CS515: Algorithms and Data Structures, Fall 2019

Homework 2*

Due: Tue, 11/19/19

Homework Policy:

1. Students should work on homework assignments in groups of preferably three people. Each group submits to TEACH one set of *typeset* solutions, and hands in a printed hard copy in class or slides the hard copy under my door before the midnight of the due day. The hard copy will be graded.
2. The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.
3. You are allowed to discuss the problems with other groups, and you are allowed to use other resources, but you *must* cite them. Also, you must write everything in your own words, copying verbatim is plagiarism.
4. *I don't know policy:* you may write "I don't know" *and nothing else* to answer a question and receive 25 percent of the total points for that problem whereas a completely wrong answer will receive zero.
5. Algorithms should be explained in plain english. Of course, you can use pseudocodes if it helps your explanation, but the grader will not try to understand a complicated pseudocode.
6. More items might be added to this list. ☺

Problem 1. **[33 pts]** Consider a treap T with n vertices.

(a) Prove that the expected number of proper descendants of any node in a treap is exactly equal to the expected depth of that node

(b) What is the expected number of nodes in T with two children?

(c) What is the probability that T has no nodes with two children?

(Hint: make sure that you understand the analysis of the treap lecture.)

Problem 2. **[33 pts]** Consider the following *randomized* algorithm for computing the kth smallest element of an array.

- Suppose that $\textsc{Random}(t)$ returns a random number from the discrete uniform distribution on $\{1, \dots, t\}$ in constant time.
- By partitioning A based on *pivot* we mean to partition A into $A[1 \dots \ell - 1]$, $A[\ell]$, and $A[\ell + 1 \dots n]$ such that (i) $A[\ell] = pivot$, (ii) $A[1 \dots \ell - 1] \leq pivot$, and (iii) $A[\ell + 1 \dots n] > pivot$.

```
SELECT(A[1...n], k)
    pivot = A[RANDOM(n)]
    partition A into A[1...ℓ-1], A[ℓ], and A[ℓ+1...n] based on pivot
    if ℓ = k then
      return A[ℓ]
    else
      if ℓ > k then
        return SELECT(A[1...ℓ], k)
      else
        return SELECT(A[ℓ+1...n], k - ℓ)
      end if
    end if
```

(a) What is the worst case running time of SELECT?

(b) What is the worst case expected running time of SELECT?

Problem 3. **[34 pts]** Let (S,T) and (S',T') be minimum (s,t)-cuts in some flow network G. Prove that $(S\cap S', T\cup T')$ and $(S\cup S', T\cap T')$ are also minimum (s,t)-cuts in G.

[*]Some of the problems are from the text book. Looking into similar problems from the book, chapters 10 and 11, as well as lecture notes on randomization is recommended.