Godfrey Yeung
Dr. Nayyeri
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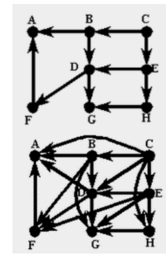
Project Proposal: Existence of a Doubled Out-Degree in any Squared Oriented Graphs – A Proof

Project Goal:
Prove <u>Seymore's conjecture:</u> Any oriented graph *O* has a node v_i *with* $d_{(o^\wedge 2)}(v_i) >= 2d_o(v_i)$.

Where $d_o(v_i)$ represents the outdegree of v_i in an oriented graph and $d_{(o^\wedge 2)}(v_i)$ represents its outdegree in the squared oriented graph. Note that $d_{(o^\wedge 2)}(v_i) >= 2d_o(v_i)$ holds if $d_o(v_i) = 0$, since 2*0 = 0.

Definitions:
<u>Oriented graph:</u> a simple directed graph (no multiple arcs between two vertices).

<u>The square</u> of an oriented graph is acquired by taking the oriented graph and adding arbitrary arc (u,w), if it does not exist, for each pair of arcs (u,v), (v,w).



Background (Why it is interesting):
Indexes to rank a team's performance (in sports or other competitive activities) often rely on what other teams a team has beaten. Yet the index may be improved by also considering what teams ***B*** a team has "indirectly beaten" by beating a team that beat those teams ϵ **B**. This is also interesting qualitative data for a team to assess themselves.

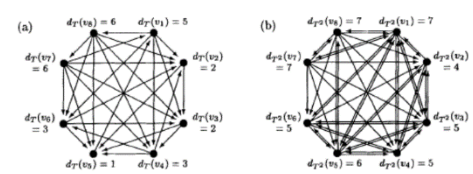
A guarantee that there will always an occurrences of "indirect winnings" in any competition would give strong reason to incorporate "indirect winnings" in competition indexes. The logical question of interest is, "can it be proved that 'indirect winnings' occur in any type of competition?"

This question can be answered by representing a competition as an <u>oriented graph</u>. Each vertex represents a team and each arc from *u* to *v*, denoted (*u,v*), represents the fact that team *u* beat team *v*.

<u>The square</u> of an oriented graph then captures "indirect winnings" of teams. The newly added arcs (*u,w*) now represents the fact that team *u* indirectly beat team *w*.

The above question can thus be answered "yes" by proving Seymore's conjecture.

Existing Work:
It turns out that the property (there exists a node whose outdegree in the square is twice that in the original oriented graph) has been proven to hold in all complete oriented graphs, called <u>tournaments.</u> In other words, there is a graph theory proof that "indirect winnings" always occur in round-robin tournament competitions, where every team plays every other team exactly once. The statement of this property is called <u>*Dean's Conjecture*</u>, which has been proven by David Fisher. He built off previous proofs that the property holds for regular and irregular tournaments, and those with a minimum outdegree of five or less.



Further considerations:
Note that "no winnings" are also considered "indirect winnings". However, "no winnings" is also meaningful data, so the proof is still of interest. Perhaps, alongside the proof, the project can also formulate an algorithm that finds the nodes whose outdegrees are doubled in the square, and then distinguish which of those are "indirect winnings" and which are "no winnings".