CS420/520: Graph Theory with Applications to CS, Winter 2017

Homework 6

Due: Thr, March/16/17

Homework Policy:

1. Students should work on homework assignments in groups of preferably three people. Each group submits to TEACH one set of typeset solutions, and hands in a printed hard copy in class or slides the hard copy under my door before the midnight of the due day. The hard copy will be graded.
2. The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.
3. You are allowed to discuss the problems with other groups, and you are allowed to use other resources, but you *must* cite them. Also, you must write everything in your own words, copying verbatim is plagiarism.
4. *I don't know policy:* you may write "I don't know" *and nothing else* to answer a question and receive 25 percent of the total points for that problem whereas a completely wrong answer will receive zero.
5. Algorithms should be explained in plain english. Of course, you can use pseudocodes if it helps your explanation, but the grader will not try to understand a complicated pseudocode.

Readings:

(A) The Wikipedia page in TSP: "https://en.wikipedia.org/wiki/Travelling_salesman_problem".

Problem 1. The Christofides heuristic approximates (Metric) TSP within a factor of 3/2 (see lectures). Give an example that shows that this bound cannot be improved, i. e., construct instances of (Metric) TSP (for general vertex numbers) such that Christofides yields a solution that asymptotically achieves this approximation ratio.

Problem 2. Consider the variant of the Metric TSP problem in which the object is to find a walk with given endpoints containing all the vertices of the graph.

(a) Design a 3-approximation algorithm for this problem.

(b) Design a 5/3-approximation algorithm for this problem.

Problem 3. Let $G = (V, E)$ be a complete undirected graph where the edge lengths $w(e)$ for every $e \in E$ are elements of $\{1, 2\}$. This graph satisfies clearly the triangle inequality. Give a 4/3 factor approximation algorithm for TSP in this special class of graphs. (Hint: Start by finding a minimum 2-matching in G, then patch cycles together. A 2-matching is a subset S of edges so that every vertex has exactly 2 edges of S incident at it.)

Problem 4. The Steiner tree problem is as follows. Given $G = (V, E)$ with positive edge weights, and whose vertices are partitioned into two sets R (required) and S (Steiner), find a minimum cost tree in G that contains all required vertices. Design a 2-approximation algorithm for the Steiner tree problem.