CS515: Algorithms and Data Structures, Fall 2014

Homework 1*

Due: Mon, Oct/13/14

Homework Policy: Students should work on homework assignments in group of preferably three people. Each group submits to TEACH one set of typeset solutions, and hands in a printed hard copy in the class. You are allowed to discuss the homework with other groups, however, you must mention their names in your submission. Also, you must cite any other source that you use.

The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.

Problem 1. Suppose you are given two sets of n points, one set $\{p_1, p_2, \ldots, p_n\}$ on the line $y = 0$ and the other set $\{q_1, q_2, \ldots, q_n\}$ on the line $y = 1$. Create a set of n line segments by connecting each point p_i to the corresponding point q_i. Suppose no three line segments intersect in a point.

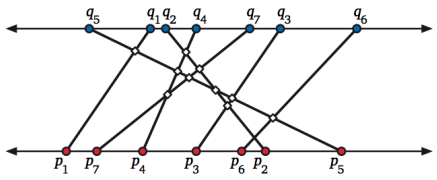

Figure 1: Eleven intersecting pairs of segments with endpoints on parallel lines.

(a) **[5 pts]** Describe and analyze an algorithm to determine how many pairs of these line segments intersect, in $O(n^2)$ time.

(b) **[20 pts]** Describe and analyze an algorithm to determine how many pairs of these line segments intersect, in $O(n \log n)$ time.

Problem 2. Suppose you are given a stack of n pancakes of different sizes. You want to sort the pancakes so that smaller pancakes are on top of larger pancakes. The only operation you can perform is a flip – insert a spatula under the top k pancakes, for some integer k between 1 and n, and flip them all over.

*Problems are from Jeff Erickson's lecture notes. Looking into similar problems from his lecture notes on recursion and dynamic programming is recommended.

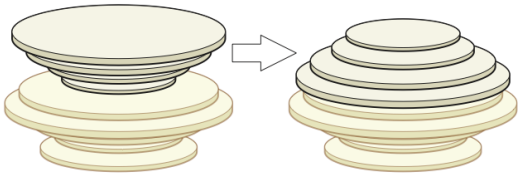

Figure 2: Flipping the top four pancakes.

(a) **[15 pts]** Describe an algorithm to sort an arbitrary stack of n pancakes using as few flips as possible. Exactly how many flips does your algorithm perform in the worst case?

(b) **[10 pts]** Now suppose one side of each pancake is burned. Describe an algorithm to sort an arbitrary stack of n pancakes, so that the burned side of every pancake is facing down, using as few flips as possible. Exactly how many flips does your algorithm perform in the worst case?

Problem 3. **[25 pts]** A palindrome is any string that is exactly the same as its reversal, like I, or DEED, or RACECAR, or AMANAPLANACATACANALPANAMA. Describe and analyze an algorithm to find the length of the longest subsequence of a given string that is also a palindrome. For example, the longest palindrome subsequence of MAHDYNAMICPROGRAMZLETMESHOWYOUTHEM is MHYMRORMYHM, so given that string as input, your algorithm should output the number 11.

Problem 4. **[25 pts]** Suppose we need to distribute a message to all the nodes in a rooted tree. Initially, only the root node knows the message. In a single round, any node that knows the message can forward it to at most one of its children. Design an algorithm to compute the minimum number of rounds required for the message to be delivered to all nodes.

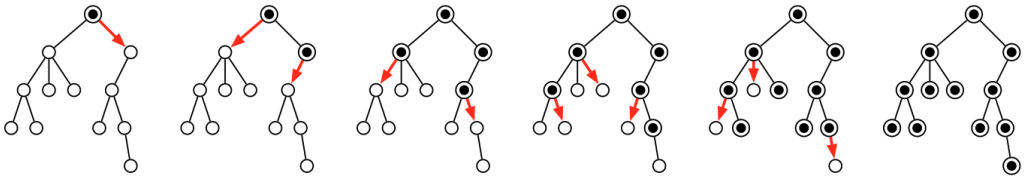

Figure 3: A message being distributed through a tree in five rounds.