMENTS
NumCpus

MAXSHIFTS