I. PROJECT PROPOSAL

The Nature exhibits many branching tree-like structures such as river networks, martian drainage basins, veins of botanical leaves, lung and blood systems, and lightening. Moreover, a number of dynamic processes can be represented as tree graphs. Such processes as spread of a disease or rumor, evolution of an earthquake aftershock sequence, or transfer of gene characteristics from parent to children also can be described by a tree. There are interesting characterizations of such trees, namely the Horton-Strahler and Tokunaga indexing schemes for tree branches.

The Horton-Strahler number of a mathematical tree is a numerical measure of its branching complexity. The Horton-Strahler indexing scheme assigns orders to the tree branches according to their relative importance in the hierarchy. In other words, for a rooted tree T consider the operation of pruning. Consecutive application of pruning eliminates any finite tree in a finite number of steps. A vertex in T is assigned order r if it is removed from the tree during the r-th application of pruning. A branch is a sequence of adjacent vertices with the same order.

A Tokunaga ordering addresses so-called side branching (merging of branches of distinct orders). A Tokunaga matrix is constructed such that each element T_ij is an average number of tributaries of order i merging into order j. Informally, the Tokunaga self-similarity implies that different levels of a hierarchical system have the same statistical structure. Formally, the Tokunaga self-similarity is characterized by special form of matrix T.

These schemes were introduced in hydrology in the middle of twenties century in order to describe the dendritic structure of river networks. After they have been rediscovered those schemes were used in other applied fields. There is an interesting phenomenon: a majority of rigorously studied branching structures is shown to be closely approximated by a simple two-parametric statistical model, a Tokunaga self-similar (TSS) tree. In other words, apparently diverse branching phenomena are statistically similar to each other, with the observed differences being related to the value of a particular model parameter rather than qualitative structural traits.

In the area of applied mathematics there are the following open questions associated with the Tokunaga and Horton-Strahler indexing schemes:

- Given a Tokunaga matrix and the Horton-Strahler number, calculate how many binary trees are there;
- Given only the Horton-Strahler number, calculate how many binary trees are there;
- Given a self-similar Tokunaga matrix, calculate how many binary trees are there;
- How much information about a tree is contained in Tokunaga matrix and HortonStrahler numbers.

The objective of this project is:

• Conduct a literature review on current topic, especially on its applications to computer science (e.g., Ershov, A. P. (1958), ”On programming of arithmetic operations”);

• Attempt to answer at least one of the questions listed above.

Reference:

- I. Zaliapin and Y. Kovchegov, Tokunaga and Horton self-similarity for level set trees of Markov chains, Chaos, Solitons and Fractals.
- Y. Kovchegov and I. Zaliapin, Horton self-similarity of Kingmans coalescent tree, Arxive.
- G. A. Burd, E.C. Waymire, R.D. Winn, A self-similar invariance of critical binary Galton-Watson trees, Bernoulli