Prob. 1-3: Find the expression for small signal $R_{in}$, $G_m$, $R_{out}$ and Gain for all the circuits. Assume all the BJTs are biased in forward active region and MOSFETs in saturation. Consider $r_o = \infty$

Prob. 1

![Circuit Diagram](a)

- $R_{in} = R_B + R_E + R_C (1 + \beta)$
- $R_{out} = R_C$
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But you can just apply voltage divider to get gain, even without knowing $G_m$ & $R_{out}$. 

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ECE 323 HW # 3

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But you can just apply voltage divider to get gain, even without knowing $G_m$ & $R_{out}$. 

---
(c) \[ R_{in} = R_E + \frac{R_B + R_E}{1 + \beta} \]
\[ R_{out} = R_E \left| \frac{R_B + R_E}{1 + \beta} \right| \]
\[ G_m = \frac{1}{R_E} \]
\[ Gain = G_m \cdot R_{out} \]
or just apply voltage divider

(d) \[ R_{in} = R_B + \frac{R_{in1} + R_E (1 + \beta)}{1 + \beta} \]
\[ R_{out} = \frac{R_{E2}}{1 + \beta_2} \left| \frac{R_C}{1 + \beta_2} \right| \]
\[ \approx \frac{1}{G_{m2}} \]
\[ G_m = -\frac{R_E \cdot G_{m1}}{R_B + R_{in1} + R_E (1 + \beta_1)} \]
\[ Gain = G_m \cdot R_{out} \]
Prob.2

(a)

\[ R_{o1} = \infty \]
\[ \downarrow \]
\[ R_{in} = \infty \]
\[ G_m = 0 \]
\[ G_{m in} = 0 \]

\[ Rout = R_C \]

(b)

\[ R_{in} = V_{in} \]
\[ Rout = R_C \parallel \frac{1}{g_m} \]
\[ G_m = \left( g_m_{Q2} + g_m_{M1} \right) \]
\[ Gain = G_m \cdot Rout \]
\[ R_{in} = \infty \quad \text{(current source)} \]
\[ R_{out} = \frac{R_i}{1 + \beta} \]
\[ G_m = \frac{1}{R_i} + g_m = \frac{1 + \beta}{R_i} \]
\[ G_{in} = G_m \cdot R_{out} = 1 \]

\[ R_{in} = \frac{R_i}{1 + \beta} + R_E (1 + \beta) \]
\[ R_{out} = 0 \quad \text{(voltage source)} \]
\[ G_m = -\frac{R_i \cdot g_m}{R_i + R_E (1 + \beta)} \]
\[ G_{in} = G_m \cdot R_{out} = 0 \]
Prob. 3

\[ R_{in} = R_1 + R_{II} \]

\[ R_{out} = R_2 \parallel \frac{1}{g_m 2} \]

\[ G_m = \frac{g_m}{R_1 + R_2} \left( g_m m_1 + g_m m_1 \right) \]

\[ G_{in} = G_m \cdot R_{out} \]
\( b_{in} = \infty \)

\[ R_{out} = \frac{1}{g_{m2}} \left| \left( R_2 + \frac{1}{g_{m3}} \right) \right| \frac{R_2}{1+\beta} \]

\( G_m = -g_{mM1} \)

\( G_{m_{in}} = G_m \cdot R_{out} \)
Prob. 4
For the circuit given below, find the expression for output resistance $R_{out}$, transimpedance $\frac{V_{out}}{I_{in}}$ and current gain $\frac{I_{z}}{I_{in}}$. Assume all the MOSFETs having same $W/L$ and operating in saturation region. Consider $r_o = \infty$

\[ V_{cc} \]

\[ \downarrow \quad I_1 \quad \uparrow \]

\[ \downarrow \quad I_2 \quad \uparrow \]

\[ \downarrow \quad I_3 \quad \uparrow \]

\[ \downarrow \quad I_4 \quad \uparrow \]

\[ I_{in} \quad \downarrow \]

\[ R_2 \]

\[ \uparrow \quad V_{out} \]

\[ I_z \]

All same $\frac{V}{I} \rightarrow I_{in} = I_1 = I_2 = I_3 = I_4 = I_Z \quad \rightarrow \quad \frac{I_Z}{I_{in}} = 1$

\[ R_{out} = R_2 \quad (r_o = \infty) \]

\[ \frac{V_{out}}{I_{in}} = 2 \cdot R_2 \]
**Prob. 5**

For the circuit given below, find the expression for $R_{in}$, $R_{out}$, $G_m$ and gain. Assume all the MOSFETs operating in saturation region and consider $r_o = \infty$. Plot the behavior of output gain vs $R_2$.

![Circuit Diagram]

$V_i$ controls $V_{gs}$ of $M_1$ and $M_2$, in parallel.

$$R_{in} = R_1 + \frac{1}{g_{m1} + g_{m2}}$$

$R_{out} = (R_1 + R_2) \left[ \frac{R_1}{R_1 + R_2} \right]^{-1} \left[ \frac{1}{R_1 + R_2} \right] \left( g_{m1} + g_{m2} \right) = \frac{R_1 + R_2}{R_1 (g_{m1} + g_{m2}) + 1}$

(Vin = 0)

$$G_m = \frac{1}{R_1 + R_2} - \frac{R_2}{R_1 + R_2} \left( g_{m1} + g_{m2} \right) = \frac{1 - R_2 (g_{m1} + g_{m2})}{R_1 + R_2}$$

(Vout shorted)

$$Gain = G_m \cdot R_{out} = \frac{1 - R_2 (g_{m1} + g_{m2})}{R_1 (g_{m1} + g_{m2})}$$
Prob. 6
Find Req2 and Req3

\[
\text{Req2} = \left[ \frac{R_1 + (R_2||\Gamma_{m1})}{R_1 + (R_2||\Gamma_{m1})} \right] \left[ \frac{(\frac{R_2||\Gamma_{m1}}{R_1 + (R_2||\Gamma_{m1})}) \cdot g_{m1}}{R_1 + (R_2||\Gamma_{m1})} \right]^{-1}
\]

\[
\text{Req3} = \left[ \frac{R_1 + R_2 + \Gamma_{m1}}{R_1 + R_2 + \Gamma_{m1}} \right] \left[ \frac{(\frac{\Gamma_{m1} \cdot g_{m1}}{R_1 + R_2 + \Gamma_{m1}})}{R_1 + R_2 + \Gamma_{m1}} \right] \left[ \frac{(\frac{R_2 + \Gamma_{m1}}{R_1 + R_2 + \Gamma_{m1}}) \cdot g_{m2}}{R_1 + R_2 + \Gamma_{m1}} \right]^{-1}
\]
Prob. 7
Find the input-to-output small-signal gain.

\[
\text{Gain} = \frac{\frac{I_{T1} + R_Z(1+\beta)}{R_1 + I_{T1} + R_Z(1+\beta)}}{R_3} \left( \frac{I_{T1} \cdot g_m}{I_{T1} + R_Z(1+\beta)} + g_m2 \right) \cdot R_3
\]