CS420/520: Graph Theory with Applications to CS, Winter 2017

Homework 5

Due: Tue, Feb/21/17

Homework Policy:

1. Students should work on homework assignments in groups of preferably three people. Each group submits to TEACH one set of typeset solutions, and hands in a printed hard copy in class or slides the hard copy under my door before the midnight of the due day. The hard copy will be graded.
2. The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.
3. You are allowed to discuss the problems with other groups, and you are allowed to use other resources, but you *must* cite them. Also, you must write everything in your own words, copying verbatim is plagiarism.
4. *I don't know policy:* you may write "I don't know" *and nothing else* to answer a question and receive 25 percent of the total points for that problem whereas a completely wrong answer will receive zero.
5. Algorithms should be explained in plain english. Of course, you can use pseudocodes if it helps your explanation, but the grader will not try to understand a complicated pseudocode.

Readings:

(A) Uri Zwick's lecture notes on all pairs shortest paths: http://www.cs.tau.ac.il/~zwick/grad-algo-13/match.pdf.

Problem 1.

(A) A matching is maximal if it does not leave any free edge. Let M be a maximal matching and M^* be a maximum matching. Prove that $|M| \geq \frac{1}{2} \cdot |M^*|$. Conclude an $O(E)$ time 2-approximation algorithm for computing the maximum matching, an $O(E)$ time algorithm that computes a matching of size at least 1/2 of the maximum matching.

(B) Suppose the degree of all vertices is smaller than a constant Δ. Design an $O(V)$ time algorithm to find a matching M' such that $|M'| \geq \frac{2}{3}|M^*|$. What is the running time of your algorithm as a function of Δ and V?

(C) Let k be a positive integer. Modify your algorithm to find a matching M'' such that $|M''| \geq \frac{k}{k+1} \cdot |M^*|$. What is the running time of your algorithm as a function of Δ and V?

Problem 2. Show that a graph is Bipartite if and only if it does not contain any cycle of odd length (Hint: Use BFS layers.)

Problem 3. Let $G = (V, E)$ be a graph and let M be a matching in G. Suppose that v is unmatched by M and that there is no augmenting path with respect to M that starts at v. Show that there exists a maximum matching M^* in which v is unmatched.