Prob-1. Determine in which of the two cases (a) $g_{mp} > g_{mn}$ (b) $g_{mp} < g_{mn}$ the circuit will be unstable and why? $g_{mp}$ and $g_{mn}$ are respective transconductances of MP and MN.

\[ L_1 = -\frac{g_{m1} R_2}{g_{m2}} \]
\[ L_2 = \frac{g_{m1} R_2}{g_{m2}} \]

($g_{mp} > g_{mn}$) yields net negative feedback loop $\rightarrow$ STABLE

($g_{mp} < g_{mn}$) yields net positive feedback loop $\rightarrow$ UNSTABLE

(only if magnitude of $\mathfrak{L}$ is greater than 1)
Prob-2. A second order system has an open loop transfer function $H(s) = \frac{2000}{(\frac{s}{10} + 1)(\frac{s}{p} + 1)}$ and feedback factor of $\beta$.

Determine the location of pole $p$ for which the loop phase margin is $45^\circ$ if (a) $\beta = \frac{1}{2}$ and (b) $\beta = \frac{1}{20}$. Show the bode plot of loop transfer function for both the cases.

- $45^\circ$ phase margin implies 2nd pole is located at 0 dB (opgain).

\[
L(s) = H(s) \cdot \beta
\]

\[
L(s) \bigg|_{\beta = \frac{1}{2}} = \frac{1000}{(1 + \frac{s}{10})(1 + \frac{s}{p})} \quad \rightarrow \quad \text{want } p = 10^4 \text{ for } 45^\circ \text{ PM.}
\]

\[
L(s) \bigg|_{\beta = \frac{1}{20}} = \frac{100}{(1 + \frac{s}{10})(1 + \frac{s}{p})} \quad \rightarrow \quad \text{want } p = 10^3 \text{ for } 45^\circ \text{ PM}
\]
Prob-3. Figure below shows a non-inverting amplifier achieved by connecting an opamp in negative feedback configuration. Assuming open loop gain of opamp $A = \infty$, the closed loop gain $\frac{V_{out}}{V_{in}}$ of the circuit is given by $1 + \frac{R1}{R2}$ which is simply $1/\beta$ where $\beta$ is the feedback factor $\frac{R2}{(R1+R2)}$. Find the expression for closed loop gain if $A$ is finite and determine % error in the closed loop gain for (a) $A=1000$ and (b) $A=10000$.

$$\beta = \frac{R2}{R1+R2}$$

$$\frac{V_{out}}{V_{in}} = 1 + \frac{A}{1+AB} = \frac{1}{\beta} = \frac{A\beta}{1+AB}$$

$$\text{Error} = 1 - \frac{A\beta}{1+AB} = \frac{1}{1+AB}$$
**Prob-4.** A system is said to be unstable if any of the poles lie in right half of the s-plane. This property can be used to convert a second order system into oscillator by using negative feedback and choosing a proper feedback function. Find the closed loop transfer function $V_{out}(s)/V_{in}(s)$ of the system and show how the location of poles change w.r.t $k$ in s-plane.

The diagram above shows the system configuration with

$$H(s) = \frac{A}{s^2 + \frac{w_n}{Q_0} s + w_n^2}$$

and the feedback function

$$\beta(s) = -k s$$

The closed loop transfer function is given by

$$\Delta(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{H(s)}{1 - H(s) \cdot k} = \frac{A}{s^2 + s \left( \frac{w_n}{Q_0} - A k \right) + w_n^2}$$

The characteristic equation is

$$s^2 + s \left( \frac{w_n}{Q_0} - A k \right) + w_n^2 = 0$$

Using the quadratic formula, we find the roots (poles) of the system.

$$s = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Where $a = 1$, $b = \frac{w_n}{Q_0} - A k$, and $c = w_n^2$.

For $b = 0$,

$$k = \frac{w_n}{A \alpha_0} \rightarrow \text{poles on jw-axis (all imaginary)}$$

For $b > 0$,

$$k < \frac{w_n}{A \alpha_0} \rightarrow \text{STABLE} \ (\text{poles in left half plane})$$

For $b < 0$,

$$k > \frac{w_n}{A \alpha_0} \rightarrow \text{UNSTABLE}$$
Prob-5. A common source amplifier with DC gain \( g_m R \) and pole location \( w_p = 1/RC \) is connected in negative feedback configuration as shown in figure below. Determine the closed loop transfer function \( V_{out}(s)/V_{in}(s) \), closed loop DC gain and closed loop -3dB bandwidth of the circuit. Show the bode plot for gain and phase for (a) \( \beta = 0 \) (b) \( \beta = 1 \) and (c) \( \beta = 1/2 \) and compare the DC gain and Bandwidth of the closed system with that of open loop gain and bandwidth of common source amplifier. Consider \( r_o = \infty \).

When \( \beta = 0 \), \( A(s) \) is same as open loop.

For \( \beta = \frac{1}{2} \), \( \beta = 1 \),

\( A(s) \) sees decrease in gain and increase in BW, by \( 1 + g_m R \beta \).

[Diagram of the circuit is shown with annotations for calculations and equations.]

\[
A(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R}{1 + g_m R \beta} \frac{1}{1 + g_m R \frac{1}{1 + sRC} \beta} = \frac{-g_m R}{1 + g_m R \beta}
\]

Gain = \( -\frac{g_m R}{1 + g_m R \beta} \)

\( BW = \frac{1}{RC} (1 + g_m R \beta) \)
**Prob-6** For the circuit shown below, considering each pole causes a 3dB drop in gain at pole frequency, find (a) minimum gain $k$ if $R=1k\Omega$ and (b) minimum value of $R$ if $k=0.25$ for which the circuit will oscillate and determine the expression for frequency of oscillation in both the cases. Consider $r_o=\infty$ and $g_m=1m\ A/V$ for all the PMOS.

Each stage to contribute 45° phase delay results in positive FB loop. 45° is at pole frequency, where gain is reduced by $\sqrt{2}$ (3dB).

Loggain at that frequency $= \frac{k\left(\frac{g_m R}{\sqrt{2}}\right)^4}{(\sqrt{2})^4}$

\( (R=1k\Omega) \rightarrow \text{want loggain } k\left(\frac{g_m R}{\sqrt{2}}\right)^4 > 1 \rightarrow k > \left(\frac{\sqrt{2}}{g_m R}\right)^4 = 4 \)

\( (k=0.25) \rightarrow k\left(\frac{g_m R}{\sqrt{2}}\right)^4 > 1 \rightarrow R > \left(\frac{\sqrt{2}}{g_m}\right) \cdot k^{1/4} = 2k\Omega \)
Prob-7. Design a CMOS realization for the following Boolean expressions

(a) 

<table>
<thead>
<tr>
<th>Select (S)</th>
<th>Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2-input NAND gate</td>
</tr>
<tr>
<td>1</td>
<td>2-input NOR gate</td>
</tr>
</tbody>
</table>

(b) \( Y = (A+B)\cdot C + D\cdot E \)

(c) \( A\cdot B + B\cdot C + C\cdot A \)

\[ Y = S \cdot (A+B) + \overline{S} \cdot (\overline{A+B}) \]

\[ (c) \quad (\overline{A+B}) + B\cdot C + C\cdot A = (\overline{A+B}) \cdot (B\cdot C + C\cdot A) \]