Prob. 1-3: Find the expression for small signal $R_{in}$, $G_m$, $R_{out}$ and gain for all the circuits. Assume all the BJTs are biased in forward active region and MOSFETs in saturation. Consider $r_o = \infty$

**Prob.1**

![Circuit Diagram](image)

(a)

(b)

\[
R_{in} = R_B + R_E + R_C(1+\beta)
\]

\[
R_{out} = R_C
\]

\[
G_m = -\frac{R_B}{R_{in}} g_m
\]

\[
Gain = G_m \cdot Rout = \frac{-\beta R_C}{R_B + R_T + R_E (1+\beta)}
\]

\[
R_{in} = R_B + R_T + R_E (1+\beta)
\]

\[
R_{out} = R_E || \frac{R_B + R_T}{1+\beta}
\]

\[
G_m = \frac{1}{R_B + R_T} + \frac{R_T}{R_B + R_T} g_m = \frac{1+\beta}{R_B + R_T}
\]

\[
Gain = G_m \cdot Rout
\]

but you can just apply voltage divider to get gain, even without knowing $G_m$ & $R_{out}$. 

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ECE 323 HW # 3
(c) \[
R_{in} = R_E + \frac{R_B + R_1}{1 + \beta} \\
R_{out} = R_E \| \frac{R_B + R_E}{1 + \beta} \\
G_m = \frac{1}{R_E} \\
Gain = G_m \cdot Rout \\
\text{or just apply voltage divider}
\]

(d) \[
R_{in} = R_B + R_{R1} + R_E (1 + \beta_1) \\
R_{out} = \frac{R_{R2}}{1 + \beta_2} \| R_C \\
\approx \frac{1}{G_m} \\
G_m = -\frac{R_1 \cdot G_m}{R_E + R_{R1} + R_E (1 + \beta_1)} \\
Gain = G_m \cdot Rout
\]
Prob.2

\[ R_{o1} = \infty \]
\[ \downarrow \]
\[ R_{in} = \infty \]
\[ G_m = 0 \]
\[ G_{ain} = 0 \]

\[ R_{out} = R_c \]

\[ R_{in} = \frac{1}{g_m} \]
\[ R_{out} = R_c \parallel \frac{1}{g_m} \]
\[ G_m = g_{m1} + g_{m2} \]
\[ G_{ain} = G_m \cdot R_{out} \]
\[ R_{in} = \infty \quad \text{(current source)} \]
\[ R_{out} = \frac{R_C}{1+\beta} \]
\[ G_m = \frac{1}{R_{in}} + G_m = \frac{1+\beta}{R_C} \]
\[ G_{in} = G_m \cdot R_{out} = 1 \]

\[ R_{in} = R_C + RE \cdot (1+\beta) \]
\[ R_{out} = 0 \quad \text{(voltage source)} \]
\[ G_m = -\frac{R_C \cdot G_m}{R_C + RE \cdot (1+\beta)} \]
\[ G_{in} = G_m \cdot R_{out} = 0 \]
Prob. 3

\[ R_{in} = R_1 + R_\| \\
R_{out} = R_2 \parallel \frac{1}{g_{m2}} \\
G_m = \frac{-R_2}{R_1 + R_\|} \left( g_{m01} + g_{m1} \right) \\
G_{in} = G_m \cdot R_{out} \]
$E_{in} = \infty$

\[ R_{out} = \frac{1}{g_{m2}} \left( R_2 + \frac{1}{g_{m3}} \right) \parallel \frac{R_2}{1+\beta} \]

\[ G_m = -g_{m1} \]

\[ G_{m,in} = G_m \cdot R_{out} \]
Prob. 4
For the circuit given below, find the expression for output resistance $R_{out}$, transimpedance $V_{out}/I_{in}$ and current gain $I_{z}/I_{in}$. Assume all the MOSFETs having same $W/L$ and operating in saturation region. Consider $r_o = \infty$

\begin{equation*}
\begin{aligned}
\text{all same } \frac{V}{I} & \rightarrow I_{in} = I_1 = I_2 = I_3 = I_4 = I_2 \\
R_{out} &= R_2 \quad (r_o = \infty) \\
\frac{V_{out}}{I_{in}} &= 2 \cdot R_2
\end{aligned}
\end{equation*}
Prob. 5
For the circuit given below, find the expression for Rin, Rout, Gm and gain. Assume all the MOSFETs operating in saturation region and consider $r_o = \infty$. Plot the behavior of output gain vs $R_2$.

$v_i$ controls $V_{gs}$ of M1 and M2, in parallel.

$R_{in} = R_1 + \frac{1}{g_{m1} + g_{m2}}$

$R_{out} = (R_1 + R_2) \left| \frac{R_1}{R_1 + R_2} \left( \frac{g_{m1} + g_{m2}}{v_i} \right) \right|^{-1} = \frac{R_1 + R_2}{R_1 (g_{m1} + g_{m2}) + 1}$ (Vin = 0)

$G_m = \frac{1}{R_1 + R_2} - \frac{R_2}{R_1 + R_2} (g_{m1} + g_{m2}) = \frac{1 - R_2 (g_{m1} + g_{m2})}{R_1 + R_2}$ (Vout shorted)

Gain = $G_m \cdot R_{out} = \frac{1 - R_2 (g_{m1} + g_{m2})}{1 + R_1 (g_{m1} + g_{m2})}$