CS 520 Winter 2015
Project Proposal

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My project proposal is on the stack number and queue number of graphs, which I found on the Open Problems Project website linked from the course web page. Yannakakis has shown that stack numbers of planar graphs are bounded above by four [1] but it is not known if queue numbers of planar graphs are bounded.

Let $G = (V, E)$ be a (simple) undirected graph. Now, let $\leq$ be a total order on V. If v and w are vertices with $v < w$, then we will denote an edge with endpoints v and w by vw. If xy and vw are edges with $x < v < y < w$, then we will say that xy and vw cross. A stack layout of G is a total order on V and a partition of E into stacks of non-crossing edges. The stack number of G is the minimum number of stacks in a stack layout of G. If xy and vw are edges with $v < x < y < w$, then we will say that xy is nested in vw. A queue layout of G is a total order on V and a partition of E into queues of non-nested edges. The queue number of G is the minimum number of queues in a queue layout of G.

Consider the following example. Let G be the complete graph with four vertices labeled a, b, c, d. A stack layout of G is given by the alphabetical ordering a, b, c, d and the partition into two stacks ab, ac, ad and bc, bd, cd. This means the stack number is at most two. Since the graph is complete and the edges ac and bd cross, it follows that there is no stack layout with one stack. Thus, the stack number is exactly two. A queue layout is given by the alphabetical ordering a, b, c, d and the partition into two queues ab, ac, ad and bc, bd, cd. This means the queue number is at most two. Since the graph is complete and the edge bc is nested in the edge ad, it follows that there is no queue layout with one queue. Thus, the queue number is exactly two.

References

[1] Mihalis Yannakakis. Embedding planar graphs in four pages. *Journal of Computer and System Sciences*, 38:36–67, 1989.