CS420/520: Graph Theory with Applications to CS, Winter 2014

Homework 4

Due: Thr, March/13/14

Homework Policy: Each student should submit his/her own set of solutions, independently. You are allowed to discuss the homework with other students, however, you need to indicate their names in your submission. Also, you are allowed to use other sources, but you must cite every source that you use.

Problem 1. Let $G = (V, E)$ be an undirected unweighted graph, and let $s \in V$.

(a) Design and analyze an $O(m)$ time algorithm to compute the shortest cycle of G that contains s.

(b) Use part (a) to find an algorithm to compute the shortest cycle of G in $O(mn)$ time.

(c) Argue that the algorithm of part (b) has running time $O(n^2)$ if G is planar.

(d) Design and analyze a faster algorithm for the case that G is planar using the separator theorem.

problem 2. Let $G = (V, E)$ be a triangulated plane graph (i.e. G is embedded in the plane, and all its faces are triangles), and let F be the set of faces of G. Observe that each simple cycle in G splits the set of faces of G into two subsets: 1) $F_{in}(C)$ the faces inside C, and 2) $F_{out}(C)$ the faces outside C.

Suppose there is a vertex $s \in V$ and a positive integer $d \in \mathbb{Z}^+$ such that all vertices in V are within distance d of s. Prove there exists a cycle C^* of length at most $2d + 1$ that splits the faces into subsets of size at most $3/4|F|$; that is $F_{in}(C^*), F_{out}(C^*) \leq 3/4|F|$.

(**Hint**: For any tree $T = (V_T, E_T)$ of maximum degree 3, there is an edge $e \in E_T$ whose removal separates T into subtrees each of size at most $3/4|V_T|$.)