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The goal of this section is to look at some of the common design and programming patterns one encounters in parallel programming and to understand some of the nuances one encounters.
The Functional Decomposition Design Pattern

Overall Problem

Thread 0 → Thread 1 → Thread 2 → Thread 3

The Functional (or Task) Decomposition Design Pattern

Climate → Animals → Plants → Money

Credit: Maxis (Sim Park)
Task Distribution Design Patterns for Parallelism

Thread-to-Thread

Broadcast

Gather

Decentralized (Peer)

Input → "Peer-threads" → Output
Manager / workers

Map-Reduce

Input

“Worker-threads”

“Accumulate-thread”

“Map-thread”

Output
Task Distribution Design Patterns for Parallelism

Pipeline

Input ➔ ➔ ➔ Output

Requires some sort of queue between the stages

Multicore Block Data Decomposition:
1D Heat Transfer Example

You have a steel bar. Each section of the bar starts out at a different temperature. There are no incoming heat sources or outgoing heat sinks (i.e., ignore boundary conditions). Ready, go! How do the temperatures change over time?

The fundamental differential equation here is:

\[ \rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \]

where:
- \( \rho \) is the density in kg/m\(^3\)
- \( C \) is the specific heat capacity measured in Joules / (kg·°K)
- \( k \) is the coefficient of thermal conductivity measured in Watts / (meter·°K)

(These units work because a Watt is a Joule/second.)

In plain words, this all means that temperatures, left to themselves, try to even out. The greater the temperature differential, the faster the evening-out process goes.
**Numerical Methods:**
**Changing a Derivative into Discrete Arithmetic**

\[
\frac{\partial T}{\partial t} = \frac{T_{t+\Delta t} - T_t}{\Delta t}
\]

\[
\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1} - 2T_i + T_{i+1}}{(\Delta x)^2}
\]

As a side note: the quantity \(k/(\rho C)\) has the unlikely units of \(m^2/sec\)!
1D Data Decomposition: Partitioning Strategies

Should you allocate the data as one large global-memory block (i.e., shared)?

Or, should you allocate it as separate arrays, each dedicated to its own core?

Does it matter?

Allocate as One Large Continuous Global or Malloc’ed Array

float Temps[ARRAYSIZE];
float *Temps = (float *)malloc(ARRAYSIZE*sizeof(float));
float *Temps = new float[ARRAYSIZE];
<< allocate a new[ ] array the same way >>

omp_set_num_threads(4);
for(int t = 0; t < NUM_TIME_STEPS; t++)
{
    #pragma omp parallel for default(none), shared(Temperatures)
    for(int i = 1; i < ARRAYSIZE-1; i++)
    {
        << compute $\Delta T_i$ using $T_{i-1}$, $T_i$, and $T_{i+1}$ >>
        new[i] = Temps[i] + $\Delta T_i$;
    }
    << copy the new[ ] array to the Temps[ ] array >>
}

What happens when computing at the boundaries? Two cores are accessing the same cache line. False Sharing!

Pick one way of allocating global or heap data
Allocate as Separate Sub-arrays

We could make each sub-array as a thread-local (i.e., private) variable. This would put each sub-array on each thread’s individual stack. But, let’s not do that just in case these arrays might be large enough to overflow the stack. Although, if we did, it wouldn’t change this story.

Be sure to start each sub-array on its own cache line boundary. (See cache notes.)

But, now when we

\[ \text{compute } \Delta T_i \text{ using } T_{i-1}, T_i, \text{ and } T_{i+1} \]

at the boundaries, \(T_{i-1}\) or \(T_{i+1}\) might be in another sub-array.

So, we need some logic to reach into the other sub-array to get the adjacent temperature. It is no longer as easy as saying \(\text{Temps}[i-1]\) or \(\text{Temps}[i+1]\).

1D Compute-to-Communicate Ratio

\[ \text{Compute} : \text{Communicate ratio} = \frac{N}{2} \]

where \(N\) is the number of compute cells per core

In the above drawing, Compute : Communicate is \(4 : 2\)
How do more Cores Interact with the Compute-to-Communicate Ratio?

In this case, with 4 cores, Compute : Communicate = 4 : 2

In this case, with 8 cores, Compute : Communicate = 2 : 2

Think if it as a *Goldilocks and the Three Bears* sort of thing. :-)

Too little *Compute : Communicate* and you are spending all your time sharing data values across threads and doing too little computing.

Too much *Compute : Communicate* and you are not spreading out your problem among enough threads to get good parallelism.

Performance as a Function of Number of Nodes

It's difficult to find the "sweet spot" without running experiments.
Performance as a Function of Number of Threads

<table>
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<th># of Threads</th>
<th># of Nodes</th>
<th>MegaNodes Computed Per Second</th>
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</tr>
</tbody>
</table>

2D Heat Transfer Equation

\[ \rho C \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]

\[ \Delta T_{ij} = \left( \frac{k}{\rho C} \left( \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} \right) \right) \Delta t \]

\[ \frac{\Delta T}{\Delta t} = k \left( \frac{\Delta^2 T}{\Delta x^2} + \frac{\Delta^2 T}{\Delta y^2} \right) \]
2D Domain (Data) Decomposition

In addition to the issues of size of the compute block, you also have issues of direction.

Direction Issue: Decomposition Order Matters (think cache)

float Array[A][B];

In 2D problems, this is often (but not always) thought of as:

float Array[NY][NX];
2D Compute-to-Communicate Ratio

Compute : Communicate ratio = $N^2 : 4N = N : 4$

where $N$ is the dimension of compute nodes per core

The 2D Compute : Communicate ratio is sometimes referred to as Area-to-Perimeter

3D Heat Transfer Equation

$$\frac{\rho C}{\partial t} \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$\Delta T_{ijk} = \left( \frac{k}{\rho C} \right) \left[ \frac{T_{i+1,j,k} - 2T_{i,j,k} + T_{i-1,j,k}}{\Delta x^2} + \frac{T_{i,j+1,k} - 2T_{i,j,k} + T_{i,j-1,k}}{\Delta y^2} + \frac{T_{i,j,k+1} - 2T_{i,j,k} + T_{i,j,k-1}}{\Delta z^2} \right] \Delta t$$

$$\frac{\Delta T}{\Delta t} = \frac{k}{\rho C} \left( \frac{\Delta^2 T}{\Delta x^2} + \frac{\Delta^2 T}{\Delta y^2} + \frac{\Delta^2 T}{\Delta z^2} \right)$$
3D Domain (Data) Decomposition

3D Block, *, *

3D *, Block, *

3D *, *, Block

3D Block, Block, Block

Direction Issue: Decomposition Order Matters (think cache)

float Array[A][B][C];

In 3D problems, this is often (but not always) thought of as:

float Array[NZ][NY][NX];
**3D Compute-to-Communicate Ratio**

Compute : Communicate ratio = $N^3 : 6N^2 = N : 6$

where $N$ is the dimension of compute nodes per core

In 3D the Compute : Communicate ratio is sometimes referred to as **Volume-to-Surface**