Nonlinear Computational Methods for Simulating Interactions of Radiation with Matter in Physical Systems

Dmitriy Y. Anistratov
Department of Nuclear Engineering
North Carolina State University

NE Seminar, Oregon State University
October 14, 2008

Introduction

- The simulation of interaction of particles with matter is a challenging problem.
- The transport equation is a basis for mathematical models of this physical phenomenon.
- Transport problems are difficult to solve
  - high dimensionality,
  - an integro-differential equation,
  - coefficients depend on the state of matter $\Longleftrightarrow$ the state of matter is affected by fluxes of particles.
- Applications:
  - reactor physics,
  - astrophysics (stars),
  - plasma physics (laser fusion),
  - atmospheric sciences.

anistratov@ncsu.edu

Introduction

- There exists a family of efficient nonlinear methods for solving the transport equation:
  Quasidiffusion method (V. Gol'din, 1964),
  Nonlinear Diffusion Acceleration (K. Smith, 2002),
  Flux methods (T. Germogenova, V. Gol'din, 1969).
- These methods are defined by a system of nonlinearly coupled high-order and low-order problems that is equivalent to the original linear transport problem.
- The Nonlinear Projective Iteration (NPI) methods possess certain advantages for their use in multiphysics applications.
- The low-order equations of NPI methods can be used to formulate approximate particle transport models.
- These low-order equations can also be utilized as a basis for development of hybrid Monte Carlo computational methods.
- The NPI methods are distinct from each other by the definition of the low-order equations which gives rise to differences in features of these methods.

Introduction

- Outline
  - A Tutorial on nonlinear methods for solving the transport equation
  - The Quasidiffusion (QD) method for transport problems in 2D Cartesian geometry on grids composed of arbitrary quadrilaterals
  - Nonlinear Weighted Flux (NWF) methods for particle transport problems in 2D Cartesian geometry on orthogonal grid
- This work was performed in collaboration with my Ph.D. students at NCSU
  - William A. Wieselquist (to graduate in 2008 $\rightarrow$ Paul Scherrer Institute, Switzerland)
  - Loren Roberts (graduated in 2008 $\rightarrow$ Baker Hughes Inc.)

Transport Problem

• Let us consider the following single-group slab geometry transport problem with isotropic scattering and source:

$$\mu \frac{\partial}{\partial x} \psi(x,\mu) + \Sigma_t(x)\psi(x,\mu) = \frac{1}{2}\left(\Sigma_s(x) \int\limits_{-1}^{1} \psi(x,\mu')d\mu' + Q(x) \right) ,$$

$$-1 \le \mu \le 1 , \quad 0 \le x \le L ,$$

$$\phi(x) = \int\limits_{-1}^{1} \psi(x,\mu)d\mu , \quad J(x) = \int\limits_{-1}^{1} \mu\psi(x,\mu)d\mu ,$$

with reflective left boundary

$$\psi(0,\mu) = \psi(0,-\mu) , \quad 0 < \mu \le 1 ,$$

and vacuum boundary condition on the right

$$\psi(L,\mu) = 0 , \quad -1 \le \mu < 0 .$$

anistratov@ncsu.edu

Low-Order Equations

- The transport equation is integrated over $-1 \leq \mu \leq 1$ with weights 1 and μ

$$\frac{dJ}{dx} + (\Sigma_t - \Sigma_s)\phi = Q\,,$$

$$\frac{d}{dx}\left(\int_{-1}^{1} \mu^2 \psi d\mu\right) + \Sigma_t J = 0\,,$$

where

$$\phi(x) = \int_{-1}^{1} \psi(x,\mu) d\mu\,, \quad J(x) = \int_{-1}^{1} \mu \psi(x,\mu) d\mu\,.$$

- What should we do with the extra moment?

$$\int_{-1}^{1} \mu^2 \psi d\mu = ?$$

anistratov@ncsu.edu

Approximate Closure: Diffusion Theory

- Diffusion theory (approximation)

$$\psi(x,\mu) = \frac{1}{2}(\phi(x) + 3\mu J(x)) \, ,$$

$$\int_{-1}^{1} \mu^2 \psi d\mu = \frac{1}{3}\phi(x) \, .$$

- P_1 equations

$$\frac{dJ}{dx} + (\Sigma_t - \Sigma_s)\phi = Q \ ,$$

$$\frac{1}{3}\frac{d\phi}{dx} + \Sigma_t J = 0 \ ,$$

- Diffusion equation

$$-\frac{d}{dx}\frac{1}{3\Sigma_t}\frac{d\phi}{dx} + (\Sigma_t - \Sigma_s)\phi = Q \ ,$$

anistratov@ncsu.edu

Approximate Closure: Variable Eddington Factor Method

$$\int_{-1}^{1} \mu^2 \psi d\mu \approx \mathcal{F}(x)\phi(x)$$

- Minerbo closure (approximation)

$$\psi(x,\mu) = \alpha(x)e^{\mu\beta(x)} \qquad \mathcal{F}(x) = 1 - \frac{2}{Z(x)}\frac{|J(x)|}{\phi(x)} \,,$$

- Kershaw closure (approximation)

$$\mathcal{F}(x) = \frac{1}{3}\left(1 + 2\left(\frac{|J(x)|}{\phi(x)}\right)^2\right)$$

- Levermore-Pomraning closure (approximation)

$$\mathcal{F}(x) = \frac{|J(x)|}{\phi(x)} coth(Z(x)) \,, \qquad coth(Z) - \frac{1}{Z} = \frac{|J|}{\phi}$$

- P_1-like nonlinear low-order equations

$$\frac{dJ}{dx} + (\Sigma_t - \Sigma_s)\phi = Q \ ,$$

$$\frac{d}{dx}(\mathcal{F}\phi) + \Sigma_t J = 0 \ , \qquad \mathcal{F} = \mathcal{F}(\phi, J)$$

Quasidiffusion (QD) Method: Exact Closure

- The Quasidiffusion method (exact)

$$\int_{-1}^{1} \mu^2 \psi d\mu = \left[\frac{\int_{-1}^{1} \mu^2 \psi d\mu}{\int_{-1}^{1} \psi d\mu} \right] \int_{-1}^{1} \psi d\mu = E(x)\phi(x) \,,$$

- The quasidiffusion factor.

$$E(x) = \frac{\int_{-1}^{1} \mu^2 \psi d\mu}{\int_{-1}^{1} \psi d\mu} = < \mu^2 >$$

- Quasidiffusion low-order equations

$$\frac{dJ}{dx} + (\Sigma_t - \Sigma_s)\phi = Q \;,$$

$$\frac{dE\phi}{dx} + \Sigma_t J = 0 \;, \qquad E = E[\psi] \,.$$

anistratov@ncsu.edu

System of Equations of the Quasidiffusion (QD) Method

- The transport (high-order) equation

$$\mu\frac{\partial}{\partial x}\psi(x,\mu)+\Sigma_t\psi(x,\mu)=\frac{1}{2}\big(\Sigma_s\phi(x)+Q\big)\,.$$

- QD (variable Eddington) factors

$$E(x)=\frac{\int\limits_{-1}^{1}\mu^2\psi(x,\mu)d\mu}{\int\limits_{-1}^{1}\psi(x,\mu)d\mu}\,,\qquad C_L=\frac{\int\limits_{-1}^{0}\mu\psi(L,\mu)d\mu}{\int\limits_{-1}^{0}\psi(L,\mu)d\mu}\,.$$

- the low-order QD equations

$$\frac{d}{dx}J(x)+(\Sigma_t-\Sigma_s)\phi(x)=Q\,,$$

$$\frac{d}{dx}\Big(E(x)\phi(x)\Big)+\Sigma_tJ(x)=0\,.$$

$$J(0)=0\,,$$

$$J(L)=C_L\phi(L)\,.$$

anistratov@ncsu.edu

QD Method: Iteration Process

- High-order problem (transport sweep)

$$\mu \frac{\partial}{\partial x} \psi^{(s+1/2)} + \Sigma_t \psi^{(s+1/2)} = \frac{1}{2} (\Sigma_s \phi^{(s)} + Q) \ ,$$

- Calculation of QD factors

$$E^{(s+1/2)}(x) = \frac{\int_{-1}^{1} \mu^2 \psi^{(s+1/2)}(x,\mu) d\mu}{\int_{-1}^{1} \psi^{(s+1/2)}(x,\mu) d\mu} \ , \qquad C_L = \frac{\int\limits_{-1}^{0} \mu \psi^{(s+1/2)}(L,\mu) d\mu}{\int\limits_{-1}^{0} \psi^{(s+1/2)}(L,\mu) d\mu} \ .$$

- Low-order QD problem

$$\frac{d}{dx} J^{(s+1)} + (\Sigma_t - \Sigma_s) \phi^{(s+1)} = Q \ ,$$

$$\frac{d}{dx} \left(E^{(s+1/2)} \phi^{(s+1)} \right) + \Sigma_t J^{(s+1)} = 0 \ ,$$

$$J^{(s+1)}(0) = 0 \, ,$$

$$J^{(s+1)}(L) = C_L^{(s+1/2)} \phi^{(s+1)}(L) \, .$$

anistratov@ncsu.edu

Nonlinear Diffusion Acceleration (NDA)

(Kord Smith)

- High-order problem (transport sweep)

$$\mu\frac{\partial}{\partial x}\psi^{(s+1/2)} + \Sigma_t \psi^{(s+1/2)} = \frac{1}{2}(\Sigma_s \phi^{(s)} + Q) \ ,$$

- Calculation of $\tilde{\mathcal{D}}^{(s+1/2)}$

$$J^{(s+1/2)}(x) = \int_{-1}^{1} \mu \psi^{(s+1/2)}(x,\mu) d\mu \, , \quad \phi^{(s+1/2)}(x) = \int_{-1}^{1} \psi^{(s+1/2)}(x,\mu) d\mu \ ,$$

$$\tilde{\mathcal{D}}^{(s+1/2)} = -\frac{1}{\phi^{(s+1/2)}}\left(J^{(s+1/2)} + \frac{1}{3\Sigma_t}\frac{d\phi^{(s+1/2)}}{dx} \right) .$$

- Low-order NDA problem

$$\frac{d}{dx}J^{(s+1)} + (\Sigma_t - \Sigma_s)\phi^{(s+1)} = Q \ ,$$

$$\frac{1}{3\Sigma_t}\frac{d\phi^{(s+1)}}{dx} + \tilde{\mathcal{D}}^{(s+1/2)}\phi^{(s+1)} + J^{(s+1)} = 0 \ ,$$

QD Method for the Multidimensional Transport Equation

- The transport (high-order) equation

$$\vec{\Omega} \cdot \vec{\nabla} \psi^{(k+1/2)}(\vec{r}, \vec{\Omega}) + \sigma_t(\vec{r}) \psi^{(k+1/2)}(\vec{r}, \vec{\Omega}) = \frac{1}{4\pi} \sigma_s(\vec{r}) \phi^{(k)}(\vec{r}) + \frac{1}{4\pi} q(\vec{r}) \,.$$

- The QD (Eddington) tensor

$$\bar{\bar{E}} = \begin{pmatrix} E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{pmatrix} ,$$

$$E_{\alpha\beta}^{(k+1/2)}(\vec{r}) = \int_{4\pi} \Omega_\alpha \Omega_\beta \psi^{(k+1/2)}(\vec{r}, \vec{\Omega}) d\vec{\Omega} \Bigg/ \int_{4\pi} \psi^{(k+1/2)}(\vec{r}, \vec{\Omega}) d\vec{\Omega} \,, \ \ \alpha, \beta = x, y \,,$$

$$E_{\alpha\beta}(\vec{r}) = \text{average value of } \Omega_\alpha \Omega_\beta \,.$$

- The low-order QD (LOQD) equations

$$\vec{\nabla} \cdot \vec{J}^{(k+1)}(\vec{r}) + \sigma_a(\vec{r}) \phi^{(k+1)}(\vec{r}) = q(\vec{r}) \,,$$

$$\vec{J}^{(k+1)}(\vec{r}) = -\frac{1}{\sigma_t(\vec{r})} \vec{\nabla} \cdot \left(\bar{\bar{E}}^{(k+1/2)}(\vec{r}) \phi^{k+1}(\vec{r}) \right) \,.$$

anistratov@ncsu.edu

Discretization of the LOQD Equations

- The unknowns: ϕ_i, $\phi_{i\omega}$, and $J_{i\omega} = \vec{J}_{i\omega} \cdot \vec{n}_{i\omega}$,
- The balance equation in ith cell

$$\sum_{\omega} \vec{J}_{i\omega} \cdot \vec{A}_{i\omega} + \sigma_{a,i}\phi_i V_i = q_i V_i \,, \quad \text{where } \omega - \text{cell face}\,.$$

- Jim Morel (2002) proposed a cell-centered discretization for the diffusion equation on meshes of arbitrary polyhedrons.

$$\{\vec{\nabla}\phi\}_{i\omega} = (\phi_{i\omega} - \phi_i)\vec{\alpha}_{i\omega} + \frac{1}{V_i}\sum_{\omega'}(\phi_{i\omega'})\vec{\beta}_{i\omega\omega'} \,,$$

where $\vec{\alpha}_{i\omega}$ and $\vec{\beta}_{i\omega\omega'}$ are based on geometry.

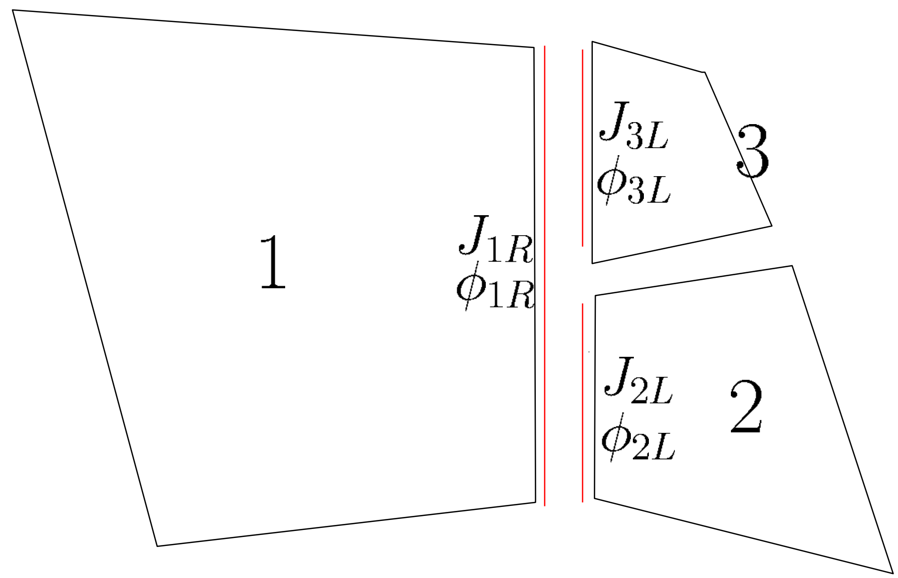
- We apply Morel's method

$$\vec{J}_{i\omega} = -\frac{1}{\sigma_{t,i}}\left\{\vec{\nabla}\cdot(\bar{\bar{E}}\phi)\right\}_{i\omega} \,.$$

$$\left\{\vec{\nabla}\cdot(\bar{\bar{E}}\phi)\right\}_{i\omega} = \vec{e}_x\left(\left\{\frac{\partial E_{xx}\phi}{\partial x}\right\}_{i\omega} + \left\{\frac{\partial E_{xy}\phi}{\partial y}\right\}_{i\omega}\right) + \vec{e}_y\left(\left\{\frac{\partial E_{xy}\phi}{\partial x}\right\}_{i\omega} + \left\{\frac{\partial E_{yy}\phi}{\partial y}\right\}_{i\omega}\right) \,.$$

Discretization of the LOQD Equations

- Conditions at interfaces between cells



- Strong current and weak scalar flux continuity conditions

$$
\begin{aligned}
J_R &= -J_{2L} \\
J_R &= -J_{3L} \\
\phi_R A_R &= \phi_{2L} A_{2L} + \phi_{3L} A_{3L}
\end{aligned}
$$

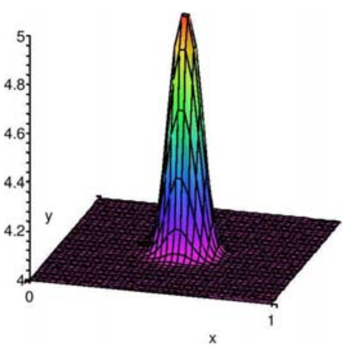
anistratov@ncsu.edu

Analytic Test Problem for the LOQD Equations

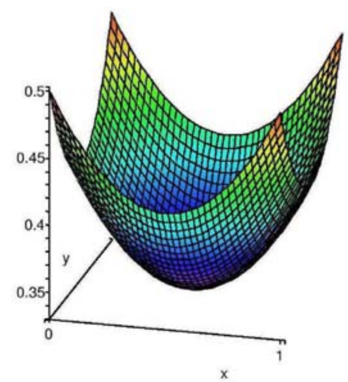
- $0 \leq x \leq 1, \;\; 0 \leq y \leq 1.$ $\sigma_t = 1$, $\sigma_a = 0.5$.
- Analytic solution

$$\phi(x,y) = 5 - \tanh\left[100\left(x - \frac{1}{2}\right)^2 + 100\left(y - \frac{1}{2}\right)^2\right]$$

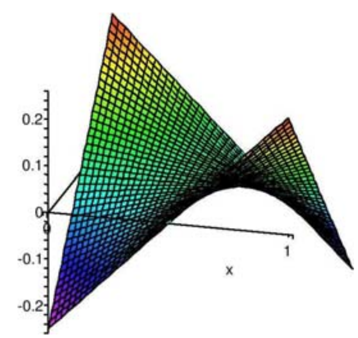
- The QD tensor ($E_{\alpha\beta}(x,y)$) is given in analytic form



$\phi(x,y)$



$E_{xx}(x,y)$

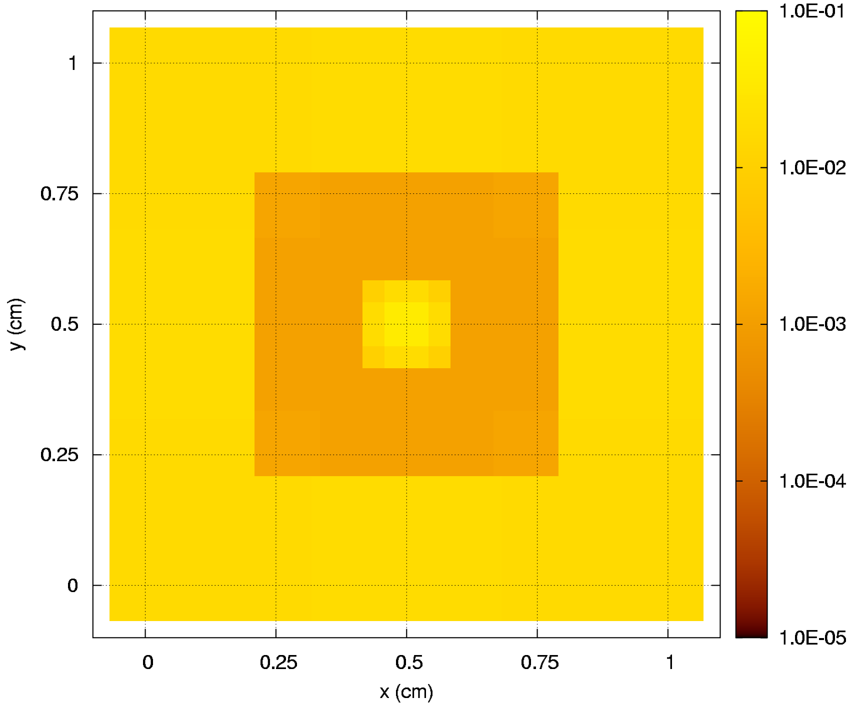


$E_{xy}(x,y)$

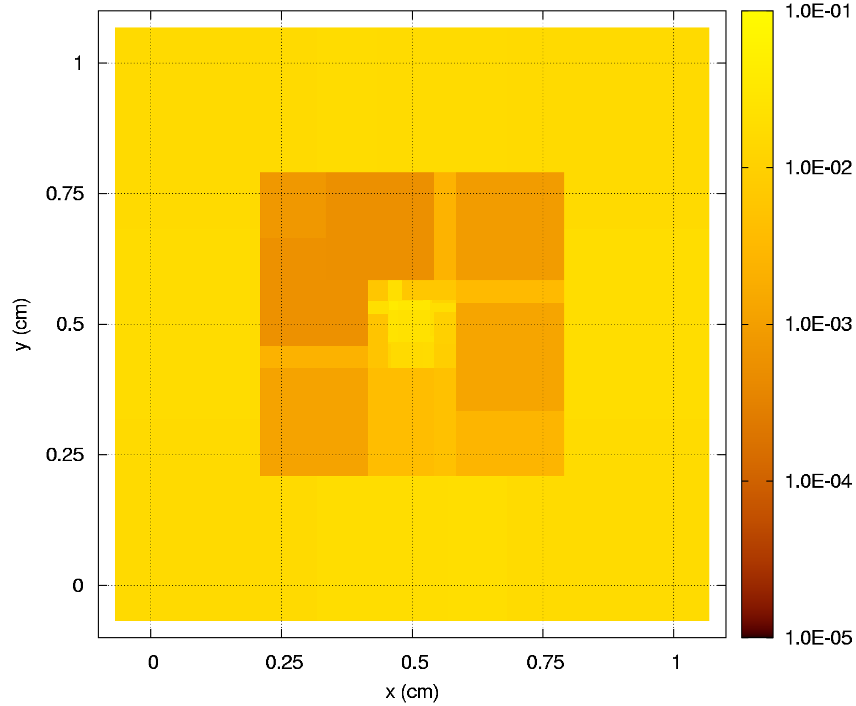
anistratov@ncsu.edu

Results on 3-Level Grids

- Relative error in the cell-average scalar flux
- Grid 1: 40 cells



orthogonal grid

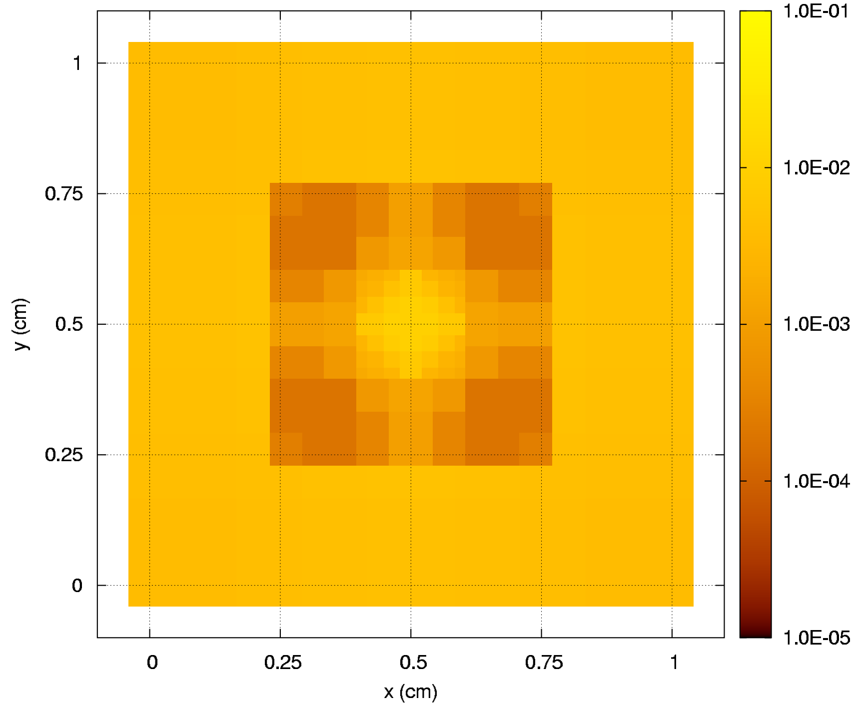


randomized (20%) grid

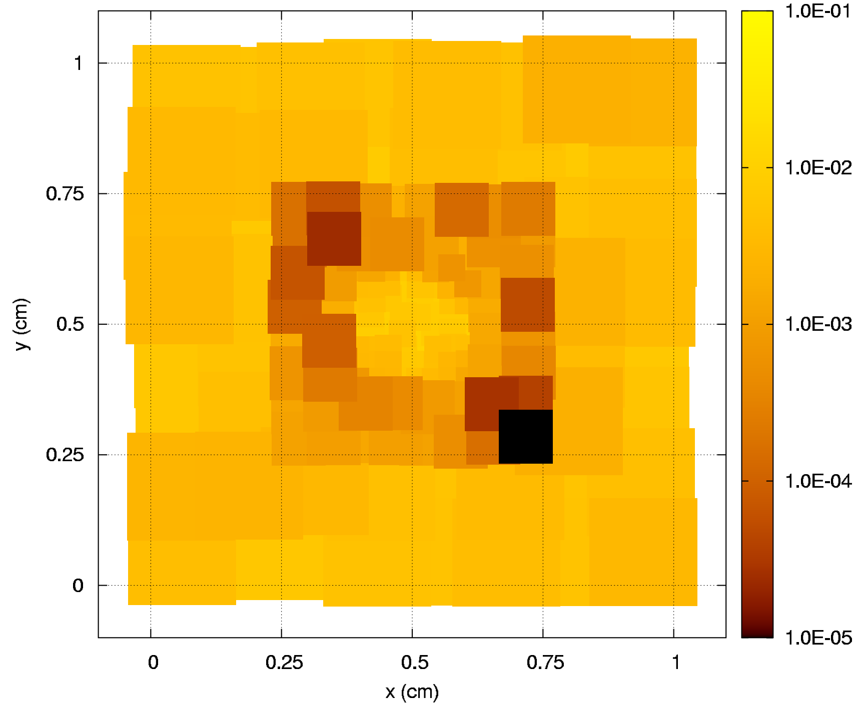
anistratov@ncsu.edu

Results on 3-Level Grids

- Relative error in the cell-average scalar flux
- Grid 2: 160 cells



orthogonal grid

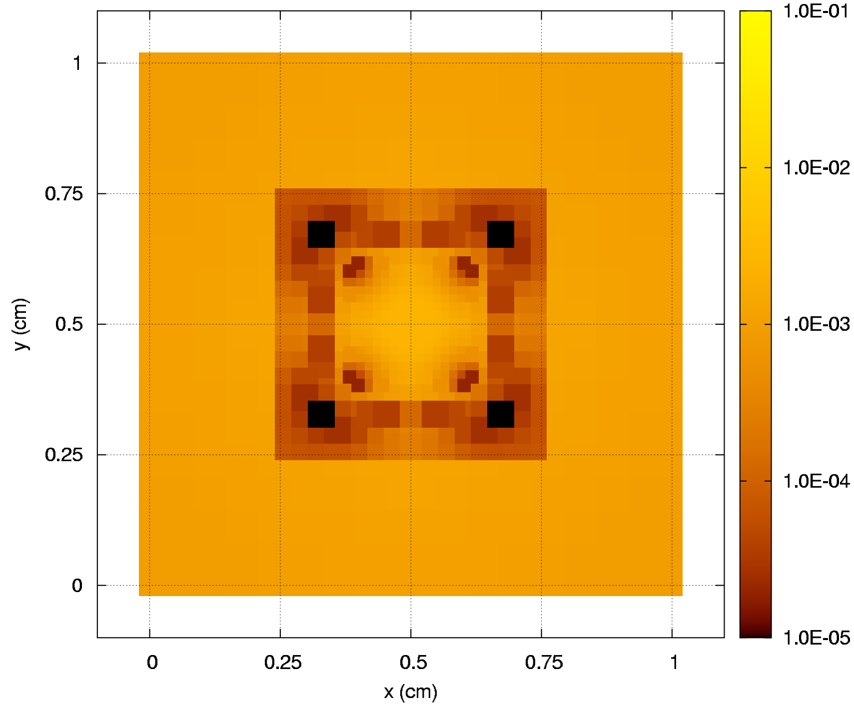


randomized (20%) grid

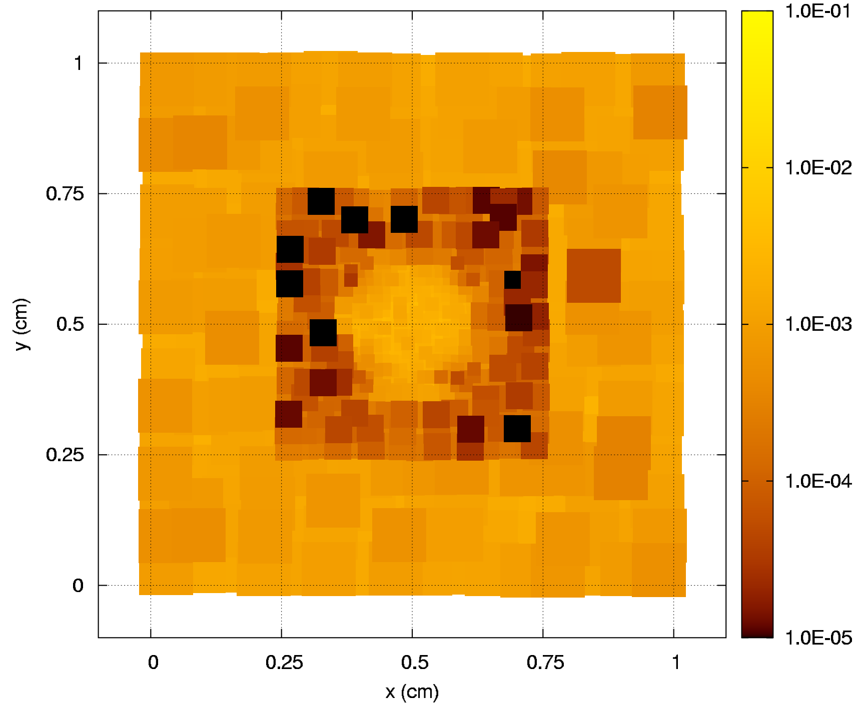
anistratov@ncsu.edu

Results on 3-Level Grids

- Relative error in the cell-average scalar flux
- Grid 3: 640 cells



orthogonal grid

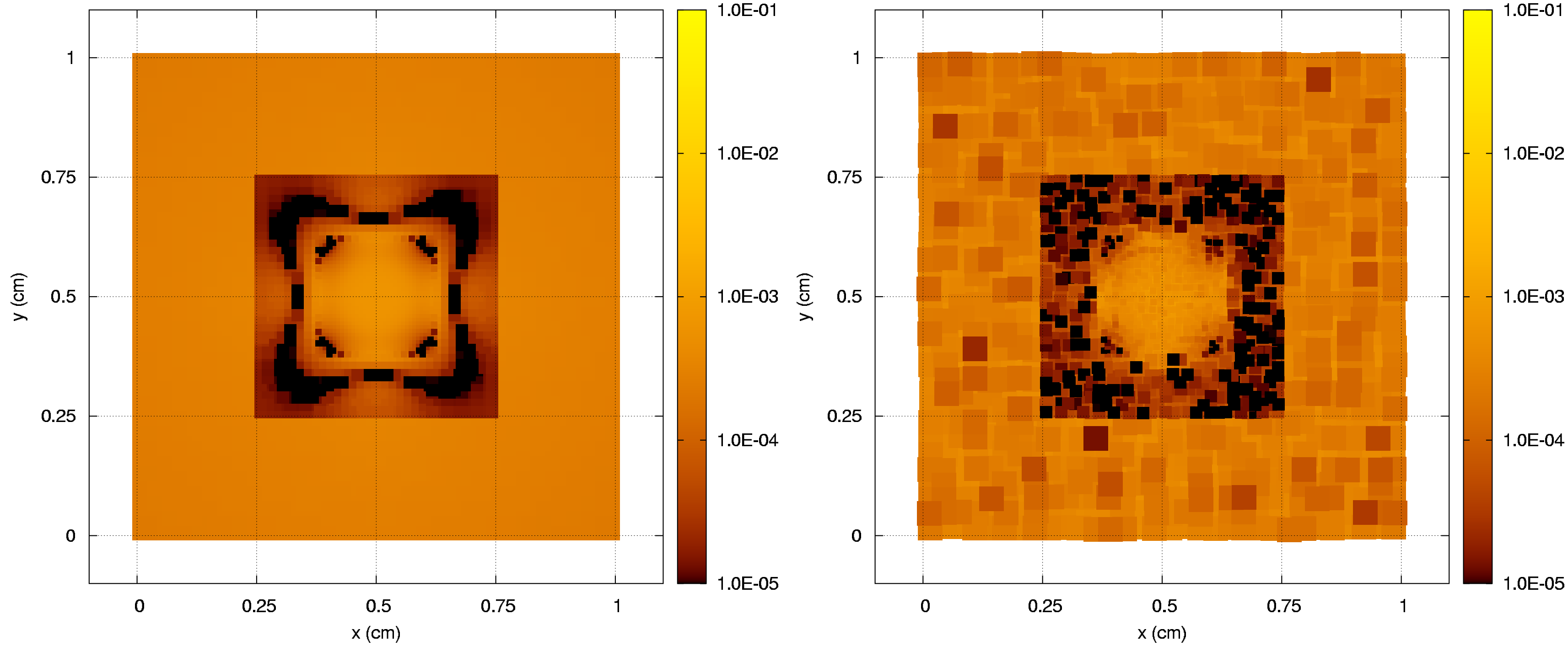


randomized (20%) grid

anistratov@ncsu.edu

Results on 3-Level Grids

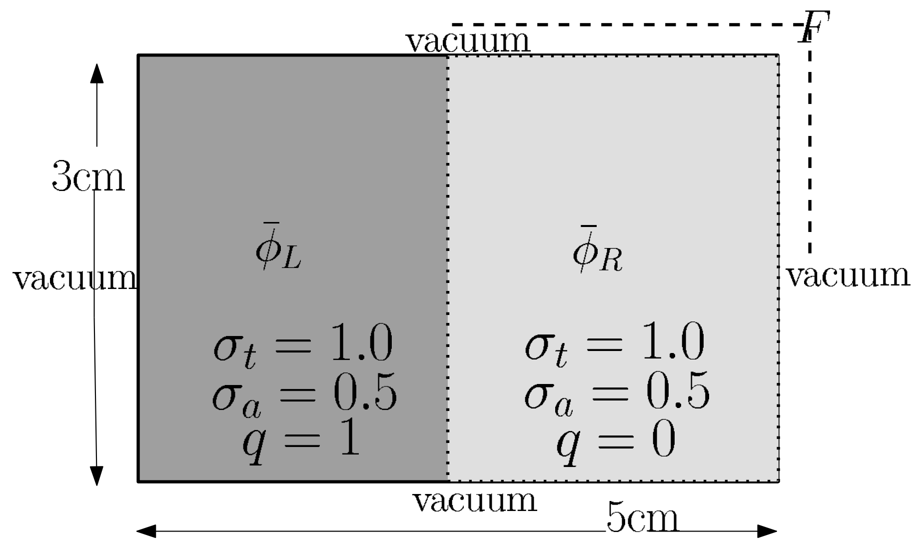
- Relative error in the cell-average scalar flux
- Grid 4: 2560 cells

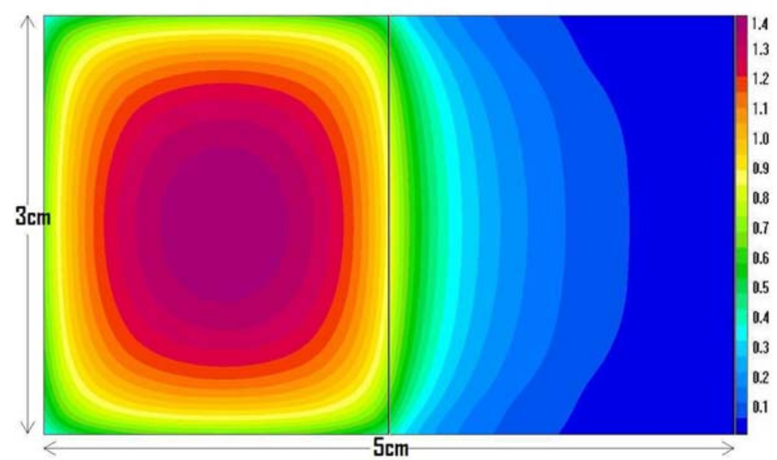


orthogonal grid | randomized (20%) grid

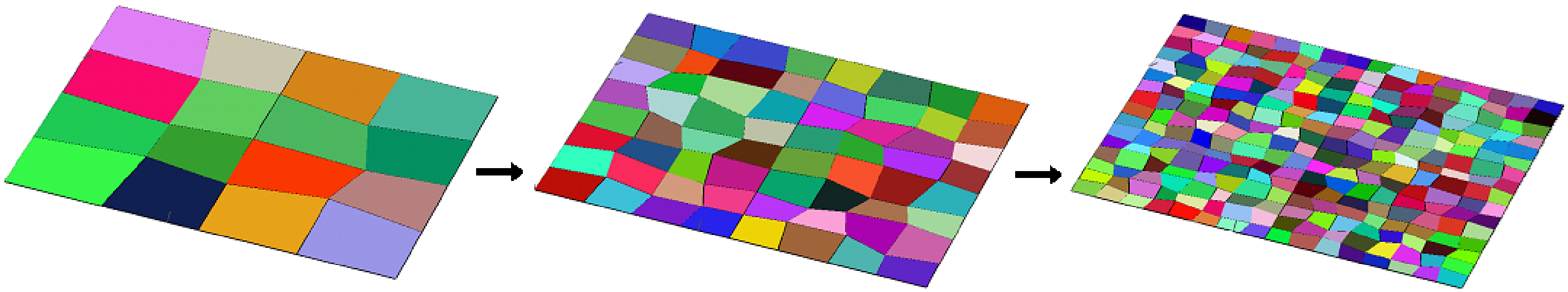
anistratov@ncsu.edu

Transport Test Problem





Numerical Results



Results of Single Level Grids

	Orthogonal Grid			Randomized Grid			Relative Difference		
	F	$\bar{\phi}_L$	$\bar{\phi}_R$	F	$\bar{\phi}_L$	$\bar{\phi}_R$	F	$\bar{\phi}_L$	$\bar{\phi}_R$
8 ×8	2.998e-2	1.1217	0.1467	2.872e-2	1.1263	0.1435	4.2e-2	-4.0e-3	2.2e-2
16 ×16	3.141e-2	1.1021	0.1564	3.124e-2	1.1045	0.1543	5.4e-3	-2.2e-3	1.4e-2
32 ×32	3.194e-2	1.0975	0.1589	3.205e-2	1.0977	0.1581	-3.4e-3	-2.4e-4	5.0e-3
64 ×64	3.209e-2	1.0965	0.1593	3.216e-2	1.0964	0.1592	-2.2e-3	1.2e-4	8.4e-4

Numerically Estimated Spatial Order of Convergence

	Orthogonal Grid			Randomized Grid		
	F	$\bar{\phi}_L$	$\bar{\phi}_R$	F	$\bar{\phi}_L$	$\bar{\phi}_R$
single-level	1.82	2.32	2.66	2.88	2.35	1.86
two-level (with refinement on the left)	1.50	1.95	1.94	2.07	1.53	1.42
two-level (with refinement on the right)	1.40	2.23	2.35	1.29	1.86	1.89

anistratov@ncsu.edu

Nonlinear Weighted Flux (NWF) Methods

- The transport (high-order) equation

$$\vec{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, \vec{\Omega}) + \sigma_t(\vec{r}) \psi(\vec{r}, \vec{\Omega}) = \frac{1}{4\pi} \sigma_s(\vec{r}) \phi(\vec{r}) + \frac{1}{4\pi} q(\vec{r}) \,.$$

- The factors

$$G_m(\vec{r}) = \Gamma_m \int_{\omega_m} w(\Omega_x, \Omega_y) \psi(\vec{r}, \vec{\Omega}) d\vec{\Omega} \Bigg/ \int_{\omega_m} \psi(\vec{r}, \vec{\Omega}) d\vec{\Omega} \,, \quad \Gamma_m = \frac{\int_{\omega_m} d\vec{\Omega}}{\int_{\omega_m} w(\Omega_x, \Omega_y) d\vec{\Omega}} \,.$$

$$F_m^{\alpha}(\vec{r}) = \Gamma_m \int_{\omega_m} |\Omega_\alpha| w(\Omega_x, \Omega_y) \psi(\vec{r}, \vec{\Omega}) d\vec{\Omega} \Bigg/ \int_{\omega_m} \psi(\vec{r}, \vec{\Omega}) d\vec{\Omega} \,, \qquad \alpha = x, y \,,$$

- The low-order NWF equations $\phi_m(\vec{r}) = \int_{\omega_m} \psi(\vec{r}, \vec{\Omega}) d\vec{\Omega}$, $m = 1, \ldots, 4$

$$\nu_m^x \frac{\partial}{\partial x} \big(F_m^x \phi_m \big) + \nu_m^y \frac{\partial}{\partial y} \big(F_m^y \phi_m \big) + \sigma_t G_m \phi_m = \frac{1}{4} (\sigma_s \phi + q) \,, \quad \phi = \sum_{m=1}^{4} \phi_m \,. \,,$$

$$\nu_1^x = \nu_1^y = 1 \,, \quad \nu_2^x = -1 \,, \quad \nu_2^y = 1 \,, \nu_3^x = \nu_3^y = -1 \,, \quad \nu_4^x = 1 \,, \quad \nu_4^y = -1 \,,$$

Asymptotic Diffusion Limit of the Transport Eqation

- We define a small parameter ε, scaled cross sections and source:

$$\sigma_t = \frac{\widehat{\sigma}_t}{\varepsilon}, \quad \sigma_a = \varepsilon \widehat{\sigma}_a , \quad q = \varepsilon \widehat{q} ,$$

- Consider that $\varepsilon \to 0$.

- We now assume that the solution can be expanded in power series of ε

$$\psi = \sum_{n=0}^{\infty} \varepsilon^n \psi^{[n]} ,$$

- The leading-order solutio of the transport the equation meets to the diffusion equation

$$-\frac{\partial}{\partial x}\frac{1}{3\sigma_t}\frac{\partial \psi^{[0]}}{\partial x} - \frac{\partial}{\partial y}\frac{1}{3\sigma_t}\frac{\partial \psi^{[0]}}{\partial y} + \sigma_a \psi^{[0]} = q \, .$$

Asymptotic Diffusion Limit of the Low-Order NWF Equations

- We consider various weights
  - $w(\Omega_x, \Omega_y) = 1$
  - $w(\Omega_x, \Omega_y) = |\Omega_x| + |\Omega_y|$
  - $w(\Omega_x, \Omega_y) = 1 + |\Omega_x| + |\Omega_y|$
  - $w(\Omega_x, \Omega_y) = 1 + \beta(|\Omega_x| + |\Omega_y|)$

Values of the diffusion coefficients (D) for specific NWF methods

Weight	1	$\|\Omega_x\|+\|\Omega_y\|$	$1+\|\Omega_x\|+\|\Omega_y\|$	$1+\beta(\|\Omega_x\|+\|\Omega_y\|)$
D	$\frac{1}{4\sigma_t}$	$\left(\frac{\pi+2}{3\pi}\right)^2 \frac{1}{\sigma_t} \approx \frac{1}{3.36\sigma_t}$	$\left(\frac{4+5\pi}{12\pi}\right)^2 \frac{1}{\sigma_t} \approx \frac{1}{3.66\sigma_t}$	$\frac{1}{3\sigma_t}$

Numerical Results

- Test problem
  - A unit square domain
  - $\sigma_t = \frac{1}{\epsilon}$, $\sigma_a = \epsilon$, and $q = \epsilon$
  - $\epsilon = 10^{-2}, 10^{-3}$, 10^{-4}, and 10^{-5}
  - Boundary conditions are vacuum.
  - A uniform spatial mesh consists of 19×19 equal cells.
  - The compatible product quadruple-range quadrature set uses 3 polar and 3 azimuthal angles per octant.

Relative errors of the cell-average scalar flux in the cell located at the center

ϵ	Weight			
	1	$\lvert\Omega_x\rvert+\lvert\Omega_y\rvert$	$1+\lvert\Omega_x\rvert+\lvert\Omega_y\rvert$	$1+\beta(\lvert\Omega_x\rvert+\lvert\Omega_y\rvert)$
10^{-2}	2.56E-1	1.01E-1	1.75E-1	7.13E-3
10^{-3}	2.68E-1	9.82E-2	1.79E-1	-2.18E-3
10^{-4}	2.69E-1	9.86E-2	1.80E-1	-1.99E-3
10^{-5}	2.69E-1	9.86E-2	1.80E-1	-1.97E-3

anistratov@ncsu.edu

Asymptotic Boundary Condition

- There exist a boundary layer with the width of order of ε.
- The scalar flux at the boundary of the diffusive domain

$$\phi^{[0]}(X,y) = 2\int_{\vec{n}\cdot\vec{\Omega}<0} W(|\vec{n}\cdot\vec{\Omega}|)\psi_{in}(X,y,\vec{\Omega})d\vec{\Omega}\,,$$

- It is possible to improve the performance of the NWF method by using

$$w(\Omega_x,\Omega_y) = 1+\lambda(|\Omega_x|+|\Omega_y|)+\kappa|\Omega_x\Omega_y|$$

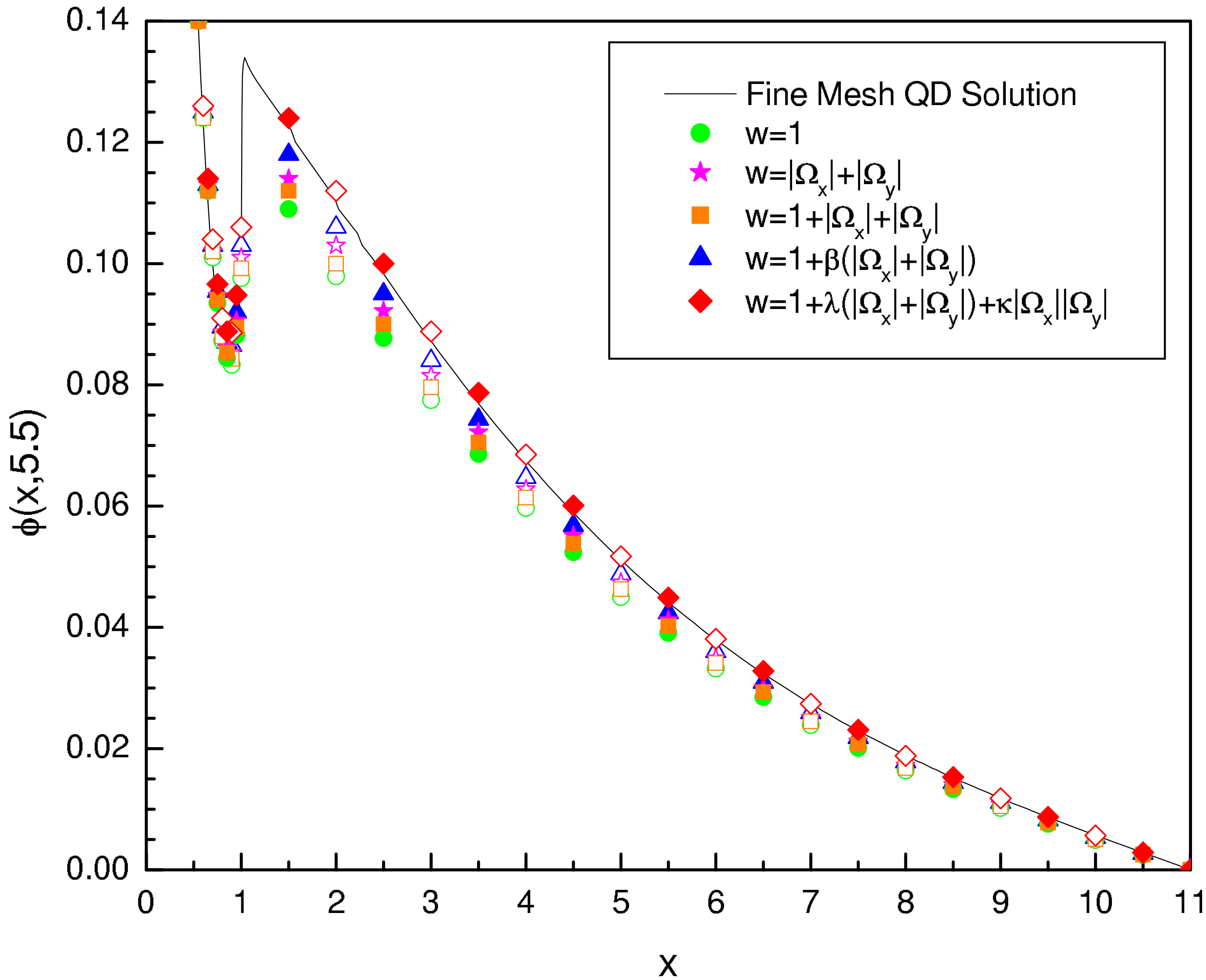
Relative errors of the asymptotic boundary conditions for the NWF methods

τ (mfp)	BLD Transport Method	Weight				
		1	$\|\Omega_x\|+\|\Omega_y\|$	$1+\|\Omega_x\|+\|\Omega_y\|$	w_β	$w_{\lambda,\kappa}$
0.0	0.033 %	0.033%	0.033%	0.033%	0.033%	-0.059%
0.2	-0.112%	-3.451%	-1.456%	-2.410%	-0.249%	-0.199%
1.0	-0.304%	-8.035%	-4.030%	-5.945%	-1.607%	0.697%
2.0	-0.423%	-10.871%	-6.557%	-8.620%	-3.948%	1.36%
4.0	-0.568%	-14.339%	-10.850%	-12.518%	-8.739%	1.25%
6.0	-0.641%	-16.089%	-13.520%	-14.748%	-11.97%	0.755%

Problem with an Unresolved Boundary Layer

- A domain $0 \leq x, y \leq 11$ with two subregions:
  1. $0 \leq x \leq 1$, $\sigma_t = 2$, $\sigma_s = 0$, $q = 0$, $\Delta x = 0.1$, and $\Delta y = 1$,
  2. $1 \leq x \leq 11$, $\sigma_t = \sigma_s = 100$, $q = 0$, and $\Delta x = \Delta y = 1$.
- There is an isotropic incoming angular flux with magnitude $\frac{1}{2\pi}$ on the left boundary
- Other boundary condition are vacuum.
- This problem enables one to test the ability of a method to reproduce an accurate diffusion solution in the interior of the diffusive region with the spatially unresolved boundary layer.

Problem with an Unresolved Boundary Layer



anistratov@ncsu.edu

Conclusions

- We have developed a new method for solving the LOQD equations on arbitrary quadrilateral and AMR-like grids
  - The resulting QD method for solving transport problems demonstrated good performance on both randomized and AMR-like grids.
  - Observe second order spatial convergence in numerical tests
- We defined a new parameterized family of nonlinear flux methods for solving the 2D transport equation.
  - The asymptotic diffusion analysis enabled us to find a particular method of this family the solution of which satisfies a good approximation of the diffusion equation in diffusive regions.
  - As a result, we developed a 2D flux method with an important feature for solving transport problems with optically thick regions.
  - Note that it is possible to formulate a linear version of the proposed methods.

anistratov@ncsu.edu