Mismatch-shaping switching for two-capacitor DAC

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A mismatch-shaping scheme is proposed for a two-capacitor digital-to-analogue converter (DAC). It uses a delta-sigma loop for finding the optimal switching sequence for each input word. Simulations indicate that the scheme can be used for the realisation of DACs with 16-bit linearity and SNR performance.

Introduction: A very economical digital-to-analogue converter (DAC) containing only two capacitors, a reference voltage source and a few switches (Fig. 1) was described by Suarez et al. [1]. It functions by charging \( C_1 \) to \( V_c \), or 0 depending on the incoming bits (starting with the least significant bit (LSB), and repeatedly sharing the charge between \( C_1 \) and \( C_2 \). It is somewhat slow; it requires \( N \) clock periods to convert an \( N \) bit digital word into an analogue voltage. However, it requires only a small chip area and little DC power, and is fully compatible with CMOS technology.

A major disadvantage of the circuit is that its linearity is limited by the matching of the two capacitors, which (even for careful layout) cannot be much better than 0.1%, corresponding to only about 12bit linearity. Recently, several papers [2, 3] have discussed methods for eliminating or reducing this nonlinearity, by using sophisticated algorithms for the operation of the switches so that \( C_1 \) and \( C_2 \) can change roles in every clock cycle, and by duplicating the DAC and combining the two resulting analogue outputs. These algorithms, however, require very complex logic and introduce new practical problems associated with the precise addition of the analogue outputs.

Fig. 1 Basic two-capacitor DAC topology

This Letter proposes a different approach which randomises the DAC error caused by mismatch, and performs a highpass filtering of the resulting noise. Thus, for oversampled operation, the inband portion of the error spectrum is suppressed. The technique does not require duplication of the analogue part of the DAC, and the digital correction system needed is much simpler than those used in the earlier schemes [2, 3].

Proposed system: The block diagram of the proposed system is shown in Fig. 2. The upper part of the Figure, \( x(n, k) \) denotes the \( k \)th bit of the \( n \)th input word \( x(n) \), \( y(n) \) is the corresponding analogue output sample, \( V_c \), \( \epsilon(n) \) is the error in \( y(n) \), and \( (n, k) = \pm 1 \) controls the choice of \( C_1 \) or \( C_2 \) when \( x(n, k) \) is converted. In the lower part, \( \epsilon(n) = 0 \) is the desired average value of \( \epsilon(n) \), \( \epsilon(n) \) is the output of the digital lowpass filter \( H(z) \), \( u(n) \) is the \( n \)th word containing the signs \( (n, k) \), and \( \epsilon(n) \) is the computed value of the error \( \epsilon(n) \) in \( y(n) \). The P/S blocks perform parallel-to-serial conversion, and the T blocks truncate the data. The digital delays needed for timing are ignored for simplicity in Fig. 2.

Fig. 2 Proposed mismatch-shaping system

Similarly, the lower part is a delta-sigma loop which attempts to keep the average value of \( \epsilon(n) \) equal to \( \epsilon(n) = 0 \) over a large range of \( n \) values. More specifically, the loop forces the \( z \)-trans-
Symmetry conditions of Boolean functions in complex Hadamard transform

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A method to identify symmetries in Boolean functions using a complex Hadamard transform is proposed. It is shown that only the half spectrum is needed to detect symmetries in Boolean functions.

Introduction: Two spectral approaches, based on either the Reed-Müller or the Hadamard-Walsh transform, have been applied to the identification of Boolean symmetries [3]. Both of these spectral approaches use the arithmetic manipulation of subsets of spectral coefficients. In this Letter, the conditions for Boolean symmetries are given for complex Hadamard transforms that are based on the complex Boolean spectra and do not require any manipulation. The analysis is limited to all possible symmetries of degree 2 for any n-variable Boolean function, and the symbols used for different types of symmetries follow the notation from [3].

Property 1: If the basis transform matrix of the complex Hadamard transform is

\[ C = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \]

then the elements of the matrix are related by

\[ c_{(2^n-1-j,k)} = i^{-j}c_{jk} \]

where 2^n defines the order of the complex Hadamard matrix, \( \gamma = n \mod 4 \).

Definition 1: Let \( f(x^2) \) be a Boolean function of \( n \) variables, with \( x_i = \{x_0, x_1, ..., x_{n-1}\} \), \( x_i \in \{0, 1\} \), \( 1 \leq i \leq n \). The truth vector \( f \) can be mapped to an \( M \)-coded \( \tilde{F} \) with encoding \([0^0 \rightarrow 1^1] \) and \([1^1 \rightarrow -1^0] \). Theorem 1: Let the encoded truth vector \( \tilde{F} = [f_0, f_1, ..., f_{2^n-1}] \) have complex Hadamard spectra of \( \tilde{M} = [m_{00}, m_{01}, ..., m_{2^n-1,2^n-1}] \). Then,

\[ m_{2^n-1-j} = m_j \exp\left(1 - \gamma \frac{\pi}{2^n}\right) \]

where \( \gamma = n \mod 4 \).

Proof: Let \( f_j = f_k \) \((1 + i)\) and \( 0 \leq k \leq 2^n-1 \) where \( f_k \in \{1, -1\} \) represents the S-coded minterms of the Boolean function [3]. By definition 1, each of the elements in \( \tilde{M} \) is expressed as \( m_j = \sum_{k=0}^{2^n-1} c_{jk} f_k \), where the complex Hadamard matrix \( C = \{c_{jk} = c_{jk} + c_{jk}^*\} \) and \( 0 \leq j \leq 2^n-1 \). Then,

\[ m_j = \sum_{k=0}^{2^n-1} f_k [(a_{jk} - \beta_{jk}) + i(a_{jk} + \beta_{jk})] \]

\[ = \sum_{k=0}^{2^n-1} f_k(a_{jk} - \beta_{jk}) + i \sum_{k=0}^{2^n-1} f_k(a_{jk} + \beta_{jk}) \]

and from property 1,

\[ m_{2^n-1-j} = i^{-j} \sum_{k=0}^{2^n-1} f_k(a_{jk} - \beta_{jk}) + i \sum_{k=0}^{2^n-1} f_k(a_{jk} + \beta_{jk}) \]

By separating summation terms, eqn. 2 is proved.

Definition 2: The reverse operator \( \tilde{R} \) on either row vector or column vector is defined as reversing the positions of all its elements. For example, if \( \tilde{F} \) as is defined in definition 1, then \( \tilde{R}(\tilde{F}) = [f_{2^n-1}, f_{2^n-2}, ..., f_0]^T \).

Property 2: From theorem 1 and definition 2,

\[ \tilde{R}(\tilde{M}(\gamma)) = \exp\left(1 - \gamma \frac{\pi}{2^n}\right) \tilde{M}(\gamma) \]

and

\[ \tilde{R}(\tilde{M}(\gamma)) = \exp\left(1 - \gamma \frac{\pi}{2^n}\right) \tilde{M}(\gamma) \]

where the conjugate operation on a column vector is defined as converting every element in the column vector to its complex conjugate equivalent. The proof of property 2 is immediate from theorem 1.

Theorem 2: Let an n-variable Boolean function \( f(x^2) \) possess symmetries of degree 2 in \( E(x_0, x_1) \). Then, the complex spectral test for this symmetry is

\[ \tilde{M}(\gamma) = \tilde{R} \left( \exp\left(1 - \gamma \frac{\pi}{2^n}\right) \tilde{M}(\gamma) \right) \]